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James Boorensdin

1805 Waterford

A

# Complete Treatiſe

ON

# ARITHMETIC,

## RATIONAL AND PRACTICAL:

*WHEREIN THE PROPERTIES OF NUMBERS ARE CLEARLY POINTED OUT;*

THE THEORY AND PRACTICE OF THE SCIENCE ARE DEDUCED FROM FIRST PRINCIPLES,

AND DEMONSTRATED IN A FAMILIAR MANNER;

WITH A GREAT VARIETY OF

PROPER EXAMPLES IN ALL THE RULES,

PERFECTLY SUITED TO

THE MAN OF BUSINESS, ACADEMIES, SCHOOLS, AND STUDENTS OF EVERY DENOMINATION,

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VOL. I.

BY P. DEIGHAN, PHILOMATH.

DUBLIN:

*PRINTED AND SOLD FOR THE AUTHOR,*

BY

J. JONES, 90, BRIDE-STREET.

1804.

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1808

# RECOMMENDATORY LETTERS.

## *To Mr. Deighan,*
## AUTHOR of the NEW TREATISE on ARITHMETIC.

" *Multi Præceptores ab hoc libro erudientur:*"

SIR,

I have, at your requeſt, looked over every Rule and Queſtion in your New Treatiſe on Arithmetic; and ſo far as my knowledge of that ſubject directs me to judge of real excellence, I think that your *modus of ſhort calculation* is ſuch as ought to be adopted in Schools and Counting-houſes, and that the publication thereof will be of general utility. You have manifeſtly far excelled Dowling, Gough, and Voſter in the extent and brevity of your Mercantile operations: much labour will be ſaved both to maſter and ſcholar; and the *man of buſineſs* who is acquainted with your rules and conciſe methods, will go through his various calculations in much leſs time, and with eaſe and delight: and indeed on viewing your Appendix on Algebra, which exceeds the other part, if poſſible, in value; it is truly *multum in parvo*, it is the ground work for the beſt ſyſtem of Algebra I ever ſaw. I feel intereſted in the ſucceſs of your book as a public benefit, permit me therefore, to recommend to you that when you get it printed, to be very careful in correcting the Preſs, as moſt of our ſchool-books, particularly Gough and Voſter, are now ſo ſhamefully incorrect, that both maſter and ſcholar are frequently more embarraſſed than aſſiſted by them. I am ſir, with wiſhing you ſucceſs, and every one who contributes his mite in the improvement of uſeful learning.

Your obedient ſervant,

Dublin, April, 18, 1803. **BENJ. WORKMAN, M.A.**

---

SIR,

I have peruſed your New Treatiſe on Arithmetic, or the Merchants' Tutors' and Pupils' guide, and think that it excels any thing of the kind that has yet been offered to the public.

I am ſir, yours,

16, Dorſet-ſtreet, Dublin, } WILLIAM JEMISON,
19th May, 1803. } *Public Accountant.*

---

SIR,

I have, at your deſire, peruſed your Arithmetic, and think that it will be a very uſeful ſchool book, calculated at once to ſave trouble to the teacher, and to give much uſeful information to the ſcholar.

Your humble ſervant,

16th July, 1803, STEPHEN JONES.
*Principal of the Academy, Aungier-ſtreet.*

---

SIR,

At your requeſt I have examined your Treatiſe on Arithmetic, which I very much approve of: your Rules are, I think, judiciouſly choſen; and the new and expeditious methods you have employed in your calculations are ſuch as will ſave much time and trouble both to maſter and ſcholar.

I am ſir, your obedient humble ſervant,

July 16, 1803. T. DALY.
*Principal of the Academy, South Great George's-ſtreet.*

Sir,

I have, at your requeſt, minutely examined your very excellent Treatiſe on Arithmetic; and, as a teacher of upwards of twenty years' experience, think it by far excels Gough, Voſter, Dowling, or any other of the kind that ever came under my inſpection, for brevity of ſyſtem in all its ſtages.

I am ſir, your obedient humble ſervant,

35, Paradiſe-row, Dublin, } WM. SHEILL,
July 18, 1083. } *Profeſſor of Geography, Accounts, &c.*

P. S. Pleaſe to ſend me 6 copies when printed.

---

Sir,

Agreeably to your deſire, I have read your Treatiſe on Arithmetic: I think it calculated to expedite buſineſs, to remove the difficulties attending on the rudiments of every ſcience, and to leſſen the trouble of the teacher.

I am, ſir, your humble ſervant,

July 17, 1803. A. NEWLAND,
*Principal of the Academy, Peter-ſtreet.*

---

Sir,

I have, at your requeſt, attentively examined your new Treatiſe on Arithmetic, and muſt beg leave to recommend it as a moſt excellent performance, and well deſerving the patronage of all candid admirers of uſeful and ſcientific learning. Your's, &c

MARK MORTON.
July 19, 1803. *Calculator and Compiler of the principal Almanacs of Dublin, and Maſter of the Mercantile and Mathematical Academy, 31, Lower Ormond-quay, Dublin.*

P. S. As you intend to print your book by ſubſcription, pleaſe to put my name down for 6 copies, as it is a more complete book than any extant on the ſubject.

---

Sir,

I have, at your requeſt, peruſed your manuſcript on Arithmetic, in theory and practice; I do recommend it as being far ſuperior to any work on the ſubject I have hitherto ſeen, which will be moſt uſeful to a learner, and conſequently of the greateſt aſſiſtance to a teacher, which both demonſtrates the rules in each ſection and will be of moſt general utility to ſchools.

Sir, I remain your friend, wiſhing you, for the ſake of the growing generation, ſucceſs, LAWRENCE LEECH,
July 21, 1803. *Principal of the Academy, Thomas ſtreet.*

---

Sir,

I have examined the manuſcript of your intended Treatiſe on Arithmetic, and am fully of opinion that it will be the moſt uſeful publication in that way which has yet appeared; being, as far as I can judge, a conſiderable improvement upon every thing of the kind which I have hitherto ſeen; I think you therefore greatly merit the countenance and ſupport of the public. I am Sir,

Dawſon-ſtreet, } Your moſt obedient humble ſervant,
July 23, 1803. } JOHN SHARMAN.
*Profeſſor of Geography.*

Sir,

According to your deſire, I have carefully peruſed your manuſcript on Arithmetic, and think it ſuperior to any book on the ſubject that ever came in my way, as your methods are more brief, and your choice of matter more ample than any of thoſe generally uſed in ſchools.

I remain ſir, your humble ſervant,
Auguſt 4, 1803. JAMES SALLY, *Mathematician.*
*Principal of the Academy, Meath-ſtreet.*

---

Sir,

I have peruſed your new Treatiſe on Arithmetic, and think it the moſt uſeful performance for the Learner, the Teacher, or the Man of Buſineſs, of any I have ſeen on that ſubject. I am ſir,
Marine School, Dublin. } Your's, &c.
Auguſt 12, 1803. } WM. M'MENAMY.
*Principal of the Marine School.*

---

Sir,

Having peruſed your new Treatiſe on Arithmetic, I think it the moſt perfect, elementary and learned ſyſtem on that uſeful ſcience, of any I have ſeen.

Your dividing it into ſuch a proper choice of chapters, your judicious arrangement of the rules, the perſpicuous manner in which you treat the various methods and operations, will lead the ſtudent, by the moſt eaſy, ſure gradations, into the knowledge of its moſt perplexed and intricate parts; and all in terms ſo plain and intelligible, as to be within the reach of the moſt common underſtandings.

I alſo think ſuch a uſeful treatiſe is much wanted by the public, therefore merits their countenance. Wiſhing ſucceſs, ſir,
Dublin, } I am your moſt humble ſervant,
Auguſt 18, 1803. } JOHN M'GAURAN, *Philomath.*

---

Sir,

I have, at your requeſt, examined your new Treatiſe on Arithmetic, and believe it to be highly deſerving of public ſupport.

Notwithſtanding the many books already extant on that ſubject, I am perſuaded that the publicare in much want of a work ſuch as yours.

I am ſir, with beſt wiſhes for your ſucceſs,
18, French-ſtreet, Dublin, } Your very humble ſervant,
Auguſt 24, 1803. } SIMON WHITE, *Mathematician.*

---

Sir,

I have at your requeſt, attentively peruſed your manuſcript Treatiſe on Arithmethic, and think it the moſt excellent on the ſubject that I have ſeen. From the judicious arrangement of the plan, I believe it to be exceedingly well calculated to ſave the teacher trouble, and to facilitate the progreſs of the ſcholar. Wiſhing you ſucceſs in your undertaking,
I remain your obedient ſervant,
September 10, 1803. TIMOTHY DILLON,
*Teacher of Mathematics, Angleſea-ſtreet.*

Mr. Deighan,

I ſaw your very ingenious and meritorious Treatiſe on Arithmetic; I believe its principal excellence is not exceeded by any that I have ſeen before. I am Sir, your very humble Servant,

Sept. 25, 1803. J. SCULLY.

*Profeſſor of Geography, Accounts, &c. Capel-ſtreet.*

---

Sir,

I have peruſed your manuſcript with as much attention as the ſmall portion of leiſure I enjoy, would admit. Your performance, as it appears to me, has great merit, and, together with uncommon application and care, ſhews you are poſſeſſed of a happy talent for figures. The inſtructive part is laid down with judgment and preciſion. The examples, by way of illuſtration are perfectly applicable and ſufficient, and the Queſtions for Exerciſe judiciouſly ſelected. Upon the whole, the work, in its original form, does you credit. If you can get the printer to do his part with ſuitable correctneſs; I am perſuaded, notwithſtanding the numerous Treatiſes on Arithmetic, already in the hands of the public, your's will be conſidered as a valuable addition to the catalogue of ſchool-books by the teachers in that line. I wiſh you, moſt heartily, ſucceſs, and am ſir,

Your obedient ſervant, SAM. WHYTE,

Oct. 15, 1803. *Principal of the Academy, Grafton-ſtreet.*

---

What Poet could refuſe his lays,
To ſo ſublime a theme;
Here every line deſerves our praiſe,
Not one deſerves our blame.

I muſt confeſs I took up Mr. Deighan's manuſcript with ſome degree of reluctance, expecting (of which I have ſeen many ſad inſtances) the common hackneyed rules of Arithmetic repeated—I have been *agreeably diſappointed*—to much new matter and an entire new method, there is an elegant arrangement, remarkable for brevity with perſpicuity: but I would be happy to impreſs on gentlemen engaged in the mechanical line, that they will meet many things new and uſeful in their profeſſional avocations. That the public may receive benefit from the book, and the author his reward from the public, is the ſincere wiſh of his friend,

Clarence-ſtreet, Nov. 1, 1803. JAMES CAMPBELL.

TO

### TO

## Mr. DEIGHAN,

### ON HIS NEW TREATISE ON ARITHMETIC, &c.

'MONGST thoſe great men who ſhone in ancient days,
And ſages wiſe, who in modern times,
Sow the rich ſeeds of erudition bright,
Teaching the ways of wiſdom to mankind,
Ere then unknown; by none reveal'd,
'Till ſcience fair her bright effulgence ſhed,
And the dark cloud of ignorance diſpell'd.
As when in former times, the argent moon,
The dark, impervious, opaque ſhadow caught
Of this our EARTH, arcanum long unknown,
'Till wiſe Mileſius to his king * foretold
The wondrous ſecret—ſo it is with thee;
The recent beauties darkneſs had obſcur'd,
Thy penetrating judgment has illum'd.
With theſe the Muſe, O DEIGHAN numbers thee,
Tho' *laſt* not *leaſt* of the enlighten'd claſs
Of learned men: the boaſt of Greece and Rome
And Athens: ſeats of ſcience and of ſenſe:
Nor let fair Egypt, nor old Chaldea's ſon,
Nor fam'd Pythagoras, the Samian ſage,
Arthimedes, Eudoxus, Plato, grave,
Of ancient Academic Grove, renown'd,
Refuſe with thee to ſhare the envy'd crown
Of ever-blooming bays in many twining wreaths—
The juſt reward to genuine merit due.
Now let the vivid youth purſue with ſpeed,
The trackleſs path of thy tranſcendant pen,
And roll with ſplendour in commercial wealth,
And all the pleaſures of his *gain* enjoy:
Or elſe to ſoar upon ſeraphic wing,
Thro' Heaven's high arch concave, to tread
*Newtonian* paths, with microſcopic eye;
*Flamſtedian* wonders to explore; or elſe
With thee to plow the briny wave propenſe,
And ſteer th' erratic barque through midnight gloom,
Of Conſtellation's friendly aid bereft:
Or tack the hoſtile fleet in battle's heat;
Or from Arabia's aromatic coaſt,
With ſwelling ſails, Ambroſial treaſures bring.—
Purſue the taſk thy genius has begun,
Nor be deterr'd by Cynic's taunting ſneer,
Or ſnarling Critic's envious gall-tongu'd ſpleen—
"You're made of ſteel which they can never bite."

* *Cyrus.*

TERENCE DWYER.

*Claſſical and Commercial School, No. 5,*
*Upper Exchange-ſtreet, April 23, 1803.*

LINES

# LINES
## Occaſioned by reading Mr. Deighan's MS.

*" Adæquat memoriam & nomen ejus cum omni poſteritate."*

O HAPPY volume! eminently great
With ev'ry vari'd excellence replete;
Fair fruit of ſkill matur'd and toil intenſe,
Enrich'd by ſcience and improv'd by ſenſe:
As I thy curious pleaſing page purſue,
New beauties crowd on my aſtoniſh'd view;
The muſe, amaz'd, enquires who could contrive
To make one digit do the work of five,
Lo here a more ſurprizing wonder's ſeen,
One figure does the duty of fifteen!
As I admire each propoſition fair,
The pronic number and the perfect ſquare,
The puzzling intricate equation ſolv'd,
As Grecia's chief the Gordian knot diſſolv'd;
Loſt in amazement, I exclaim "'Tis he
Fair Alexandria's ſubtle ſage I ſee
Adorn'd with bays and dignify'd by truth
In all the bloom of renovated youth."
How many ſophs, to ſenſe and ſcience blind,
Range through the realms of nonſenſe unconfin'd,
Unaw'd by ſhame and unreſtrain'd by law,
Their labour chaff, and their reward a ſtraw;
Neglected and deſpis'd, they ſink in ſhame
To that oblivion whence, unſought, they came.
The muſe, indignant, oft with grief has ſeen
An author led by ignorance and ſpleen,
With ſnail-pac'd ſpeed, but unremitting toil,
In attic chamber waſte the midnight oil,
With waſte of paper, loſs of ink combin'd,
And pens from public offices purloin'd.—
'Tis done! and the Herculean taſk is o'er,
The pupil is no wiſer than before.
But Deighan of a more enlighten'd mind,
More innate genius, talents more refin'd,
More ſkill by aſſiduity increas'd,
A ſounder judgment and a juſter taſte,
To him a more exalted taſk is due,
To teach the pupil and *the maſter too.*
Proceed my friend, nor dread the cynic's ſneer,
Can worth like your's from *ſuch* have cauſe to fear;
And as the paths of ſcience you purſue
This little maxim always keep in view,
Tho' ſpite and envy all their arts exert,
FAME FROM TRUE MERIT NEVER WILL DEPART.

*Drumcondra*, 17 *May*, 1804. JOHN BARTLEY.

TO

ALL THOSE

**Who think that a Knowledge of**

**Accounts,**

*IS USEFUL TO MANKIND,*

FROM

THE KING ON THE THRONE,

TO THE

*LOWEST SUBJECT,*

THIS

*New Treatise on Arithmetic*

**Is most humbly**

DEDICATED,

BY

Their Most Obedient and

HUMBLE SERVANT,

*The Author,*

P. DEIGHAN.

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——— Kelly, Eſq. do.
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# PREFACE.

THE Author conceives it not only unneceſſary, but abſurd in the extreme, to offer any apology for, the preſent attempt of adding his mite of improvement to the general ſtock of Arithmetical knowledge. Arithmetic is, as it were, the *Primum Mobile* of Mathematical ſciences: it is the *Initium* to the whole; its operations are of the laſt importance to to each; its calculations are the moſt uſeful to *man;* in fact, its concluſions are the ultimate object of ſcientific inquiries, when mathematical certainty and numeral deductions are ſought for.

We are told by Joſephus, that, when by the command of God, Abraham went forth from Chaldea, through Paleſtine, into Egypt, he carried Arithmetic with him; and finding the Egyptians capable of improvement, he there employed himſelf in the arduous taſk of an inſtructor, and firſt taught Arithmetic and Aſtronomy to that ancient people; and hence the antiquity of this moſt excellent art can ſcarce be diſputed.

But this noble branch wants not the aſſiſtance of diſtant authority or ancient hiſtory to recommend it; it needs not the aid of remote ages called in to point out its value, nor the help of refined modern philoſophy to give it more *luſtre;* nay, the univerſal conſent of mankind, at this preſent moment, is of itſelf ſimply ſufficient to declare, that *no man* ever employed his time in the cultivation of a ſcience more generally uſeful than that of Arithmetic.

By Arithmetic, the Geometer calculates the ratios and and ſeveral dimenſions of his different magnitudes and figures; the Aſtronomer determines the preciſe times of eclipſes, and all the other celeſtial phenomena; the Navigator diſcovers his exact place in the extended ocean, and his courſe and diſtance to his wiſhed-for port; the ſurveyor makes out the quantity of land in his ſurrounded tract; the merchant regulates his various profits, exchanges, &c.; the avaricious uſurer ſettles his rates per cent; and the uſeful and indigent mechanic and labourer come at the balance of the ſmall pittance ſtill remaining due: in ſhort, Arithmetic is ſo extenſive in its operations and important in its ſeveral calculations, that the perſon who has a competent knowledge therein is, by common conſent, allowed to be a *good ſcholar*, however ſlender his other literary acquirements may be.

The author of this eſſay by no means wiſhes to depreciate the labours of other writers on this ſubject; many have gone far and deſerve well of the public; all his ambition is, to think that he has made ſome important improvements to this valuable art—an art that can never be exhauſted! He does not pretend that he has brought the ſcience to its *ne plus ultra* of perfection; but he hopes it will be found, that he has done much that way; and that his fellow-labourers who may come after him will diſcover that the greater part of the rubbiſh is removed, and that the path is now rendered ſtraight and tolerably level.

P. D.

# TO THE READER.

AS books of this kind contain ſeveral articles, of which there is no abſolute neceſſity that every reader ſhould make himſelf maſter, and as theſe works may fall into the hands of thoſe who have neither proper opportunity, nor proper aſſiſtance; the whole indiſcriminately is intended to aſſiſt the young ſtudent in the ground-work of Arithmetic, as well as the practice. We have therefore carefully ſtudied what directions could be given in a matter of ſuch importance, and now we truſt, that the precepts and examples ſuit the object, viz. the import and matter of the improvement of the pupil, and our own character depending thereon. We truſt that in the following ſheets ſuch a ſyſtem of ſcience and practice ſhall be found, that all our readers and pupils never met with any thing more ſuitable to their avocations and wants. We do not wiſh to depreciate the excellent Arithmeticians who have gone before us, particularly Malcolm and Emerſon, who are the two modern authors of real abilities, that have kept cloſe to a proper ſyſtematic plan and manner of writing; but then they are by no means calculated in other reſpects for common uſe: Malcolm, propoſing to give a complete view of every particular of the ſcience, enters ſo deeply into it, and makes ſo much uſe of Algebra, that it requires a tolerable ſkill in that branch to be able to read him; the principles of which he alſo intermixes with thoſe of Arithmetic, as a part of his plan.

In Emerſon, is attempted a brevity in the method, which we think has rendered his book not properly fit for a mere *novice;* the rules are very few of them demonſtrated, nor is it ſeemingly in other reſpects well calculated for an aſſiſtant to teachers, and the uſe of ſchools. However, in juſtice to theſe authors, it muſt be obſerved of the firſt, that his book will be a laſting monument of his learning and induſtry, and that the ingenious theory of numbers, given us in the other's work, will intitle it to the peruſal of all lovers of Arithmetic.

We do not intend to diſcourage the numerous and excellent accountants who have received their inſtructions from theſe excellent authors (or from any other writer on Arithmetic); but we hope in the following treatiſe, that we ſhall lay down ſuch rules and principles and *ſhort methods*, that every *ſchool-boy*, and thoſe of the moſt dull apprehenſions, ſhall receive that benefit and inſtruction, which every book of Arithmetic ſhould afford.—What follows for the aſſiſtance of the young Arithmetician in the courſe of his ſtudies together with the explanation of ſome mathematical terms, he may ſometimes meet with in his progreſs, even in the ſecond volume of this work, is our intention here.—After our young friend has made himſelf maſter of Numeration and its dependent principles, we then recommend to his moſt ſerious peruſal, the definitions in the firſt part of the work; theſe we aſſert, if properly attended to, will ſerve him in the whole courſe of his ſtudies; many excellent accountants have been wandering in the dark, merely through want of ſuch aſſiſtance. If the pupil's time and genius afford the opportunity of his reading our demonſtrations, ſo much the more will he profit; but if theſe favourable circumſtances occur not, then let him carefully attend to the tables, and we aſſert roundly, ſo complete a ſet of tables of coins, weights and meaſures, &c. were never before publiſhed. A perfect knowledge of theſe tables will ſerve every ſtudent, be his intended profeſſion what it may. In the courſe of the work, many contractions of great importance in the reſpective rules are to be met with; the pupil may therefore, according to his ability and taſte, attend to them in the firſt going over, or in a ſecond peruſal; but we moſt ſeriouſly recommend that it be his *ultimatum*, not to be deficient therein. Surely, ſhort methods in accounts, give the pre-eminence; and, indiſputably, the whole ſyſtem contains more thereof than ever were before publiſhed together. The firſt volume, we apprehend, has abundance of matter for the beginner; but the complete Arithmetician will not ſtop ſhort there, until he has gone through the ſecond, and made himſelf acquainted with Decimal Arithmetic, and the finiſhing rules for mercantile profeſſions; then the theory of proportions, we expect, will cloſely engage his conſideration, after which he may apply himſelf to the extraction of the Square and Cube Roots, and of powers and roots in general, with their applications. If the ſtudent's time and abilities authoriſe, we adviſe that he ſhould read our Appendix of Algebra, there he will find more important matter, than probably ever was comprized in the ſame compaſs. We have attentively read Simpſon, Emerſon, Ward, &c. on that ſubject, and we hope that we have extracted the moſt uſeful parts, and added our own improvements thereon; we do not aſſert that we have given a finiſhed and extenſive ſyſtem of Algebra; but certain it is, that any one after reading this work with due attention, may encounter the greateſt difficulty in that ſcience with courage and certainty of ſucceſs.

In the firſt volume there will be found a ſyſtem of practical rules, better ſuited to buſineſs than ever before were publiſhed; theſe are under the head Compendiums, after Compound Diviſion. Merchants, Traders, Bankers, &c. will find many uſeful hints and methods contained therein, well worthy of their peruſal; and theſe methods when fully underſtood by the pupil, will qualify him to paſs through the general *routine* of common buſineſs, with that adroitneſs and facility, which every good accountant ſhould make his principal object.

From theſe general obſervations on the work, we apprehend, that our readers will be better qualified to go through the ſame than if they were left entirely to their own judgment. True, indeed, the work ſpeaks for itſelf; but ſtill we think that there is an abſolute neceſſity in pointing out a few of the numerous improvements and original excellencies in which we think our ſyſtem exceeds others; in order to direct the attention to thoſe beauties ſo neceſſary in theory and practical affairs.

We hope that the judicious *preceptor* who has his pupil's intereſt at heart, will carefully direct him to ſtudy theſe ſhort rules and methods, with the moſt ſerious attention. Theſe, he muſt ſee, will aſſiſt him in his moſt ordinary concerns, and give him that taſte for proficiency in numbers, which nothing but ſuch *brevity* of ſyſtem can produce.

Teachers, we hope, will find this work of the laſt importance to themſelves and ſcholars; the multitude of examples ſuited to all ranks and profeſſions, with the ſhorteſt methods poſſible of ſolution, and the whole deduced from proper precepts, will leave nothing wanting for their uſe and information. The author does not aſſert that all his queſtions are original, but he hopes that the ſelection of thoſe which he has borrowed from Clare and others, is of the beſt quality.

In reſpect to his own, he leaves the reader to judge of their worth and utility; this, however, he believes, that if they are not in the beſt dreſs poſſible; they are more completely adapted to buſineſs, than any other to be met with.

Before we diſmiſs this ſubject it may not be improper to obſerve, that ſtudents of the Univerſity may find ſeveral matters in the notes worthy of their moſt careful peruſal. Truly, the theory of numbers, is there pretty fully exhibited; and if to a complete knowledge thereof, they ſubjoin a perfect acquaintance with our Appendix of Algebra, perhaps no ſubject in their Mathematical courſe, will be found too difficult.

R. S. If our Errata appear large, we think that the reader has the advantage thereof: and it is the ſame to him whether the true anſwer is found at the end of the queſtion, or in the errata. Every poſſible care will be taken in a ſubſequent edition, to expunge errors; at the ſame time, it is hoped, that every one in poſſeſſion of the firſt edition, will correct the errata with the pen, for a mutual advantage, if others appear; every one will be attended to and thankfully received by the Author.

# PROEM.

THE education of youth is conſidered, in all civilized countries, as a ſubject of the higheſt importance, not only to individuals and families, but to the common-wealth at large. Accordingly we find the moſt eminent men in all ages engaged therein. To enumerate all the Sages, Philoſophers, Heroes, and even Princes, who have ſubmitted to the arduous taſk, would be vain; but in vindication of that honourable profeſſion of a *Preceptor*, we ſhall notice a few. Joſephus ſays, that Adam carefully inſtructed the poſterity of Seth, and as a faithful prophet and ſervant of the Lord, ſhewed them, that in conſequence of his fall, this world ſhould undergo a twofold deſtruction; the one by a deluge and the other by fire.

The deſcendants of Seth dwelling in Chaldea, where, by the clearneſs of the air, they had an opportunity of obſerving the various motions and changes of the celeſtial bodies; aſtronomy and its concomitant branches of Arithmetic and Geometry, began to be ſtudied. Abraham, the father of the faithful, being initiated into thoſe myſteries, removed from thence (by divine direction) into Egypt, and there followed the profeſſion of a teacher: as Joſephus further teſtifies.

Theſe ſciences paſſing over the Levant were brought into Greece: accordingly we find that Thales the Mileſian, who taught about 584 years before Chriſt, was the firſt among the Greeks who foretold an eclipſe of the moon to king Cyrus; and out of his ſeminary many great ſcholars aroſe, as Ariſtotle and Hippias write.

Pythagoras is the next among the Greeks of whom mention is made, as keeping an academy. He firſt of all abſtracted Geometry from matter, and applied that ſcience to the intellect; and puſhed Arithmetic to a perfection unknown before. The greater part of his reaſoning was taken from the property of numbers and magnitude. In thoſe times no one was reckoned a good citizen, or an honourable man, among the Greeks, who was unacquainted with Arithmetic and Geometry.

Theodorus, the maſter of Plato, is not to be forgotten: in his academy, many great men were educated; their writings, except ſome of Plato's, are now loſt; but his pupil Plato honourably acknowledged, that to him he owed much, and that the foundation of all he knew, was laid in the firſt inſtructions he received from his excellent preceptor. He frequently and candidly owned, that Theodorus ought to have the honour and not himſelf. "If," ſays he, "the "public are in anywiſe benefited by my acquirements, it is "not me, but my maſter Theodorus to whom they are "obligated; he began the work and ſketched out the uſeful "plan, and I have endeavoured to finiſh it." Plato was an excellent teacher, and a man of dignified humanity and morals; and the Greeks were ſo conſcious of his merits, that they wiſely encouraged him: and had not luxury and corruption preponderated at laſt among them, learning and liberty would no doubt have remained in full vigour in Greece, to this day.

The inſcription over Plato's academy is well known to all men of ſcience, viz. "Let no one ignorant of Geometry enter here."

Archytas, one of his ſcholars, was a moſt profound teacher; he firſt brought down the ſciences to human uſes: he was alſo a general, and in five battles which were fought againſt the enemies of his country he commanded and came off victorious. Thus, a ſchoolmaſter in Greece, was not only an humble teacher, but a hero and a ſaviour of his country.

Xenocrates, another of Plato's pupils, was the preceptor of Ariſtotle, whoſe name, as being the tutor of Alexander the Great, ſufficiently evinces him to have been a man whom poſterity muſt revere. What remains of Ariſtotle's writings, clearly ſhews that he was a lover of learning and a man of great wiſdom: it is an acknowledged truth, that, had his pupil, Alexander, been directed by his ſapient inſtructions, the dreadful diſmemberment and extinction of the conquered eaſtern empire would not have followed his death ſo immediately.

But why attempt to mention all the great men who have flouriſhed among the ancients, in the department of the inſtructors of youth? It would fill a large volume: for there is ſcarce a ſage, philoſopher, &c. whoſe name has been handed down to poſterity, as deſerving well of the common wealth of letters, whom we do not find to have been a teacher. Euclid, Archimedes, Diophantus, &c. &c. were profeſſors, and taught public ſchools; and even Dionyſius, king of Sicily, when baniſhed from his throne, became an humble ſchoolmaſter in Greece.

Dionyſius the *leſs*, a monk, in the 6th century, who ſettled the Chriſtian Æra or Epoch, was a ſchoolmaſter; and notwithſtanding he lived in the dark ages, when the Goths and Vandals had over-run the Roman empire, and had nearly obliterated every trace of ſcience and chronology, yet his adjuſtment of that Æra was found to be ſo accurate, that it is followed to this day.

We may now come to modern times; and here we ſhall find the names of profeſſors, whoſe labours, as actual teachers or writers, will make them eſteemed and admired to the lateſt times. Newton, Whiſton, Barrow, Simpſon, Emerſon, Maſkelyne, &c. are names whom poſterity muſt ever revere. The celebrated Dr. Hutton, at preſent employed in the education of youth at Woolwich, and his excellent aſſiſtant Bonnycaſtle, do honour to the profeſſion of teachers. And if we take a view of the profeſſors of our own country, we ſhall find abundance of eminent names to add luſtre to the catalogue. Starrit, Roynane, Fenn, Hamilton, M'Gee,* Uſher, Davenport, Naſh, and all the preſent Fellows, and Profeſſors of Trinity College, take an exalted rank in the liſt. And we are juſtified in cloſing it, by adding thereto the names of the worthy and learned teachers of the city of Dublin, who have ſo patiently examined the following work, and after the moſt minute inveſtigation of its perfections and defects, have candidly given their opinions thereof. And we truſt, that the youth for whoſe improvement it was written tem, will receive that aſſiſtance from it, which theſe gentlemen expect.

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* The opinion of the author, of the Profeſſors of Trinity College, is founded on incontrovertible facts; namely, he judges from the number of eminent men whom they have qualified to fill, in this and other countries, the moſt exalted ſtations, in every learned department, whether it be the ſenate, pulpit, bar, or the more humble character of a merchant and private gentleman.

Aſk the learned judges on the bench, or the moſt eloquent counſellors at the bar, who laid the foundation of that knowledge which has raiſed them to their preſent dignified rank? or who inſtilled into their tender minds the firſt rudiments of that learning upon which the ſuperſtructure of their profeſſional reputation has been erected? Candour anſwers; my ſchoolmaſter, my old, faithful friend. He it was who performed the kind paternal office, and conducted my mind from the unlettered chaos, to the temple of ſcience, of reaſon, and perfection.

Let us turn to the divine, and put the ſame queſtion to him? As a miniſter of the Goſpel, an advocate for Chriſtian verity, he is compelled to anſwer, that his eſteemed tutor, his kind preceptor, tenderly opened to his view, the firſt dawnings of ſcience, of reaſon, of virtue, of the perceptions of the perfections of the *Deity*, and the truths of the Goſpel.

Again, if we look to the ſplendid courtier, the eloquent ſenator, the prime miniſter of the ſtate, and aſk them are they not as much indebted to the foſtering inſtructions of their old ſchoolmaſter as thoſe? The brilliant figure which they now make on the public ſtage, received its firſt ſketches from the teacher, who blocked out the piece and pointed out the beauties of the firſt principles of learning, of eloquence, of virtue, and patriotiſm.

We preſume not to cenſure the great, or to charge them with ingratitude towards their former teachers; that many benevolent gentlemen have made *ample* proviſion for their tutors in the decline of life, is an undoubted fact: yet that too many worthy ſchoolmaſters are neglected by their former pupils, and ſuffered to die in obſcurity and wretchedneſs, is a melancholy truth, which humanity ſhudders at relating. This is an evil, which we hope cannot exiſt much longer, in the midſt of enlightened *Chriſtianity*. Cicero, though a Pagan, ſays, "What greater or more excellent "gift can we offer to the republic, than to teach the youth "carefully." And if ſuch were the ſentiments of a Roman orator, unaſſiſted by the light of the Goſpel; how much more ſtrong and benevolent towards teachers, ſhould be both the theory and practice of the followers of the *merciful Redeemer*?

Royalty itſelf is ornamented by Literature; whence from the Monarch on the throne to the moſt inferior profeſſional character, all are more or leſs indebted to the ſchoolmaſters' inſtructions for their advancements in life and future proſpects; and of conſequence it is the bounden duty of every man of virtue and patriotiſm to cheriſh and befriend Teachers: ſince the happineſs and ſucceſs of their children and thoſe of the ſtate greatly depend thereon. On the inſtructions firſt inſtilled into the puerile mind, the future exertions of virtuous purſuits, or vicious habits are greatly to be attributed. Let Profeſſors of uſeful learning, therefore, meet with that liberal encouragement and receive that honour and reſpect to which their important truſt and utility in the commonwealth entitle them; and the empire itſelf muſt reap the benefit; from our beloved *Sovereign* on the throne to the humble cottager, all muſt feel the ſalutury effects of a humane attention to ſchoolmaſters. And when the ſneer of ignorance, or the ridicule of unlettered inſolence is unjuſtly levelled againſt the profeſſion; every man of taſte and virtue muſt ſpurn the illiberal and contemptible fellow: from the clown, indeed, men of ſcience can expect nothing but ungenerous and baſe treatment; but gentlemen of eruditon and politeneſs will ever patronize and encourage teachers of uſeful learning.

From theſe few obſervations on what ought to be the general conduct of every one in the ſeveral departments of life, towards ſchoolmaſters; we truſt that our intention ſhall not be miſconſtrued into any partial prejudice for, or againſt any particular claſs of learned gentlemen, or any other body of men; for we ſolemnly declare, that for our own part we ſhould do violence to our feelings and injuſtice to truth, if we did not acknowledge in the moſt unequivocal manner that the Writer's encouragement, while he was in the line of a teacher, flowed ſo equally and liberally from all ranks, that he was never able to make a diſcrimination; and as an Author, gratitude and honour compel him publicly to own that he has found numerous patrons and friends in all orders and degrees of men: for which he returns his unfeigned thanks to the Nobility, Clergy, Gentry, &c. particularly his brethren, the worthy and generous teachers of the City of Dublin, for the public and handſome teſtimony of his laborious work, which was the reſult of twenty years cloſe application. And when he reflects on the patience they have ſhewn, in accurately peruſing a work ſo voluminous, he feels himſelf more gratified at their diſintereſted and philantropic conduct, than he poſſibly could be, were the higheſt worldly honours to be conferred on him by them.

Perhaps it may be enquired, Why has the author of a ſyſtem of Arithmetic ſtepped forward as the oſtenſible advocate for ſchoolmaſters? In anſwer, he replies, that his motives, he hopes, are of the pureſt kind. He has the public good alone at heart; and one way to promote the ſame, he humbly conceives, is by a proper attention among all learned and influential characters, to the reſpectability and comfort of all worthy teachers. As one who has ſpent many years in that department, he feels for the happineſs of his fellow-labourers in the ſame vine-yard; and he verily believes, that the more this claſs of men are encouraged, the more ſhall the whole community at large be benefitted. And although he has for the preſent removed from the profeſſion, and is not actually employed at this moment in teaching a ſchool, yet, as an author of a ſchool-book, intended ſolely for the inſtruction of youth, he apprehends that the line in which he is now engaged, and that of a teacher, can ſcarce be conſidered as any thing but one and the ſame honourable employment, namely, that of a ſchoolmaſter.

Which of the arts, ſciences, languages, &c. taught by the laborious tutor, is the moſt uſeful to mankind, we pretend not preciſely to determine; each having its own peculiar utility; but this much we may venture to aſſert, that in the common concerns of life, Arithmetic and the concomitant branches of the mathematics, take the firſt place. In our preface we have noticed ſome inſtances of the importance of Arithmetic to civilized man; and here we ſhall juſt obſerve, that every one who is inclined to advance in life, ſhould make himſelf maſter of figures, before he ſets forward on his journey. Some knowledge of accounts is neceſſary for every man. If the king wants to know the amount of his revenue; the man of landed eſtate, the aggregate of his rent-roll; the merchant, the ſtate of his affairs and property; Arithmetic exhibits the object of enquiry; this ſcience alone determines the matter with perfect certainty. Let every one therefore impreſs it on the mind of youth, that Arithmetic is a ſcience abſolutely neceſſary to be underſtood by all ranks and degrees of men; nor is a knowledge of figures unimportant to the female character; that woman who is qualified to keep accounts, is beſt adapted for the care of a family.— May Providence therefore, befriend the teachers of this and every other uſeful ſcience, is the fervent prayer of

*THE Author, to prevent piracy, offers a reward of* TWO HUNDRED POUNDS *to any perſon who will give information of any attempt made to print or publiſh this work, without his approbation under his Hand and Seal firſt obtained, agreeably to the Act of Parliament in that caſe provided: Alſo, a further reward of* ONE HUNDRED POUNDS *for any perſon who has taken out of the work, and publiſhed the number of pages ſpecified in ſaid Act. Moreover a reward of* FIFTY POUNDS *will be given to any perſon who may give private information of any piracy thereof, that may lead to conviction, and his name ſhall be kept ſecret.*

121 Capel-ſtreet.
July 30th, 1804.

P. Deighan.

# ARITHMETIC.

## DEFINITIONS.

ARITHMETIC is a ſcience which teaches the properties of numbers, and by them deduces precepts of computation from certain data (requiſites or conditions given) relative to the affairs of the buſy part of mankind, and the enquiries of the curious.

And in ſtrictneſs there are but two primary operations, from which the reſt are drawn, *i. e.* Addition and Subtraction, as will appear further on: For Multiplication is but a compendious method of Addition, and Diviſion the ſame of Subtraction.

2. The moſt part of the objects of our knowledge may be conſidered as capable of augmentation and diminution; and our ideas of things, as far as they have that capacity, is what we call quantity; by which word may be comprehended whatever can properly be ſaid to have parts.

Under this definition we may rank extenſion, (a multitude of units), weight, motion, time, &c.

The one being taken, greater or leſs, heavier or lighter, ſwifter or ſlower, in relation to one another of the ſame kind: And, ſince the primary and moſt conſiderable property of quantity is, the being capable of more or leſs, therefore quantities may be added to, ſubtracted from, or multiplied by one another, and divided into the parts which they contain.

3. Unity is that by which every thing that is called one, is conſidered. Unity is the beginning of every number.

4. Number, is a multitude compoſed of units.

5. One number is ſaid to meaſure another, when the leſſer being taken ſome number of times, is exactly equal to the greater; as 4 meaſureth 12, becauſe three times 4 make 12. Unity meaſureth all numbers.

6. One number is a multiplex or multiple of another, when the leſs meaſureth the greater, or when the greater containeth the leſs ſome number of times preciſely.

7. An aliquot part of a number is that which meaſureth the ſaid number without a remainder. The number 2 is an aliquot part of 10, becauſe it meaſureth 10, being taken 5 times; but 3 is an *aliquant* part of 10, becauſe it does not meaſure 10, for being taken three times it makes 9, and taken 4 times it makes 12.

8. Numbers are various, as digits, articles, compounds, whole, broken, mixed, &c.

9. Numbers are ſaid to be equal, unequal, even, odd, evenly even, evenly odd, oddly odd, compoſite, plain, ſolid, perfect, harmonic, ſquare, cube, &c. &c.

10. Equal numbers are ſuch as contain an equal number of units.

11. Unequal numbers are thoſe whoſe number of units differ.

12. An even number is that which may be divided into two equal parts, therefore ſome number meaſureth every even number by 2.

13. An odd number is that which cannot be divided into two equal parts, or that which differeth from an even number by an unit.

14. A number evenly even, is that which an even number meaſureth by an even number: ſuch is 24, which is the even number 6 meaſured by the even number 4.

15. A number evenly odd, is that which an even number meaſureth by an odd number: ſuch is 12, which the even number 4 meaſured by the odd number 3.

16. A number oddly odd, is that which an odd number meaſureth by an odd number; ſuch is 21, which an odd number 7 meaſureth by an odd number 3.

17. A compoſite one, is that which ſome certain number beſides an unit meaſureth, and conſequently hath ſeveral

aliquot parts; ſuch are 4, 6, 8, 9, 10, 12, 14, 16, and infinite others.

18. Plain numbers are ſuch as are made of the multiplication of two, as $6\times2$ is 12, conſequently every plain is compoſite.

19. A ſolid number is that which is produced from the multiplication of three numbers, and the numbers that multiply one another are called the ſides of the ſaid ſolid number.

Therefore every ſolid number is compoſite. 24 is a ſolid number, becauſe it is made by the multiplication of three numbers, 2, 3, and 4, for $2\times3=6$, and $6\times4=24$.

20. Perfect numbers are ſuch, whoſe aliquot parts added together are equal* to themſelves; as 6, whoſe parts are 3,

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* If a ſeries of numbers, continually proportional from unity in a duplicate ratio, be continued until their ſum be a prime number, the ſum being multiplied into the greateſt term ſhall produce a perfect number.

Hence by the above, may be found all the perfect numbers; becauſe the ſum of 1 and 2 is 3, a prime number, $3\times2=6$ the firſt perfect number, whoſe aliquot parts are 1, 2, and 3; and becauſe the ſums of 1, 2, and 4 are 7, a prime number, 7 multiplied by the greateſt aliquot part 4, make 28, the ſecond perfect number, whoſe aliquot parts are 1, 2, 7, and 14.

Again, the ſum of 1, 2, 4, 8, and 16, is 31, a prime number $=31\times16=496$ the third perfect number; the aliquot of the next are 1, 2, 4, 8, 16, 31, 62, 124, 248, the ſum will be eaſily had, if the following number be leſſened by an unit. (ſee Quintin, on the ninth book of Euclid.) And the aliquot parts of any perfect number may be thus found: ſeek how many numbers are taken doubled from unity, ſecluding unity, take ſo many doubled from the prime or ſumnumering the prime, theſe with thoſe, double from unity, and the unit itſelf are the aliquot parts of the required perfect number. Moreover, the manner of finding the perfect numbers is briefly exhibited (by the foregoing writer) thus, let the ratio of 1 to 2 be continued infinitely from an unit; ſee what numbers of the progreſſion, by taking away an unit, make or become prime numbers; for theſe being drawn or multiplied into the next preceding them, will give the perfect numbers.

Of the multitude of perfect numbers already found, Mar. Martennus, in his preface, ſect. 9, writes thus: that there are but 11 yet found, to wit, 6, 28, 496, 8, 128, 550, 336, 8589, 869, 056, 137, 431, 691, 328, 2385, 843, 008, 139, 952, 128, and three other, the laſt whereof is made of the 257th term of a double progreſſion, leſſened by an unit, and multiplied into 256 terms of thoſe 28 which Bungus, in the 28th chapter of his book of the Myſteries of Numbers, rehearſes, only the 8 firſt are perfect numbers, (to wit, thoſe delivered above) the others are imperfect.

But the reaſon that more have not been yet found, is becauſe in a ſeries of a duple progreſſion, the intervals of the numbers, which by leſſening by an unit, become primes, are ſo very great, that the 11th of the ſaid numbers, 3, 2, 1=6; the ſecond perfect number is 28, for all the aliquot parts thereof are 1, 2, 4, 7, 14, which added together make 28.

21. Harmonic numbers are ſuch, that the aliquot parts of the one collected, are equal to thoſe of the other number.

22. A ſquare number is that which is made by the multiplication of two equal numbers, or by the multiplication of any number by itſelf, which is called a ſquare root. The firſt ſquare in whole numbers is 4, which is made of 2 by 2, or of 2 multiplied into itſelf; the ſecond is 9, which is made of 3 into itſelf, and ſo on in infinitum.

23. A cube number is that which is made by the multiplication of three equal numbers, or of the ſame number thrice taken in multiplication; the number itſelf is called a cube root. The firſt cube except 1 is 8, which is made by the multiplication of 2 thrice taken; 2×2=4, and 4×2=8; the ſecond is 27, which is made by the multiplication of 3 thrice taken, 3, 3, 3, for 3×3=9, and 9×3=27.

24. Numbers are ſaid to be aliquot, aliquant, prime, compoſite, &c. to each other.

Aliquot, when one meaſures the other, as 6 to 18.

Aliquant, when one does not meaſure the other without a remainder, as 6 to 19.

Prime, when unity meaſures both, as 11 and 15.

Compoſite, when a number meaſures both exactly, as 12 and 36.

25. ‘ In numbers, the ratio or proportion, is the mutual habitude of two numbers to oneanother, and is two-fold, either in reſpect of quantity or quality. In reſpect of quantity, it is conſidered between two numbers, the firſt called the antecedent, the ſecond the conſequent, and is equal, as 3 to 3, or unequal, as the greater to the leſs, 6 to 4, or the leſs to the greater, as 4 to 6.

26. ‘ Reaſon or ratio, as well of the greater to the leſs, as the leſs to the greater, is five-fold, viz. 1ſt. Multiple, 2d. Superparticular, 3d. Superpartiens, 4th. Multiple ſuperparticular,

numbers is the 25th term from unity, excluſive (as is ſaid above) and conſequently the number is very great, and the labour to find whether or not it be a prime, is extremely tedious; if the number conſiſts of twenty figures, perhaps a whole age is not ſufficient to try it, let one uſe any way already found.

ticular, 5th. Multiple ſuperpartiens. The three firſt are called ſimple, the two laſt mixed reaſon, or habitude. To name their oppoſites, join the prepoſition ſub thereto, as ſubmultiple, &c.

‘ 1ſt. Multiple reaſon, is when the antecedent, or greater number, contains the conſequent ſome number of times without a remainder, as 6 to 3, called duple, 21 to 7, called triple reaſon, &c. Their oppoſite is of the leſs to the greater, as 3 to 6, 7 to 21, called ſubduple, ſubtriple reaſon, &c.

‘ 2d. Superparticular reaſon is, when the antecedent contains the conſequent but once, with a fraction, whoſe numerator is always unity; ſuch are 3 to 2, 4 to 3, 5 to 4, &c. called ſeſquialtera, ſubſeſquitertia, ſubſeſquiqarta, &c.

‘ 3d. Superpartient reaſon, is when the antecedent contains the conſequent once, with a fraction, whoſe numerator is always more than unity, ſuch as 5 to 3, 7 to 4, &c. called ſuperdupartiens tres, and ſubſupertriartipens, &c. its oppoſite ſuperpartiens, as 3 to 5, 4 to 7, or ſuperdupartiens tres, ſubſupertripartiens quartas, &c.

‘ 4th. Multiple ſuparparticular reaſon, is when the antecedent contains the conſequent divers times with a fraction whoſe numerator is unity, as 9 to 4, or duplaſeſquiequarta, 9 to 2, or quadruplaſeſquialtera, 26 to 5, &c. its oppoſite is ſubmultipleſuperparticular, as 4 to 9, &c.

‘ 5. Multipleſuperpartiens reaſon, is when the antecedent contains the conſequent divers times with a fraction, whoſe numerator is always greater than unity, as 8 to 3, called duplaſuperdupartiens tertia, 19 to 5, called triplaſuperquadripartiens quinta, &c. its oppoſite is ſubmultipleſuperpartiens, 3 to 8, 5 to 19, &c. Under ſome of theſe five ſpecies are comprehended all the varieties that can happen between two numbers in reſpect of quantity, and holds good alſo in fractions as well as mixed numbers.

In reſpect of quality, which is a ſimilitude of reaſons called proportion, it is conſidered between more than two numbers; for though the reaſon of two numbers may be had as before, yet a ſimilitude of reaſons cannot be found, unleſs the numbers be more than 2, and is three-fold; 1ſt. in reſpect of their difference, 2d. of their quote, 3d. in reſpect of both.

Of the firſt ariſeth arithmetical proportion; of the ſecond, geometrical proportion; of the third, harmonical proportion.

1. Arithmetical progreſſion is an equality of difference, (i. e.) when a rank of numbers have one and the ſame difference; and is two-fold, continued or diſcontinued.

Continued, when of ſeveral, the ſecond exceeds or is leſs than the firſt, by the ſame number of units as the third exceeds or is leſs than the ſecond, or the fourth the third, &c. ſo 1, 3, 5, 7, 9, and 13, 10, 7, 4, 1, are numbers in arithmetical progreſſion, the firſt term increaſing by 2, and the ſecond decreaſing by 3: and this is what is commonly called arithmetical progreſſion.

Diſcontinued, i. e. when there is the ſame difference between the firſt and ſecond, as between the third and fourth; but not as between the ſecond and third: ſo 1, 3 ÷ 7, 9 are four numbers in arithmetical progreſſion, the difference of the firſt and ſecond and third and fourth being 2, which is not the difference between 3 and 7.

2. Geometrical proportion is an equality of ratios, (i. e.) when ſeveral numbers being divided by one another, have their ſeveral quotients equal; and is continued or diſcontinued.

Continued, when of ſeveral numbers the firſt bears the ſame ratio or proportion to the ſecond as the ſecond does to the third, and as the third does to the fourth, &c. Thus 2, 3, 4, 6, are proportionals continued, ſince there is the ſame reaſon of 2 to 3 as of 4 to 6, each being ſubſeſquialtera: 1, 2, 4, 8, 16, 32, &c. are numbers in Geometrical proportion for the ſame reaſon; and this is called Geometrical progreſſion.

Diſcontinued, when the proportion of the firſt to the ſecond is the ſame of the third to the fourth; thus 3:6 :: 16:32 are Geometrical proportionals diſcontinued, 3 being contained as often in 6 as 16 in 32, viz. twice; and this is commonly called the Rule of three direct.

3. Harmonic or Harmonical proportion is when the firſt term is to the laſt as the difference of the firſt and ſecond to the difference of the two laſt; ſo theſe three numbers, 1, 2, 3, 6, are in muſical proportion, ſince 2:6 :: 1:3 = the difference of the two laſt; thus alſo theſe 4 numbers are in harmonical proportion, viz. 2, 3, 6, 12; ſince the firſt is to the laſt, as the difference of the two firſt to the difference of the two laſt.

A Theorem is a propoſition whoſe truth is to be demonſtrated.

A Problem is a propoſition of ſomething to be done or diſcovered.

A Lemma is a theorem inſtructive to ſome ſubſequent propoſition, to ſhorten the proof or practice of it.

A Corollary is a propoſition gained in conſequence of another, whoſe truth is evident, from the truth or demonſtration of the former.

A Demonſtration is an infalliable proof of the truth or falſity of propoſitions.

## POSTULATES.

1. Let it be granted that a number given may be increaſed by unity and the number known.

2. That a given number may be diminiſhed by unity and the remainder.

## AXIOMS.

1. Theſe things which are equal to the ſame thing are equal to one another.

2. If equal things be added to equals the whole ſhall be equal.

3. If equals be taken from equals the remainders ſhall be equal.

4. If equals be multiplied by equals the products ſhall be equal.

5. If equals be divided by equals the quotients ſhall be equal.

6. The whole is equal to all its parts.

*Note,* A Poſtulate is ſomething to be granted on which to found a proof.

An Axiom is a propoſition whoſe truth is ſelf-evident.

A Propoſition is whatſoever is offered or propoſed either as matter of aſſent, practice, or ſpeculation.

The Enunciation of a propoſition is that which is declared therein.

Propoſitions are divided into Theorems, Problems, Lemmas, and Corollaries.

# CHAP. I.

## NOTATION AND NUMERATION OF INTEGERS.

NUMERATION is the firſt principal part of Arithmetic, teaches how to read, write, value, or expreſs any number of figures written or propounded, and conſiſts of two parts.

1. The due order of placing down figures.
2. The true valuing of each figure in its place.

Several methods for that purpoſe have, in the courſe of time, appeared; the more remarkable are,

Firſt, the method of the ancient Romans by the initial letters, I, V, X, L, C, D, M.

Secondly, that of the Arabians by theſe characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0; called the decedal method.

Thirdly, the method of the great Leibnitz, by theſe characters 1, 0, only, called the binary method. But experience ſhews that no method hitherto known are ſo appoſite to the various purpoſes of ſcience and human affairs, as the decedal method of the Arabians. The uſe of theſe characters was introduced into England about the year 1130. The firſt of theſe characters is called unity, and repreſents one of any kind of ſpecies or quantity; as, one world, one ſtar, one man, &c. Unity is that by which every thing that is, is called one, (E. 7. L. D. 1.) and is the beginning of all numbers, *i. e.* Number is a multitude of units, (E. 7. L. D. 1.) Then the formation and names of the Arabic digits will run thus,

1 = 1 one
1+1 = 2 two
1+1+1 = 3 three
1+1+1+1 = 4 four
1+1+1+1+1 = 5 five
1+1+1+1+1+1 = 6 ſix
1+1+1+1+1+1+1 = 7 ſeven
1+1+1+1+1+1+1+1 = 8 eight
1+1+1+1+1+1+1+1+1 = 9 nine

By the various arrangements of which, and the cypher (0) all numbers whatſoever are eaſily deſignable.

The method of diſpoſing the figures is ſuch, that any character is encreaſed in its value in a tenfold proportion towards the left hand; the firſt is the place of units, the ſecond the place of tens, the third the place of hundreds, &c.

See the table.

1 . . . . . . . . . . Units
1 2 . . . . . . . . . Tens
1 2 3 . . . . . . . . Hundreds
1 2 3 4 . . . . . . . Thouſands
1 2 3 4 5 . . . . . . Tens of Thouſands
1 2 3 4 5 6 . . . . . Hundreds of Thouſands
1 2 3 4 5 6 7 . . . Millions
1 2 3 4 5 6 7 8 . . Tens of Millions
1 2 3 4 5 6 7 8 9 . Hundreds of Millions.

### Roman Characters.

| | | | | | |
|---|---|---|---|---|---|
| One | 1 | I | Thirty | 30 | XXX |
| Two | 2 | II | Thirty-five | 35 | XXXV |
| Three | 3 | III | Forty | 40 | XL |
| Four | 4 | IV | Forty-five | 45 | XLV |
| Five | 5 | V | Fifty | 50 | L |
| Six | 6 | VI | Fifty-five | 55 | LV |
| Seven | 7 | VII | Sixty | 60 | LX |
| Eight | 8 | VIII | Sixty-five | 65 | LXV |
| Nine | 9 | IX | Seventy | 70 | LXX |
| Ten | 10 | X | Seventy-five | 75 | LXXV |
| Eleven | 11 | XI | Eighty | 80 | LXXX |
| Twelve | 12 | XII | Eighty-five | 85 | LXXXV |
| Thirteen | 13 | XIII | Ninety | 90 | XC |
| Fourteen | 14 | XIV | Ninety-five | 95 | XCV |
| Fifteen | 15 | XV | One Hundred | 100 | C |
| Sixteen | 16 | XVI | TwoHundred | 200 | CC |
| Seventeen | 17 | XVII | Three Hund. | 300 | CCC |
| Eighteen | 18 | XVIII | FourHundred | 400 | CCCC<br>or CD |
| Nineteen | 19 | XIX | | | |
| Twenty | 20 | XX | Five Hundred | 500 | D |
| Twenty-one | 21 | XXI | Six Hundred | 600 | DC |
| Twenty-two | 22 | XXII | Seven Hund. | 700 | DCC |
| Twenty-three | 23 | XXIII | Eight Hund. | 800 | CCM |
| Twenty-four | 24 | XXIV | Nine Hund. | 900 | CM |
| Twenty-five | 25 | XXV | OneThouſ. | 1000 | M or CIↃ |

| | | |
|---|---|---|
| Two Thouſand | 2000 | MM |
| Three Thouſand | 3000 | MMM |
| Four Thouſand | 4000 | $\overline{\text{IV}}$ |
| Five Thouſand | 5000 | $\overline{\text{V}}$ |
| Six Thouſand | 6000 | $\overline{\text{VI}}$ |
| Seven Thouſand | 7000 | $\overline{\text{VII}}$ |
| Eight Thouſand | 8000 | $\overline{\text{VIII}}$ |
| Nine Thouſand | 9000 | $\overline{\text{IX}}$ |
| Ten Thouſand | 10000 | $\overline{\text{X}}$ |
| One Hundred Thouſand | 100000 | $\overline{\text{C}}$ |
| Two Hundred Thouſand | 200000 | $\overline{\text{CC}}$ |
| Three Hundred Thouſand | 300000 | $\overline{\text{CCC}}$ |
| Five Hundred Thouſand | 500000 | $\overline{\text{D}}$ |
| Seven Hundred Thouſand | 700000 | $\overline{\text{DCC}}$ |
| Nine Hundred Thouſand | 900000 | $\overline{\text{CM}}$ |
| One Million | 1000000 | $\overline{\text{M}}$ |

To be brief, all that is requiſite to a farther knowledge of numeration may be obtained by the following table, which contains a ſummary of the whole doctrine.

| *Periods.* | Quadrillio. | | Trillions | | Billions | | Millions | | Units. | |
|---|---|---|---|---|---|---|---|---|---|---|
| *Half Per.* | thouſands | units. | thouſands | units. | thouſands | units. | thouſands | units. | thouſands | units. |
| *Figures.* | 167, | 345. | 162, | 346. | 604, | 372. | 543, | 201. | 763, | 870. |

The names of the ſucceeding periods are Quintillions, Sextillions, Septillions, Octillions, Nonillions, &c.

Write down in words the following figures,

$139\underset{\bullet}{1}72428\underset{\bullet}{8}88725\underset{\bullet}{2}99942\underset{\bullet}{5}12849\underset{\bullet}{3}402200$

*Rule.* Begin at units, ſet a point under the ſeventh place, reckoning that as one; count forward and ſet another under the ſeventh place; and ſo continue to the end.

One thouſand three hundred and ninety one quintillions, ſeven hundred and twenty-four thouſand two hundred and eighty-eight quadrillions, eight hundred and eighty-ſeven thouſand two hundred and fifty-two trillions, nine hundred and ninety-nine thouſand four hundred and twenty-five billions, one hundred and twenty-eight thouſand four hundred and ninety-three millions, four hundred and two thouſand two hundred.

Which vaſt number (comprehending all the words, ſignificant and inſignificant, that can be made out of the 24 letters) is here given as an inſtance of the power of letters and numbers, and to ſhew a form of the moſt eaſy method of Numeration Table.

1. Write in figures, one thouſand eight hundred and two.

2. Expreſs in figures, twelve thouſand ſix hundred and thirty-five.

3. Set down in figures, one hundred and thirty two thouſand one hundred and nineteen.

4. Expreſs in figures, ſix hundred and nine thouſand five hundred and one.

5. Write down in figures, ten hundred thouſand.

6. Write in figures, one million, two hundred and twenty ſeven thouſand and eleven.

7. Expreſs in figures, fifty millions, twenty thouſand and nine.

8. Set down in figures, five hundred millions, three hudred and forty-nine.

9. Expreſs in figures, nine hundred and eighty four millions and twenty-ſeven.

10. Write down in figures, a thouſand millions.

## CHAP. II.

## SIMPLE ADDITION.

SIMPLE Addition, or Addition of Whole Numbers, is the ſecond eſſential part of Arithmetic, and teaches, of ſeveral numbers of the ſame denomination to make one total, called their ſum.

*Rule.** Set down all the numbers to be added, as in the following examples, wherein, obſerve to ſet no figure in the ſame column, but what is of the ſame name, value, or place, and draw a line under them.

---

* Becauſe ten units in the firſt column towards the right hand, make an unit in the next row towards the left; the reaſon of carrying one for every ten is evident, and the method of placing the figures is no leſs evident, be-

2. Then begin at the place of units, add up that column, find how many tens are contained therein.

3. Set down what remains above the tens, or if nothing remains, a cypher, and carry as many ones to the next column as there were tens in this.

4. Add up the ſecond column together with the number carried, in the ſame manner as the firſt, and proceed thus till the whole is finiſhed.

### EXAMPLES.

cauſe any other diſpoſition of the figures would entirely alter their value. The latter part of the rule is founded on the known axiom "The whole is equal to all its parts."

### DEMONSTRATION.

The method of proof: 1ſt. Draw a line below the uppermoſt number, and ſuppoſe it cut off; 2d. add all the reſt together, and ſet their ſum under the number to be proved; 3d. add this laſt-found number and the uppermoſt line together, and if the ſum be the ſame as that found by the firſt addition, the work is right. Or thus: Add the ſum with all the parts again, and if the ſum be double the former, the work is right.

Beſides theſe two laſt methods here given, for proving Simple Addition, there is another very ingenious one, by caſting out the nines, thus:

Rule 1ſt. Add the figures in the uppermoſt row together, and find how many nines are contained in their ſum.

2d. Reject the nines, and ſet down the remainder directly even with the firſt figures in the row.

3d. Do the ſame with each of the other rows; and ſet all theſe exceſſes of nines together in a line, and find their ſum; then if the exceſs of the nines in this ſum found as before, be equal to the exceſs of nines in the total ſum, the work is right.

### EXAMPLES.

| | | |
|---|---|---|
| 3782 | Exceſs of nines | 2 |
| 5766 | | 6 |
| 8755 | | 7 |
| 18303 | | 6 the exceſs of nines in the total equal 6 proof. |

This method depends upon a property of the number 9, which, except to the number 3, belongs to no other digit whatever; that any number divided by 9, will leave the ſame remainder as the ſum of its figures or digits divided by 9, which may be thus demonſtrated:

### DEMONSTRATION.

Let there be any number as 3467, this ſeparated into its ſeveral parts, is $3000+400+60+7$; but $3000=3\times1000=3\times(999+1)=3\times999+3$.

## EXAMPLES.

| i) | ii) | iii) | iv) | v) |
|---|---|---|---|---|
| 786378 | 3454 | 4683 | 4563 | 8163 |
| 110937 | 2367 | 1334 | 1376 | 1453 |
| 821169 | 1378 | 5671 | 3687 | 1234 |
| 162914 | 5674 | 6841 | 1684 | 5678 |
| 319301 | 5908 | 3681 | 4168 | 9123 |
| 162717 | 3716 | 6716 | 1361 | 4567 |
| 216271 | 1371 | 9324 | 1111 | 8912 |

2579687

1793109

2579687 *Proof by firſt method.*

2579687

5159374 *Proof by ſecond method.*

Proof

In like manner, $400=4\times99+4$, and $60=6\times9+6$. Therefore $3467=3\times999+4\times99+6\times9+3+4+6+7$ & $3467\div9=(3\times999+4\times99++6\times9+3+4+6+7)\div9$ but $3\times999+4\times99+6\times9$ is evidently diviſible by 9. Therefore if 3467 be divided by 9, it will leave the ſame remainder as $3+4+6+7\div9$; and the ſame will hold of any other number whatever. Q. E. D.

The ſame may be demonſtrated thus univerſally.

**Demonstration.** Let $N=$ any number whatever, $a$, $b$, $c$, &c. the digits of which it is compoſed, and $n=$ as many ciphers as $a$ the higheſt digit, is places from unity. Then $N=a$ with $n$, o's $+b$ with $(n-1)$ o's $+c$ with $(n-2)$ o's &c. by the nature of notation; $=a\times(\overline{n-1})$ 9's $+a+b\times(\overline{n-2})$ 9's $+c\times(\overline{n-3})$ 9's &c. $+a+b+c$ &c. but $a\times(\overline{n-1})$ 9's $+b\times(\overline{n-2})$ 9's $+c\times(\overline{n-3})$ 9's &c. is plainly diviſible by 9; and therefore $N$ divided by 9 will leave the ſame remainder as $a+b+c$ &c. divided by 9. Q. E. D.

In the ſame manner this property may be ſhewn to belong to the number 3; but the preference is uſually given to the number 9, on account of its being more convenient in practice. Now from the demonſtration here given the reaſon of the rule itſelf is evident; for the exceſs of 9's in two or more numbers being taken ſeparately, and the exceſs of 9's taken alſo out of the ſum of the former exceſſes, it is plain this laſt exceſs muſt be equal to the exceſs of the nines contained in the total ſum of all theſe numbers; the parts being equal to the whole.

This rule was given by Dr. Wallis in his Arithmetic, publiſhed Anno 1657, and is a very eaſy ſimple method; though it is liable to this inconvenience, that a wrong operation may ſometimes appear to be right; for if we change the places of any two figures in the ſum, it will ſtill be the ſame; but then a true ſum will always appear to be ſo by this proof; and to make a falſe one appear to be true, there muſt be at leaſt two errors, which are exactly oppoſite to each other; and if there be more than two errors, they muſt balance among themſelves; but the chance againſt this particular circumſtance is ſo great, that we may as ſafely truſt to this as any other; except indeed when a perſon who knows the method, is minded to tranſpoſe the figures in the manner above mentioned, which muſt be always guarded againſt.

*Proof by the nines, which is the third method.*

Nines rejected out of the firſt line =3
Ditto ſecond line . . . . . =3
Ditto third line . . . . . . =0
Ditto fourth line . . . . . =5
Ditto fifth line . . . . . . =8
Ditto ſixth line . . . . . . =6
Ditto ſeventh line . . . . . =1

Ditto of ſum=8 Proof =8

## TABLE OF ADDITION,

To be got by heart by beginners in this ſcience.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 2 |  | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 3 |  |  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 4 |  |  |  | 8 | 9 | 10 | 11 | 12 | 13 |
| 5 |  |  |  |  | 10 | 11 | 12 | 13 | 14 |
| 6 |  |  |  |  |  | 12 | 13 | 14 | 15 |
| 7 |  |  |  |  |  |  | 14 | 15 | 16 |
| 8 |  |  |  |  |  |  |  | 16 | 17 |
| 9 |  |  |  |  |  |  |  |  | 18 |

The manner is thus: take the greater of two digits, whoſe ſum is ſought, in the upper line, and the leſs on the left hand column; in the ſame line with this, and underneath the other ſtands the ſum.

As, ſuppoſe I wanted the ſum of 9 and 7; I look for 9 at the head of the table, and in the ſame line with 7 on the left hand ſide ſtands 16, the ſum.

### THE USE.

1. A merchant on ſettling his accounts finds he owes A. 60*l.* B. 150*l.* C. 240*l.* and to D. 100*l.* I demand how much he owes in all? *Anſw.* 550*l.*

EXAMPLE. £. 60
150
240
100

Anſwer, £. 550 required.

2. A merchant is indebted to A. 4600*l.* to B. 370*l.* to C. 6000*l.* to D. 1267*l.* to E. 7640*l.* to F. 60*l.* I demand the whole ſum? *Anſ.* 19937*l.*

3. A man born in the year 1803, I demand when will he be ſixty years old? *Anſ.* 1863.

4. A merchant receives the following ſums, 200*l.* 317*l.* 315*l.* 10*l.* 172*l.* 513*l.* and 9*l.* what is the total ſum? *Anſ.* 1536*l.*

5. What is the total weight of theſe ſeven caſks of merchandize, viz. No. 1 weighing 960lb. No. 2, 725lb. No. 3, 830lb. No. 4, 798lb. No. 5, 697lb. No. 6, 569lb. and No. 7, 987lb. *Anſ.* 5566lb.

6. A man borrowed a ſum of money, and paid in part 302*l.* and the remainder is 30*l.* I demand the ſum borrowed? *Anſ.* 332*l.*

7. At the Cuſtom-houſe, the 1ſt of May, were entered 1200 yards of linen; on the 16th, 1479 yards; on the day following 1941 yards; again 6195 yards; on the four laſt days of the ſame month 1236 yards each day; how many yards were entered during the month? *Anſ.* 15759 yards.

8. How many days in twelve calendar months, being a common * year? *Anſ.* 365.

9. What interval of time will elapſe between a tranſaction which happened fifty-ſix years ago and one that may happen fifty-ſix years hence? *Anſ.* 112.

10. An army conſiſting of 4000 foot ſoldiers, 4006 cavalry or horſe, of 3093 light, of 1224 gunners, 1400 pioneers, and 200 miners; required the number of men in the whole army? *Anſ.* 13923.

11. An old man's age was required, he anſwered; I have five ſons and three daughters, between the birth of each of my ſons were two years, between my laſt ſon and firſt daughter four years, and four years each between the reſt of my daughters;

* Thirty days have November,
April, June, and September;
February hath twenty-eight alone,
And all the reſt have thirty-one.
Note; every Leap year February hath twenty-nine.

daughters; in my twentieth year my firſt ſon was born, and that is the age of my youngeſt daughter. I demand the old man's age? *Anſw.* 60.

12. How many days are there from the 1ſt June to the 27th January following? *Anſw.* 240 days.

13. Suppoſe a man was born in the year of our Lord 1803, in what year will he be ninety-nine years of age? *Anſ.* 1902, the year required.

14. A gentleman left his elder daughter one thouſand pounds more than the younger, whoſe fortune was eleven thouſand eleven hundred and eleven pounds: what was the elder daughter's fortune, and what did the father leave them? *Anſw.* younger, 12111*l.* elder, 13111*l.* both 25222*l.*

15. How much will A (born ſixteen years ago,) be older than B, at the time of his birth, who will come into the world fourteen years hence? *Anſw.* 30 years.

16. A ſheepfold was robbed three nights ſucceſſively; the firſt night half the ſheep were ſtolen, and half a ſheep more; the ſecond night half the remainder and half a ſheep more; and the laſt night they took half what were left, and half a ſheep more; by which time they were reduced to twenty; how many were there at firſt? *Anſw.* 167.

left 3d 2d 1ſt nights.

Solution. $20+21+42+84=167$. the anſwer required.

17. What is the ſum of ſix dozen dozen? *Anſw.* 864.

18. How many years elapſed from the creation of Adam to the univerſal deluge in the days called Noah's flood, by the 5th chapter, and 6th verſe of the 7th chapter of Geneſis? *Aſw.* 1656 years.

18. In a Biſſextile or Leap year how many days? *An.* 366.

19. In the year 1802 the value of goods exported to Sweden was fifty-nine thouſand ſix hundred and ſeventy-eight pounds, to Ruſſia one hundred and nine thouſand nine hundred pounds, to Spain one million ſeventy-eight thouſand ſeven hundred and thirty-one pounds, to Portugal ſix hundred and ſixty-ſeven thouſand one hundred and one pounds, to Holland one million ſix hundred and two thouſand nine hundred and twenty-four, and to Germany one million eight hundred and eleven thouſand two hundred and ſixty-eight pounds; required the amount? *Anſw.* 5329602£.

# CHAP. II.

## SIMPLE SUBTRACTION.

SUBTRACTION of Whole Numbers, or Simple Subtraction, is the finding how much one ſingle number exceeds another, or the taking a leſs number from a greater.

The number to be ſubtracted is called the ſubtrahend, and that out of which it is to be taken is called the minuend: alſo, the number remaining after the one is taken out of the other, is named, their difference.

**Rule.* Set down the numbers as directed in Addition, and ſet the greater uppermoſt: begin at the units' place of the ſubtrahend, and take the difference between it and the figure that ſtands in the ſame place with it in the minuend, which difference ſet under that place for the remainder; do ſo with each place to the left hand; but if the figure in

---

* Demonstration. The rule is evident; for ſince the remainder is the difference of the ſubtrahend and the minuend, or the exceſs of the minuend above the ſubtrahend, that difference or exceſs added to the ſubtrahend, which is the leſſer number, muſt make the minuend, which is the greater, (by the 6th axiom) it plainly follows from the nature of Addition and Subtraction,

1ſt. That Addition alone finds the ſum of many particular ſums, and if it be applied to a man's account, it ſhews in what ſum he is debtor or creditor. Or if it be applied to the inventory of a man's eſtate, it ſhews in groſs what he is worth.

2d. Subtraction alone, only finds the difference between any two numbers or ſums of money, and if it be applied either to an account of money or goods, that difference is called the balance of the account, and theſe ſums may be either original or ſimple ſums.

3d. When any figure of the greater number is leſs than its correſponding figure in the leſſer, the ten which is added by the rule, is the value of an unit in the next place, by the nature of Notation; and as the one which is added to the next place of the leſſer number, diminiſhes the correſpondent place of the greater. Accordingly, this is only taking from one place, and adding as much to another, by which the total is never changed. So that by this means, the greater number is reſolved into ſuch parts as are each greater than, or equal to the ſimilar parts of the leſſer; and therefore the difference of the correſponding figures, taken together, will make up the difference of the whole, as was before.

The truth of the method of proof is evident; for the difference of two numbers added to the leſſer is equal to the greater.

any place of the ſubtrahend be greater than its correſponding figure in the minuend, then add 10 to the deficient figure, and ſubtract; when you come to the next place, add one to the figure of the ſubtrahend, and take the ſum from the correſponding one of the minuend; do this as often as you have occaſion to add 10, and the differences ſet down orderly in this manner make the total difference or remainder ſought.

Proof. To the ſubtrahend add the remainder; if this ſum be equal to the minuend the work is right.

EXAMPLES.

| | i. | ii. | iii. | iv. | v. |
|---|---|---|---|---|---|
| From | 4763 | 8765 | 7812 | 9782 | 476423 |
| Take | 3584 | 1234 | 5843 | 5476 | 459875 |
| Rem. | 1179 | | | | |
| Proof | 4763 | | | | |

THE USE.

1. Suppoſe John was born in the year of our Lord one thouſand ſeven hundred and ninety, and James in the year eighteen hundred and three, what is the difference of their ages? *Anſ.* 13 years.

2. A man born in the year 1780, what was his age in 1803? *Anſ.* 23 years.

3. There are two numbers, the greater is 1795, and the difference is 1695, what is the leſſer number? *Anſ.* 100.

4. Suppoſe I lent my friend 419*l.* and that he died 403*l.* in my debt, how much did he pay me? *Anſ.* 16*l.*

5. A merchant bought 14 cwt. of tobacco for 15400l. and ſold 8 cwt. for 8800l. how many hundreds remain on hand, and what does he want to make up the firſt coſt? *Anſ.* 600 cwt. and it ſtands him in 6600*l.*

6. In 5 bags were different ſums of money, to the amount of 1000*l.* in the firſt 100*l.* in the ſecond 314*l.* in the third 143l. and in the fourth 209*l.* what did the fifth contain? *Anſ.* 234*l.*

7. Suppoſe a gentleman has an eſtate worth 3090l. a year, and pays for land tax 150l. for quit-rent 95l. what is his neat eſtate worth per annum? *Anſ.* 2845l.

8. A ſnail getting up a May-pole only twenty feet high, was obſerved to climb eight feet every day, but every night it came down again four feet; in what time, by this method, did he reach the top of the pole? *Anſ.* The night of the fourth day.

9. The ſemi-diameter of the Earth's orbit, or annual path 'round the ſun, is about 81000000 of miles; that of Venus 59000000, when they are both on the ſame ſide of the ſun they are in perigeo: when on different ſides, in apogeo; what is the difference of their diſtances in both theſe poſitions? *Anſ.* 118000000.

*Solution*, $81000000-59000000=22000000$, the diſtance of the Earth from Venus when in perigeo; and $81000000+59000000=140000000$ the diſtance the Earth is from Venus when in apogeo; therefore $140000000-22000000=118000000$ miles, the anſwer.

10. What time is there between the eclipſe which happened in the year 1715, the 22d day of April, 30 minutes paſt 7 in the morning, and the eclipſe which happened in the year 1724, the 11th of May, at 9 minutes paſt 5 in the afternoon? *Anſ.* 9 years, 20 days, 9 hours, 39 minutes.

11. If the mean diſtance between the Earth and Sun be 81 million of miles, and between the Earth and Moon 240 thouſand, how far are theſe two luminaries aſunder in an eclipſe of the Sun, when the Moon is lineally between the Earth and Sun? and in another of the Moon, when the Earth is in a line between her and him?

*Anſ.* { In an eclipſe of the Sun, 80760000 miles.
In an eclipſe of the Moon, 81240000 miles.

Solution, $81000000-240000=80760000$ miles, the diſtance theſe two luminaries are aſunder in an eclipſe of the Sun. Likewiſe $81000000+240000=81240000$ miles, the diſtance theſe two luminaries are aſunder in an eclipſe of the Moon.

# CHAP. III.

## SIMPLE MULTIPLICATION.

SIMPLE Multiplication, or Multiplication of Whole Numbers, is a manifold addition, or the repeating a given quantity as often as required, that is, to take a quantity ſo many times, part or parts of a time, as is repreſented by another.

| | | | |
|---|---|---|---|
| The number | to be repeated | is called | the multiplicand. |
| | of repetitions | | the multiplier. |

The ſum of the number ſo often repeated is called the product. Both the multiplier and multiplicand are called factors.

### GENERAL RULE.

Make either of the given factors the multiplicand, and the other the multiplier.

### RULE.[*]

1. Place the multiplier under the multiplicand, ſo that units may ſtand under units, tens under tens, &c. and draw a line under them.

2. Begin

---

[*] **Demonstration.** The reaſon of placing the firſt figure of every particular product under the multiplying figure, is evident from the nature of Numeration; if it be conſidered, that as we begin with the units' figure of the multiplier, the ſame, or any of the figures ſtanding in the tens' place, muſt produce a number of ten times as great as if it ſtood in the place of units; and if it ſtood in the place of hundreds the product would be one hundred times as great, and ſo on for all the other figures in the multiplier: but there is no other way to make thoſe products to bear thoſe names, but by placing the firſt figure of each product under that figure of the firſt product, which is the ſame diſtance from the unity that your multiplying figure is; therefore theſe ſeveral products added together, will be equal to the whole required product. Q. E. D.

**Proof.** Make the former multiplicand the multiplier, and the multiplier the multiplicand; and if the product found from this operation is the ſame as before, the work is right.

Beſides the method of proof given above, there is another very convenient and eaſy one, by the help of that peculiar property of the number 9, mentioned in addition, which is performed thus:

RULE.

1. Add all the digits of the multiplicand together, divide their ſum by 9, ſet

2. Begin at the right hand, and multiply every figure in the multiplicand by each of the figures in the multiplier.

3. Reckon how many tens there are in the product of every two ſimple figures, and ſet down the remainder directly under the figure you are multiplying by, and if nothing remains, a cypher.

4. Carry as many units as there were tens to the product of the next figure, and proceed, in like manner, till the whole is finiſhed.

5. Add all the products together, and the ſum will be the anſwer required.

The

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ſet down the remainder; do the ſame with the multiplier, and put the remainder under the former.

2. Multiply the two remainders together, and if the product is leſs than 9, ſet it down, if greater, divide it by 9, and ſet down the remainder.

3d. Add together the digits of the product, divide their ſum by 9, and if the remainder be equal to the laſt remainder, the work is right.

**Example.** Multiply 896546
by 803

2689638
71723680

719926438

4
2 X 2
4

Demonſtration of this. Let M and N be the number of 9's in the factors to be multiplied, and $a$ and $b$ what remains, then $M+a$ and $N+b$ will be the numbers themſelves, and their products is $(M \times N)+(M \times b)+$ $+(N \times a)+(a \times b)$; but the three firſt of theſe products are each a preciſe number of 9's, becauſe one of their factors is ſo: theſe therefore being caſt away, there remains only $a \times b$; and if the 9's are alſo caſt out of this, the exceſs is the exceſs of 9's in the total product; but $a$ and $b$ are the exceſſes in the factors themſelves, and $a \times b$ their product; therefore the rule is true. Q. E. D. This method is liable to the ſame inconvenience with that in Addition.

In all Multiplication, as 1 is to the multiplier, ſo is the multiplicand to the product; whence it is evident, that in Multiplication of Integers the product muſt be greater than either of the factors (for ſo the multiplier and multiplicand are called, becauſe between themſelves they make up the product) becauſe either of the factors is greater than unity; but in Multiplication of Fractions (which will be fully explained in its proper place) the product muſt be leſs in value than either of the factors, becauſe they are both leſs than unity, thus, 24, the product of 6, multiplied by 4, is greater than either 6 or 4, but $\frac{12}{20}$ the product of $\frac{3}{4}$ multiplied by $\frac{4}{5}$ is leſs in value than either $\frac{3}{4}$ or $\frac{4}{5}$.

The following table* to be learned by heart before the learner can proceed any farther.

## TABLE.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 120 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

*Caſe* 1. When the multiplier is any figure from 2 to 12, proceed thus:

| EXAMPLES. | Multiply | 34678946 |
|---|---|---|
| | by | 2 |
| | | 69357892 |

2. Multiply 374328756432 by 3. *Anſ.* 1122986269296.
3. Multiply 5806342748 by 4. *Anſ.* 23225370992.

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* The inventor of this eſſential aid, is ſaid to have been the great Pythagoras.

4. Multiply 8435674 by 5. *Anſ.* 42178370.
5. Multiply 274567546473 by 6. *Anſ.* 1647405278838.
6. Multiply 54328432847 by 8. *Anſ.* 434627462776.
7. Multiply 8643597 by 9. *Anſ.* 77792373.
8. Multiply 796534289 by 11. *Anſ.* 8761877179.
9. Multiply 3274656461 by 12. *Anſ.* 39295877532.

*Caſe* 2. When the multiplier is the product of two or more numbers, each not greater than 12,

*Rule.* Multiply continually by the component parts of the multiplier.

EXAMPLE. 1. Required, the product of 376 by 21

376
3
———
1128
7
———
7896

2. Required, the product of 12345678 by 32. *Anſ.* 395061696.
3. Multiply 9378964 by 42. *Anſ.* 393916488.
4. Multiply 1345894 by 49. *Anſ.* 65948806.
5. Multiply 576784 by 64. *Anſ.* 36914176.
6. Multiply 596875 by 144. *Anſ.* 85950000.
7. Multiply 46123101 by 72. *Anſ.* 3320863272,
8. Multiply 61835720 by 132. *Anſ.* 8162315040
9. Multiply 718328 by 96. *Anſ.* 68959488.
10. Multiply 7128368 by 1440. *Anſ.* 10264849920.

*Caſe* 3. When the multiplier and multiplicand conſiſt of many figures, proceed as in the general rule.

EXAMPLE 1. Multiply 762768
by 426
———
4576608
1525536
3051072
———
324939168 product, the anſwer.

0
0 X 3 Proof.
0

2. Multiply 79650479068 by 70604.
*Anſ.* 5623642424117072.
3. Multiply 140601282 by 912356784.
*Anſw.* 128278533471797088.
4. Multiply 4161381684 by 631. *Anſ.* 2625831842604.
5. Mult. 16358724 by 704006. *Anſ.* 11516639848344.
6. Mult. 38015732 by 40070065. *Anſ.* 1523292852262580.
7. Multiply 123456789 by 1440. *Anſ.* 177777776160.
8. Multiply 921760035 by 520007091.
*Anſw.* 479321754400408185.
9. Multiply 402097316 by 195. *Anſ.* 78408976620.
10. Mult. 4161381684 by 6314. *Anſ.* 26274963952776.

### *Contractions in Multiplication.*

1. If your multiplier be between 10 and 20.

*Rule.* Multiply each figure in the multiplicand by the units of the multiplier adding to each product its own back figure, and to the laſt figure add the tens you carry if any.

EXAMPLES.

| | | | | | | |
|---|---|---|---|---|---|---|
| Mult. | 385 | 679 | 4873 | 6958 | 7956 | 7685 |
| by | 13 | 16 | 18 | 17 | 15 | 19 |
| Anſ. | 5005 | 10864 | 87714 | 118286 | 119340 | 146015 |

2. If the multiplier be between twenty and thirty.

*Rule.* Multiply by the units of the multiplier as above, and take the back figure double, and add the tens you carry to the laſt figure doubled.

EXAMPLES.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Multip. | 798 | 567 | 395 | 487 | 6784 | 123 | 158 |
| By | 22 | 23 | 25 | 27 | 27 | 28 | 29 |
| *Anſ.* | | 13041 | 9875 | 13149 | 183168 | 3444 | 4582 |

3. If your multiplier be compoſite, (*i. e.*) the product of two numbers.

*Rule.* Multiply the multiplicand by one of the compoſites, and the product by the other.

EXAMPLES.

Multiply 456 by 32. 789 by 44. 874 by 96. 876 by 144.

$$\begin{array}{r}456\\ 4\\ \hline 1824\\ 8\\ \hline 14592\\ \hline\end{array}$$

Multiply 3489 by 45.

4. If your multiplier exceed, or fall ſhort of the next tens by $\frac{1}{10}$ of thoſe next tens.

*Rule.* Multiply your multiplicand by thoſe tens, ſet down the product, then ſet ſaid product under itſelf ſo many places from the unit's place as the multiplying figure ſtands.

If your multiplying figure exceed thoſe tens, the ſum, elſe the difference, is the anſwer.

EXAMPLES. Multiply 3792 by 33 or 27

$$\begin{array}{r}30\\ \hline 113760\\ 11376\\ \hline\end{array}$$

$$\left.\begin{array}{r}\text{Sum } 125136\\ \hline \text{Difference } 102384\end{array}\right\}\text{Product for}\left\{\begin{array}{l}33\\ \\ 27\end{array}\right.$$

Multiply 24362 by 88 or 72

$$\begin{array}{r}80\\ \hline 1948960\\ 194896\\ \hline\end{array}$$

$$\left.\begin{array}{r}\text{Sum } 2143856\\ \hline \text{Difference } 1754064\end{array}\right\}\text{Product for}\left\{\begin{array}{l}88\\ \\ 72\end{array}\right.$$

5. If your multiplier exceed or fall ſhort of the next tens, hundreds, thouſands, &c. the ſum, or difference, of that product and the product is the anſwer.

EXAMPLES.

Multiply 76543 by 51 or 49

$$\begin{array}{r}382715\\ \hline\end{array}$$

$$\left.\begin{array}{r}\text{Sum } 3903693\\ \hline \text{Difference } 3750607\end{array}\right\}\text{Product for}\left\{\begin{array}{l}51\\ \\ 49\end{array}\right.$$

Multiply 26482 by 1001 or 999.

$$\begin{array}{rr} & 26482 \\ \hline \text{Sum} & 26508482 \\ \hline \text{Difference} & 26455518 \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\end{array}}\right\}\text{Product for}\left\{\vphantom{\begin{array}{c}1\\1\\1\end{array}}\begin{array}{l} 1001 \\ \\ 999 \end{array}\right.$$

6. If your multiplier be either an aliquot part of a hundred, thouſand, &c. or the ſum or difference of ſaid aliquot parts, if of a hundred add two cyphers, if of a thouſand add three, to the multiplicand, which divide by the denominator of ſuch part: if the multiplier be only an aliquot part, the quotient is the anſwer: if it be the ſum of the aliquot part of a hundred, thouſand, &c. add, if the difference, ſubtract that quotient from the multiplicand ſo increaſed, and the ſum, or difference, is the anſwer.

**EXAMPLES.** Multiply 3746 by 25, 75, and 125.

$$\begin{array}{rr} & 4)374600 \\ \hline \text{Quotient} & 93650 \\ \hline \text{Difference} & 280950 \\ \hline \text{Sum} & 468250 \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\right\}\text{for}\left\{\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\begin{array}{l} 25 \\ \\ 75 \\ \\ 125 \end{array}\right.$$

Multiply 71963 by 125, 875, and 1125.

$$\begin{array}{rr} & 8)71963000 \\ \hline \text{Quotient} & 8995375 \\ \hline \text{Difference} & 62967625 \\ \hline \text{Sum} & 80958375 \end{array}\left.\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\right\}\text{for}\left\{\vphantom{\begin{array}{c}1\\1\\1\\1\\1\end{array}}\begin{array}{l} 125 \\ \\ 875 \\ \\ 1125 \end{array}\right.$$

7. If the units of the multiplier be any aliquot part of the other figures of the multiplier,

*Rule.* Multiply the multiplicand by the units' figure of the multiplier, and multiply the product by the denominator of ſuch aliquot part, ſetting the product one place nearer the left, the ſum of thoſe two products is the anſwer.

**EXAMPLES.** 3762 by 279 4716 by 486

$$\begin{array}{rl} 3762 & \\ \hline & 9 \\ 33858 & \\ 101574\phantom{0} & 3 \\ \hline 1049598 & \end{array}\qquad\qquad\begin{array}{rl} 4716 & \\ \hline & 6 \\ 28296 & \\ 226368\phantom{0} & 8 \\ \hline 2291976 & \end{array}$$

Or, more generally, multiply the multiplicand by the unit figure of the multiplier, and divide the remaining part of the multiplier into parts which may be multiples, either of the unit figure, or of one another, and multiply the product of the units' figure by ſuch multiple, and thoſe products by their multiples, &c.

**Examples.** 27624 by 2947

193368 2.140.2800
386736
773472

81407928

2d. 43762 by 3756
218810
1094050
1531670

164370072

3d. Multiply 2712 by 3848
8
21696
173568
86784

10435776

8. To multiply by 111, 112, 113, &c. to 119, incluſive, ſo as to have the product in one line: Multiply the multiplicand by the unit figure only of the multiplier, and add to the product the two figures which ſtand next on the right hand of that which you multiplied, and to the two laſt figures, ſeparately add what you carry.

Examples.

| i. | ii. | iii. | iv. | v. | |
|---|---|---|---|---|---|
| 2183 | 4296 | 5589 | 6273 | 7182 | |
| 111 | 112 | 113 | 114 | 115 | |
| 242313 | 481152 | 659807 | 715122 | 107730 | *Anſwers.* |

| vi. | vii. | viii. | ix. | |
|---|---|---|---|---|
| 83716 | 3295 | 8183 | 7295 | |
| 116 | 117 | 118 | 119 | |
| 9711056 | 385515 | 1783894 | 868105 | *Anſwers.* |

9. To multiply by any number, in which a cypher is inſerted betwixt an unit on the left hand, and any other digit on the right, at 101, 102, 103, &c.

### RULE.

Multiply by the units' digits, and when you have found two figures in the product, take in each time a figure in the multiplicand, beginning with the firſt.

### EXAMPLES.

4321
106
458026

904150
109
65852350

10. To multiply by any digit repeated, as 11, 111, 1111, &c. 22, 222, 2222, &c. 33, 333, 3333, &c.

### RULE.

Multiply by the repeating figure, and out of the product make the total product thus:

Begin at the place of units, and firſt take one figure, then two, then three, &c. (ſetting down the ſums under or over tens and carrying the tens) repeating the operation ſtill from the firſt place, as often as the number of places in the multiplier, and if there is not another figure to take in at every operation, you muſt continue to take in the whole multiplicand ſo often, till your number of operations be finiſhed.—Then begin at the ſecond place, and third place, &c. ſucceſſively, taking in from each as many on the left as the number of places of the multiplier, as long as you can find as many, and when they fail, take in all that remain, till the laſt figure is taken in alone, (minding always to carry the tens from every operation to the next.)

### EXAMPLES.

8739
444
Product by 4 34956
3880116 *Anſ.*

4538797
7777777
Product by 7 31771579
35301750914269 *Anſ.*

Multiply 784325634 by 7777777

.......
784325634
1111111

871472839519374×by 7=6100309876635618 *Anſwer.*

Or thus; a more compendious method:

7843256340000000
7

54902794380000000
5490279438 ſubtract

9)54902788889720562

6100309876635618 the ſame as before.

The above operations are performed with leſs than half the figures made uſe of the common way, and will be found very uſeful in decimals, where one of the factors, or part of it, is a ſeries of ſimilar figures, which the ingenious artiſts well know how to apply.

11. To multiply by a number of nines, as 99, 999, 9999, &c.

### RULE

Put down as many cyphers to the right hand of the multiplicand as there are nines in the multiplier, and from the reſult ſubtract the multiplicand; and what remains is the product required.

### EXAMPLES.

| | | | |
|---|---|---|---|
| Multiply | 2378 by 999 | and | 37568 by 999999 |
| | 2378000 | | 37568000000 |
| Subtract | 2378 | Subtract | 37568 |
| | 2375622 *Anſ.* | | 37568062432 *Anſ.* |

12. To multiply by 21, 31, 41, 51, 61, 71, 81, and 91, in one line; firſt bring down the unit figure of the multiplicand, which will always be the unit figure of the

product, then multiply the tens' figure of the multiplier by every figure of the multiplicand, and to each product add the figure which ſtands next on the left hand of that which you multiplied.

| | | | |
|---|---|---|---|
| 7086 | 80713 | 4286 | 5394 |
| 21 | 31 | 41 | 51 |
| 148806 | 2502103 | 175726 | 275094 |

| | | | |
|---|---|---|---|
| 8276 | 9718 | 74826 | 8327 |
| 61 | 71 | 81 | 91 |
| 504836 | 689978 | 6060906 | 757757 *Anſwers.* |

13. To multiply by 22, 23, 24, 25, 26, 27, 28, and 29, ſo as to have the product in one line: Multiply every figure of the multiplicand by the unit figure of the multiplier, and add, to each product, twice that figure which ſtands next on the right hand of that which you multiplied, and to twice the laſt figure of the multiplicand add what you carry.

| | | | |
|---|---|---|---|
| 3178 | 61729 | 1742 | 8375 |
| 22 | 23 | 24 | 25 |
| 69916 | 1419767 | 41808 | 209375 |

| | | | |
|---|---|---|---|
| 96842 | 31875 | 3789 | 4263 |
| 26 | 27 | 28 | 29 |
| 2517892 | 860625 | 106092 | 123627 |

14. To multiply any number of figures by any number of figures, at once, in one line, without the aid of intermediate lines.

*Rule.* Multiply the units of the multiplicand by the units of the multiplier, ſetting down the units of the product, and carry the tens; next, multiply the tens in the multiplicand by the units of the multiplier, to which add the product of the units of the multiplicand, multiplied by the tens in the multiplier, and the tens carried; then multiply the hundreds in the multiplicand by the units of the multiplier, adding the product of the tens in the multiplicand, multiplied by the tens in the multiplier, and the units in the multiplicand by the hundreds in the multiplier ; and ſo proceed till you have multiplied the multiplicand all through, by every figure in the multiplier.

**Example.** Multiply 35234
by 52424

1847107216

35234
52424

140936
70468
140936
70468
176170

Proof 1847107216

☞ Perhaps it might retard the progreſs of many learners to engage with theſe contractions before they underſtand the common method perfectly ; but after having advanced ſomewhat farther, we would ſtrongly recommend them to return to our mode, examine its utility, and then being better judges, they will the more eaſily explore its beauty, admire the improvement, and underſtand its conciſeneſs.

## CHAP. IV.

## SIMPLE DIVISION.

SIMPLE Diviſion, or Diviſion of Whole numbers, is the finding how often one number is contained in another ; or the dividing of any given number into any propoſed number of equal parts.

The number to be divided is called the *Dividend.*

The number you divide by is called the *Diviſor.*

The number of times the dividend contains the diviſor is called the Quotient. If the dividend contain the diviſor any number of times, and ſome part or parts remain, thoſe parts are called the Remainder.

RULE.

### RULE.*

Firſt write down your dividend, making two crooked lines or lunales at the end thereof, that on the left hand to contain the diviſor and that on the right for the quotient; and having placed your diviſor on the left, diſtinguiſh with a point off ſo many places of your dividend from the left towards the right hand as will contain the diviſor once at leaſt, and aſk how often the diviſor is contained in the ſaid ſum, the anſwer muſt be placed in the quotient; then multiply the diviſor by the figure laſt placed in the quotient, ſetting your product under the aforeſaid diſtinguiſhed ſum, and drawing a line under both, take the lower from the higher, and to the remainder bring down the next figure of the dividend, with which proceed as you did with your diſtinguiſhed number; and ſo on, till you have brought down all the figures of the dividend, and the work is done.

*Remarks.*

---

* According to the rule, we reſolve the dividend into parts, and find by trial the number of times the diviſor is contained in each of thoſe parts; the only thing then which remains to be proved is, that the ſeveral figures of the quotient, taken as one number according to the order in which they are placed, is the true quotient of the whole dividend by the diviſor; which may be thus demonſtrated.

DEMONSTRATION. The complete value of the firſt part of the dividend, is, by the nature of notation, 10, 100, or 1000, &c. times the value of which it is taken in the operation; according as there are 1, 2, 3, &c. figures ſtanding before it, and conſequently the true value of the quotient figure belonging to that part of the dividend, is alſo 10, 100, or 1000, &c. times its ſimple value. But the true value of the quotient figure belonging to that part of the dividend, as found by the rule is alſo 10, 100, or 1000, &c. times its ſimple value; for there are as many figures ſet before it as the number of remaining figures in the dividend; and therefore this firſt quotient figure taken in its complete value from the place it ſtands in, is the true quotient of the diviſor in the complete value of the firſt of the dividend. For the ſame reaſon all the reſt of the figures of the quotient, taken according to their places, are each the true quotients of the diviſor in the complete value of the ſeveral parts of the dividend belonging to each; becauſe, as the firſt figure on the right hand of each ſucceeding part of the dividend has a leſs number of figures by one ſtanding before it, ſo in like manner have their quotients; and conſequently taking all the quotient figures in order, as they are placed by the rule, they make one number, which is equal to the ſum of the true quotients of all the ſeveral parts of the dividend; and this therefore, is the true quotient of the whole dividend by the diviſor. Q. E. D.

To leave no obſcurity in this demonſtration, I ſhall illuſtrate it by an example; in which I ſhall ſet down the ſeveral parts of the dividend and quotient, according to their true values; for this purpoſe let 8560 be divided by 36, and the work will ſtand thus:

Diviſor

*Remarks.* 1. Every remainder muſt be leſs than the Diviſor. 2. There will be as many figures in the quotient as there are points in the dividend, more one. 3 If in any caſe the remainder be ſo ſmall, that when the figure of the dividend joined with it makes a ſum leſs than the diviſor, then a cypher is to be placed in the quotient, and another figure brought down, and then proceed as before.

*Proof.* Multiply the quotient by the diviſor, to the product add the remainder, if any, the ſum will be equal to the dividend, when the work is right.

Second method of proof, by addition.

*Rule.* Add your laſt remainder and thoſe products which ariſe by multiplying your quotient figures and diviſors together, as they ſtand in their proper places as you do a ſum in addition of integers, which ſum will be the ſame as your dividend, if right.

| | | | |
|---|---|---|---|
| Diviſor 36 ) | 8560 | dividend | |
| Firſt part of the dividend | 8500 | | |
| 36×200= | 7200 | ---- 200 | Firſt quotient |
| Firſt remainder | 1300 | | |
| add | 60 | | |
| Second part of the dividend | 1360 | | |
| 36×30= | 1080 | ---- 30 | Second quotient |
| Second remainder | 280 | | |
| add | 0 | | |
| Third part of the dividend | 280 | | |
| 36×7= | 252 | ---- 7 | Third quotient. |
| Laſt remainder | 28 | Anſwer 237 | Sum of the quotients. |

**Note.** When there is no remainder to a diviſion, the quotient is the abſolute and perfect anſwer to the queſtion: but when there is a remainder it may be obſerved that it goes ſo much towards another time as it approaches the diviſor.

In all Diviſions, as one is to the diviſor, ſo is the quotient to the dividend; wherefore in whole numbers the quotient muſt always be leſs than the dividend; yet in Fractions it always muſt be greater when the diviſor is a proper fraction: thus 36 divided by 4, the quotient is 9, leſs than 36; becauſe ſince 1 : 4 :: 9 : 36; 1 being leſs than 4, 9 muſt be leſs than 36; but if $\frac{4}{5}$ were to be divided by $\frac{3}{4}$ the quotient would be $\frac{16}{15}$, which is greater than the dividend $\frac{4}{5}$, becauſe, as $1 : \frac{3}{4} :: \frac{16}{15} : \frac{4}{5}$. (i. e) As 1 is to the diviſor, ſo is the quotient to the dividend; but 1 is greater than $\frac{3}{4}$ or $\frac{1}{2}$, wherefore $\frac{16}{15}$ muſt alſo be greater than $\frac{4}{5}$.

Third method of proof, by diviſion.

*Rule.* Subtract the remainder, if any, from the dividend, divide the difference by the quotient; the new quotient will be equal to the diviſor, if right.

Fourth method of proof, by caſting out the nines.

*Rule.* Caſt out the nines from the diviſor and quotient, place the exceſſes at the right and left ſides of a croſs; reject the nines out of the product of theſe exceſſes, and add the remainder, if any, to this laſt exceſs, reject the nines out of the ſame, and place this at the top of the croſs, then caſt the nines out of the dividend, place the exceſs at the bottom of the croſs, if this be equal to the figure at the top, the work is right.

## EXAMPLES IN SHORT DIVISION.

| | *Anſwer,* | *Rem.* |
|---|---|---|
| 1. Divide 3763436724540 by 2. | 1881718362270. | |
| 2. Divide 176894376346376 by 3. | 58964792115458 | 2. |
| 3. Divide 123456789013767 by 4. | 30864197253441 | 3. |
| 4. Divide 367345678914675 by 5. | 73469135782935. | |
| 5. Divide 2345678964 by 6. | 390946494. | |
| 6. Divide 2345678964 by 7. | 335096994 | 6. |
| 7. Divide 9876543210 by 8. | 1234567901, | 2. |
| 8. Divide 1357975305 by 9. | 150886145. | |
| 9. Divide 123456789 by 10. | 12345678, | 9. |
| 10. Divide 65491732 by 11. | 5953793, | 9. |
| 11. Divide 570196382 by 12. | 47516365, | 2. |

When the diviſor is any number not exceeding twelve,

*Rule.* Find how often it is contained in the firſt figure or number of the dividend; ſet it down under the figure you divided, and carry the overplus, if any, to the next in the dividend, as ſo many tens; then find how often the diviſor is contained therein, ſet it down, and continue the ſame till you have gone through the dividend.

### EXAMPLES IN LONG DIVISION.

Diviſor 426 ) 7890123 ( 18521 Quotient.

426···· 426×

3630 111133

3408 37049

74085

2221

2130 7890123 = dividend, proof.

Proved by Addition thus:

912 426

852 3408

2130

603 852

426 426

177 laſt remainder.

remains 177

The ſame proved by Diviſion, 7890123 = dividend,

7890123 dividend Proof.

Subtract 177 laſt remainder.

18521 ) 7889946 ( 426 = diviſor, Proof.

74084

48154

37042

111126

111126

The ſame Example proved by rejecting the nines.

3

exceſs of nines in the quotient 8 X 3 exceſs of nines in the diviſor.

exceſs of nines in the dividend 3 proof.

exceſs of nines of 3×8=6 to which add

the laſt remainder 177 *See the rules for theſe methods, pages* 12 & 21.

the nines out of the ſum 183=3 place at the top.

| | *Anſwer,* | *Rem.* |
|---|---|---|
| 2. Divide 7210473 by 37. | 194877 | 24. |
| 3. Divide 736473575 by 25. | 29458943. | |
| 4. Divide 35730972 by 84. | 25368 | 60. |

5. Divide

| | *Anſwer,* | *Remainder.* |
|---|---|---|
| 5. Divide 63463902247 by 3065. | 20706003 | 3052. |
| 6. Divide 1204530760 by 7489. | 160840. | |
| 7. Divide 30891829676 by 759. | 40700697 | 653. |
| 8. Divide 392628787 by 61745. | 6358 | 54077. |
| 9. Divide 293839455936 by 8405. | 34960078 | 346. |
| 10. Divide 352607193214 by 210472. | 1672940, | 165534. |

It is ſometimes difficult to determine the quotient figure without many trials, we ſhall therefore give an eaſy and infallible rule to determine the quotient figure by inſpection.

*Rule.* Firſt, try how often the firſt figure of the diviſor is contained in the firſt figure or firſt two figures of the dividend, allowing a ſufficient remainder to be placed mentally before the next figure of the dividual, to admit the ſecond figure of the diviſor to go ſo often in this as the firſt figure was taken, ſtill allowing a ſufficient remainder here to be joined mentally to the following figure of the dividual as before, to admit the third figure of the diviſor to go as often as the firſt was taken; and ſo on.

Note, if after the firſt figure of the diviſor or any ſucceeding one has been tried, if the remainder be equal to or greater than the number of times or quotient figure you make trial with, it will go ſo often, and the trial may ceaſe there; and ſo long as the remainder or remainders keep leſs, you are never ſure until you have made trial with every figure of the diviſor.

A ſecond rule by which you may certainly know what figure to ſet in your quotient, and never take one too great or too little, but that which will juſt ſerve; and alſo how to perform, with eaſe and certainty, the moſt difficult ſum that can be propoſed in diviſion, without the aſſiſtance of multiplication, only by addition and ſubtraction, not burthening the memory.

In the practice of diviſion there is nothing more difficult to beginners than in large ſums (eſpecially if the firſt figures of the diviſor be either 1, 2, or 3, and the laſt figures, 7, 8, 9,) to know certainly what figure to put in the quotient, when it is demanded how often the diviſor may be had in the dividend, for the certain finding thereof, (a little pains being taken before you begin your work) do thus; ſuppoſe you are to divide any ſum, as 1097909 by 309; firſt ſet down the 9 digits 1, 2, 3, &c. one under another, and againſt the figure 1 ſet 309 your diviſor, which doubled is

618, and ſet againſt 2, theſe added together make 927, which ſtands againſt 3; add the diviſor 309 to to 927, it makes 1236, which is againſt 4; to this add the diviſor, and it makes 1545, which ſtands againſt 5, and thus to every laſt number ſtill add the diviſor till you have gone through all the nine digits, then will they be as follows.

Dividend.
Diviſor 309 ) 1097909 ( 3553 Quotient.
927
1709
1545
1640
1545
959
927
Remainder 32

| | |
|---|---|
| 1 | 309 |
| 2 | 618 |
| 3 | 927 |
| 4 | 1236 |
| 5 | 1545 |
| 6 | 1854 |
| 7 | 2163 |
| 8 | 2472 |
| 9 | 2781 |

| | Anſwer, | Remainder. |
|---|---|---|
| 1. Divide 38161732196 by 784617. | 48637, | 315167. |
| 2. Divide 7456818 by 486. | 15343, | 120. |
| 3. Divide 2762634 by 642. | 4303, | 108. |
| 4. Divide 363914567345 by 32041. | 11357778, | 2447. |
| 5. Divide 79165238 by 238. | 332627, | 22. |
| 6. Divide 29137062 by 5317. | 5479, | 5219. |
| 7. Divide 62015735 by 7803. | 7947, | 5294. |
| 8. Divide 74855092410 by 949998. | 78795. | |
| 9. Divide 1467286 by 6425. | 228, | 2386. |
| 10. Divide 4673876496 by 6743. | 693144 | 6504. |

Having prepared this table, ſet your dividend and diviſor down as in the firſt method in diviſion pointing the dividend, and draw a line under it as there directed, and as you ſee in the above work, then lay your little table before you; look in it for 1097, the firſt four figures in the dividend, which you cannot exactly find there, but the neareſt number leſs, (which you muſt always take when you cannot find the juſt number you look for) is 927 againſt which ſtands 3; ſet 3 in the quotient, and ſubtract 927 from 1097 and there will remain 170, to which bring down 9, the next figure of your dividend, and it is 1709; look for this number in your table, which you cannot find, but the next leſs is 1545, againſt which ſtands 5; ſet 5 in the quotient, and ſubtract 1545 from 1709, and there will remain 164, to which bring down the next figure of your dividend, (which here is 0) making it 1640; look for this 1640 in the table, which you cannot find, but the next leſs is 1545, againſt which ſtands 5, ſet 5 in the quotient, and ſubtract 1545 from 1640, and there will remain 95; to which bring down the laſt figure in your dividend, which is 9, making it 959; look for this number in the table, or the next leſs 927, againſt which ſtands 3; ſet 3 in the quotient, and ſubtract 927 from 959, the remainder is 32, ſo is your dividend ended, and the quotient is $3553\frac{32}{109}$, and with what eaſe and certainty this is effected, no multiplication being uſed, is left to the reader to judge.

## *Contractions in Diviſion.*

*Caſe* 1ſt. When there are cyphers on the right hand of the diviſor.

*Rule.* Point from the right hand of the dividend as many places as there are cyphers in the diviſor, as alſo the cyphers on the right hand of the diviſor.

Find the quotient ariſing from the places on the left of the points for the quotient required.

To the remainder annex the figures cut off in the dividend for the true remainder.

### EXAMPLES.

Divide 376732139 by 20.

2|0)37673213|9

18836606 Remainder 19.

| | *Anſ.* | *Rem.* |
|---|---|---|
| 2. Divide 310869017 by 7100 | 43784 | 2617 |
| 3. Divide 7380964 by 2300 | 3209 | 264 |
| 4. Divide 2304109 by 5800 | 397 | 1509 |
| 5. Divide 9674867486 by 367000 | 26362 | 13486 |

*Caſe* 2d. When the diviſor is an unit with cyphers.

*Rule.* Point off in the dividend as many places as there are cyphers in the diviſor; thoſe on the left of the point will be the quotient, and thoſe on the right thereof the remainder.

EXAMPLES.

### EXAMPLES.

| | *Anſ.* | *Rem.* |
|---|---|---|
| 1. Divide 987654321 by 100 | 98765432 | 21 |
| 2. Divide 123456789 by 1000 | 123456 | 789 |

*Caſe* 3d. When the diviſor is any number not exceeding 12, with any number of cyphers annexed, the operation may be performed mentally, and the quotient ſet under the dividend, ſo that the left hand figure may ſtand under the right of its dividual.

### EXAMPLES.

| | *Anſ.* | *Rem.* |
|---|---|---|
| 1. Divide 416131321 by 300 | 1387104 | 121 |
| 2. Divide 45678912 by 400 | 114197 | 112 |
| 3. Divide 987654321 by 1200 | 823045 | 321 |

*Caſe* 4th. When the diviſor is the product of two or more numbers, each not greater than 12, it will be found more eaſy and expeditious to divide continually by thoſe numbers, than by the whole diviſor at once.

*Note.* It is neceſſary we ſhould ſhew how the laſt or true remainder in queſtions ſolved by this method is found.

*Rule.* Multiply the diviſor by the quotient, and ſubtract that product from the dividend; the reſult will be the true remainder. Or thus:

If there be any remainders after ſuch diviſions, multiply the laſt remainder by the preceding diviſor, and to the product add the remainder belonging to the ſame, then multiply the ſum by the next preceding diviſor, and to the product add its correſponding remainder; proceed in the ſame manner throughout all the diviſors and remainders; ſo ſhall the laſt ſum be the remainder, the ſame as if the diviſion had been performed by one operation.

### EXAMPLES.

1. Divide 42901685 by 96

8)4290685

12)536335 $\frac{5}{8}$

44694 $\frac{61}{96}$

*Note.* The component parts are 8 and 12.

2. Divide

| | Anſ. | Rem. |
|---|---|---|
| 2. Divide 5840 by 48, or 6 times 8 | 121 | 32 |
| 3. Divide 31046835 by 56, or 7 and 8 | 554407 | |
| 4. Divide 5130652 by 132, or by 12 and 11 | 38868 | 76 |
| 5. Divide 83016572 by 240 | 345902 | 92 |
| 6. Divide 7014596 by 72, or 9 and 8 | 97424 | 68 |
| 7. Divide 2143434 by 36, or by 4 and 9 | 59539 | 30 |
| 8. Divide 5645785 by 315, or by 9, 5, and 7 | 17923 | 40 |
| 9. Divide 76849 by 45, or 5 and 9 | 1707 | 34 |
| 10. Divide 654369 by 27, or by 3 and 9 | 24235 | 24 |

## CHAP. V.

## TABLE OF COINS, WEIGHTS, AND MEASURES.

THE things expreſſed by numbers, are Money, Weight, Meaſure, Time, &c. of the three firſt of theſe, there are almoſt infinite kinds and varieties, according to the diverſity of the ſeveral nations wherein they are uſed, which would be uſeleſs to introduce in this place; we ſhall therefore, for the preſent, content ourſelves with the money, weights, and meaſures which are generally uſed in the united kingdoms, viz. England, Ireland, and Scotland.

### COINS.*

In theſe countries, accounts are kept in pounds, ſhillings, pence, and farthings.

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* In the firſt ages of commerce there was little occaſion for computation, as one commodity was bartered for another by the bulk, a cuſtom which even at this day prevails among the ſavage unpoliſhed nations of Chili,

A farthing is the leaſt coin current in Ireland, marked thus, *q*.

An half-penny, equal in value to two farthings,

*(a)* A penny, equal in value to four farthings, marked thus, *d*.

*(a)* A ſhilling, equal in value to twelve pence, marked thus, *s*.

An Engliſh ſhilling paſſes for 13*d*. Iriſh.

An half-crown, value two ſhillings and eight pence halfpenny.

A crown, value five ſhillings and five pence.

An half-guinea, value eleven ſhillings and four pence halfpenny.

A guinea, value twenty-two ſhillings and nine pence.

*(a)* A pound ſterling, value twenty ſhillings, marked thus, £.

*(a)* Thus marked, are imaginary coins, there being no pieces of money now current of their exact value.

## TABLE OF COIN.

4 Farthings make one penny.
12 Pence one ſhilling.
20 Shillings one pound.

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Chili, on the South Sea, in the land of Jeſſo, on the Pacific Ocean, and other barbarous countries; but by degrees, as improvements were made in the world, ſomething new was added daily to the conveniencies of life; and as ſuch a method of bartering commodities was found to be difficult and inconvenient, it was agreed among mankind to make choice of one commodity, which being in general and conſtant eſteem, an equivalent quantity of it might always remove the difficulty of bartering in kind. To determine, therefore, this ſubſtance that ſhould be in univerſal and conſtant eſteem, they made choice of gold and ſilver, not only becauſe diviſible and portable, but becauſe they were more valuable than any other metals.—— Since there was a conſiderable difference in the nature of theſe two metals, and gold was more precious than ſilver, both on account of its rarity and intrinſic worth; beſides the expenſe in working gold far exceeds the charge which attends the working of ſilver, as appears by the tax paid upon each to the ſovereigns of mints, that upon gold being only five of the hundred, and that on ſilver twenty; it was therefore juſt to aſcribe the greater value to gold; and becauſe the baſer metal ought to be given in greater meaſure, that what was wanting in quality might be made up in quantity.

### *Pence Table.*

| d. | s. | d. | d. | s. | d. |
|---|---|---|---|---|---|
| 12 — | 1 | 0 | 70 — | 5 | 10 |
| 20 — | 1 | 8 | 80 — | 6 | 8 |
| 24 — | 2 | 0 | 84 — | 7 | 0 |
| 30 — | 2 | 6 | 90 — | 7 | 6 |
| 36 — | 3 | 0 | 96 — | 8 | 0 |
| 40 — | 3 | 4 | 100 — | 8 | 4 |
| 48 — | 4 | 0 | 110 — | 9 | 2 |
| 50 — | 4 | 2 | 120 — | 10 | 0 |
| 60 — | 5 | 0 | 240 — | 20 | 0 |

### *Pounds Table.*

| s. | l. | s. |
|---|---|---|
| 20 — | 1 | 0 |
| 30 — | 1 | 10 |
| 40 — | 2 | 0 |
| 50 — | 2 | 10 |
| 60 — | 3 | 0 |
| 70 — | 3 | 10 |
| 80 — | 4 | 0 |
| 90 — | 4 | 10 |
| 100 — | 5 | 0 |

### *Table of Guineas.*

| | | l. | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Guinea | 1 | 2 | 9 | 11 | Guineas | 12 | 10 | 3 |
| 1½ | —— | 1 | 14 | 1½ | 11½ | —— | 13 | 1 | 7½ |
| 2 | —— | 2 | 5 | 6 | 12 | —— | 13 | 13 | 0 |
| 2½ | —— | 2 | 16 | 10½ | 12½ | —— | 14 | 4 | 4½ |
| 3 | —— | 3 | 8 | 3 | 13 | —— | 14 | 15 | 9 |
| 3½ | —— | 3 | 19 | 7½ | 13½ | —— | 15 | 7 | 1½ |
| 4 | —— | 4 | 11 | 0 | 14 | —— | 15 | 18 | 6 |
| 4½ | —— | 5 | 2 | 4½ | 14½ | —— | 16 | 9 | 10½ |
| 5 | —— | 5 | 13 | 9 | 15 | —— | 17 | 1 | 3 |
| 5½ | —— | 6 | 5 | 1½ | 15½ | —— | 17 | 12 | 7½ |
| 6 | —— | 6 | 16 | 6 | 16 | —— | 18 | 4 | 0 |
| 6½ | —— | 7 | 7 | 7½ | 16½ | —— | 18 | 15 | 4½ |
| 7 | —— | 7 | 19 | 3 | 17 | —— | 19 | 6 | 9 |
| 7½ | —— | 8 | 10 | 7½ | 17½ | —— | 19 | 18 | 1½ |
| 8 | —— | 9 | 2 | 0 | 18 | —— | 20 | 9 | 6 |
| 8½ | —— | 9 | 13 | 4½ | 18½ | —— | 21 | 0 | 10½ |
| 9 | —— | 10 | 4 | 9 | 19 | —— | 21 | 12 | 3 |
| 9½ | —— | 10 | 16 | 1½ | 19½ | —— | 22 | 3 | 7½ |
| 10 | —— | 11 | 7 | 6 | 20 | —— | 22 | 15 | 0 |
| 10½ | —— | 11 | 18 | 10½ | 30 | —— | 34 | 2 | 6 |

*Troy*

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It was likewiſe found neceſſary to fix a proportion between them by ſome certain and determined rule; hence it is, that, in the practice of commerce, (though formerly the proportion of gold to ſilver was ſettled as ten to one, yet

# TROY WEIGHT.[*]

## *The Table.*

|  |  |  |  | marked |
|---|---|---|---|---|
| 24 | Grains | make | 1 penny-weight. | *dwt.* |
| 20 | Penny-wts. | —— | 1 ounce. | *oz.* |
| 12 | Ounces | —— | 1 pound. | *lb.* |

Grs.
24 = 1 dwt.
480 = 20 = 1 oz.
5760 = 240 = 12 = 1 lb.

*Note* 1ſt. In philoſophical experiments, the proportion of the gravity of bodies is tried by Troy weight.

*Note* 2d. One grain of Troy weight is found by experiment to be equal in weight to one grain and an half of ſound dry wheat.

By this weight, gold, ſilver, jewels, electuaries, and all liquors, are weighed.

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yet the matter is ſo ſettled at preſent, throughout the greateſt part of Europe) that one ounce of gold is worth about fifteen ounces of ſilver. When theſe metals were agreed upon at firſt to be a common equivalent for any of the conveniences of life, the particular quantity of them to be given as the value of any thing elſe, was determined by the bulk and weight; but afterwards, to ſave the trouble of proving this weight upon every occaſion, it was coined in certain forms by public authority, and impreſſed with a mark of diſtinction, expreſſing the quantity each piece contained, ſo that it ſhould always have the ſame determined value, and be every where the ſame both for matter and weight.

[*] A pound, Troy weight, regulates and preſcribes a form how to keep the money of this realm at a certain ſtandard. About 200 years before the Conqueſt, Oſbright, a Saxon, being then King of England, cauſed an ounce Troy of ſilver to be divided into twenty pieces, at the ſame time called pence, and ſo an ounce of ſilver at that time was worth no more than 20 pence, or 1s. 8d. which continued at the ſame value until the time of Henry VII. who (in regard of enhancing of money in foreign parts) valued the ſame at thirty pence, ſo that then there were accordingly thirty pieces made out of the ounce; and the old pieces went then for three halfpence till the time of Edward IV. who valued the ounce at 40d. and then the old pieces at 2d. each. After this, Henry VIII. valued the ounce of Sterling ſilver at 45d. which value continued till Queen Elizabeth's time, who valued the ſame old pieces at 3d. each; ſo that all three pences coined by the ſame queen weighed but a penny-weight, and every 6d. two penny-weights, and the ſhillings and the other pieces proportionally, which made the ounce Troy of ſilver to be valued at 60d. or 5s. as it remains at this day.—See ſtatutes made and provided in this caſe.

Gold, when perfectly pure is found to be nearly 19½ times heavier than rain water, and much heavier than any other metal, it is then ſaid to be twenty-four carats fine; (a carat being eſtimated as the twenty-fourth part of any thing or weight,) but ſtandard or ſterling gold contains twenty-two parts of pure gold, and two of copper; or a pound of ſterling gold contains eleven ounces of pure gold and one of copper; and the ſame proportion of pure ſilver and copper, produces ſterling ſilver. By ſterling gold or ſilver is meant, that of which our gold and ſilver coins, as well as utenſils offered for ſale in goldſmiths' ſhops, are, or ſhould be made. A carat among jewellers is twenty-four grains, as in diamonds, pearls, &c. at other times it is the one twenty-fourth part of an ounce, = 20 grains, as in gold, &c. but when the jewellers would be very exact, they ſubdivide the grain into twenty mites, each mite into twenty perrots, and each perrot into twenty-four blanks.

## APOTHECARIES' WEIGHT.

The only difference between this and Troy weight conſiſts in the diviſion of the ounce; the grain, ounce, and pound being the ſame in both.

*The Table.*

| | | | | *Thus marked.* |
|---|---|---|---|---|
| 20 | Grains | make | 1 Scruple. | ℈ |
| 3 | Scruples | — | 1 Dram. | ʒ |
| 8 | Drams | — | 1 Ounce. | ℥ |
| 12 | Ounces | — | 1 Pound. | *lb.* |

Their value and proportion.

Grains.

20 = 1 Scruple.

60 = 3 = 1 Dram.

480 = 24 = 8 = 1 Ounce.

5760 = 288 = 96 = 12 = 1 Pound.

Apothecaries

As the ſecurity of Commerce depends much on the juſtneſs of weights, moſt nations have taken care to prevent their being falſified. The ſtandard of weights in Great Britain is kept in the Exchequer, by a particular officer called the Clerk or Comptroller of the Market. By the 27th chapter of Magna Charta, the weights are to be the ſame all over England; but as commerce flouriſhed and introduced a greater variety of commodities, it was found convenient that beſides the original weights others ſhould be invented better calculated for diſpatch in buſineſs, which hath introduced a diverſity of weights in almoſt every different county or province. The firſt of the weights uſed in Great Britain and Ireland was a grain of wheat picked out of the middle of the ear, which being well dried became the leaſt denomination of Troy-weight, now uſed for gold, ſilver, jewels, ſeeds, liquors, bread, and the ſale of medicines.

Apothecaries in making up their medicines uſe this weight but buy their drugs by Avoirdupoiſe weight.

## AVOIRDUPOISE WEIGHT.*

| *The Table.* | | | | | *Thus marked.* |
|---|---|---|---|---|---|
| 16 | Drams | make | 1 | Ounce. | *oz.* |
| 16 | Ounces | —— | 1 | Pound. | *lb.* |
| 14 | Pounds | —— | 1 | Stone. | *ſt.* |
| 16 | Pounds | —— | 1 | Stone of wool. | |
| 7 | Stone | —— | 1 | Hundred wt. do. | |
| 2 | Stone, or 28*lb.* | | 1 | Quarter. | *qr.* |
| 4 | Quarters, or 112*lb.* | | 1 | Hundred wt. | *cwt.* |
| 20 | Hundred | —— | 1 | Ton. | *Ton.* |
| 20 | Stone of wheat, rye, oatmeal, &c. | | 1 | Barrel. | |
| 16 | St. of bere or barley | | 1 | Barrel. | |
| $7\frac{1}{2}$ | Hundred of ſtones | | 1 | Load. | |

The proportions are

Drams.

16= 1 Ounce.

256= 16= 1 Pound.

7168= 448= 28= 1 Quarter.

28672= 1792= 112= 4= 1 Hundred.

573440=35840=2240=80=20=1 Ton.

## LIQUID MEASURE.†

| *The Table.* | | | | | *Thus marked.* |
|---|---|---|---|---|---|
| 4 | Naggins | make | 1 | Pint. | *Pt.* |
| 2 | Pints | —— | 1 | Quart. | *Qt.* |
| 4 | Quarts | —— | 1 | Gallon. | *Gal.* |
| 63 | Gallons | —— | 1 | Hogſhead. | *Hhd.* |
| 4 | Hogſheads | —— | 1 | Ton. | *Ton.* |
| $18\frac{1}{2}$ | Gallons | —— | 1 | Runlet. | |

31 Gallons

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* This weight was rather introduced by chance, and confirmed by cuſtom, than fixed by any law. The Troy weight was in practice found to be too ſmall for coarſe and heavy goods, ſuch as grocery ware, pitch, tar, roſin, wax, tallow, flax, hemp, &c. copper, tin, iron, lead, ſteel, fiſh, fleſh, butter, cheeſe, ſalt, &c. for which, and other ſuch goods, it was thought proper to allow a greater weight than the law had provided, which in this weight exceeds the Troy by one ſixth, one pound Avoirdupoiſe being equal to one pound two ounces eleven penny-weights fifteen grains and a half Troy. In lead they give only nineteen hundred weight and a half to the ton or fodder.

† All meaſures of capacity, both liquid and dry, were at firſt made from Troy weight. Vide ſtatutes, 9 & 51 Henry III. 12 Henry VII. &c. wherein it is enacted, that eight pounds Troy weight of wheat, gathered out of the middle of the ear, and well dried, ſhould make one gallon of wine meaſure;

and

| | | | | *Thus marked.* |
|---|---|---|---|---|
| 31 | Gallons | make | 1 Barrel or quarter caſk of wine, oil, or honey. | |
| 42 | Gallons | —— | 1 Tierce, | *Tier.* |
| 84 | Gallons | —— | 1 Puncheon, | *Pun.* |
| 40 | Gallons | —— | 1 Barrel of ale, | *Bar.* |

The proportions are as follow:

Naggins.
4= 1 Pint.
8= 2= 1 Quart.
32= 8= 4= 1 Gallon.
2016= 504= 252= 63=1 Hhd.
8064=2016=1008=252=1 Ton.

## DRY MEASURE.

| | | *The Table.* | | *Thus marked.* |
|---|---|---|---|---|
| 2 | Pints | make | 1 Quart, | *Qt.* |
| 2 | Quarts | —— | 1 Pottle, | *Pot.* |
| 2 | Pottles | —— | 1 Gallon, | *Gal.* |
| 2 | Gallons | —— | 1 Peck, | *Peck.* |
| 4 | Pecks | —— | 1 Buſhel, | *Buſh.* |
| 4 | Buſhels | —— | 1 Barrel, | *Bar.* |
| 2 | Barrels | —— | 1 Quarter, | *Qr.* |
| 8 | Barrels | —— | 1 Ton of coals, | *Tn.* |
| 4 | Quarters | —— | 1 Chaldron, | *Chal.* |
| 5 | Quarters | —— | 1 Whey or load | *W.* |
| 2 | Wheys | —— | 1 Laſt, | *Laſt.* |

By

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and that there ſhould be but one meaſure for wine, ale, and coin throughout this realm. Vide ſtat. 14 Edward III. 15 Richard II. But time and cuſtom have altered meaſures, as they have done weights, (and perhaps for one and the ſame reaſon,) for now we have three different meaſures, viz, one for wine, one for ale or beer, and one for coin.

The common wine gallon, ſealed at Guild-hall in London, by which all wines, brandies, ſpirits, ſtrong waters, mead, perry, cyder, vinegar, oil, honey, &c. are meaſured and ſold, is ſuppoſed to contain 231 cubic inches; and from thence the reſt are computed, as in the above table.

Note, $31\frac{1}{2}$ gallons are a wine or vinegar barrel, and 236 gallons are a ton of ſweet oil. The beer or ale gallon (which are both one) is much larger than the wine gallon; it being probably made at firſt to correſpond with Avoirdupoiſe weight, as the wine gallon did with Troy weight; for one pound Avoirdupoiſe weight being nearly equal to 14 ounces, 12 pennyweights Troy, and as one pound Troy is in proportion to the cubic inches in a wine gallon, ſo is one pound Avoirdupoiſe weight to the cubic inches in an ale gallon; that is, As $12 : 14\frac{1}{2}\frac{2}{15} :: 231 : 282$ nearly. A barrel of ale ſhould contain 40 gallons only, yet it uſually holds 42.

The gallon appointed to be uſed for meaſuring all kinds of liquids in Ireland is $217\frac{6}{10}$ cubic inches; in England the wine gallon is 231, and the beer or ale gallon 282 cubic inches, as was ſaid before.

By the ſtandard in his Majeſty's Exchequer, every round buſhel with a plain and even bottom, $18\frac{1}{2}$ inches wide throughout, and 8 inches deep, is eſteemed a legal corn buſhel, and will contain $2150\frac{21}{50}$ cubic inches; conſequently the corn gallon contains $268\frac{4}{5}$ cubic inches.

## CLOTH MEASURE.

Cloth meaſure is a ſpecies of Long Meaſure; the higheſt denomination is an ell, although the yard is the moſt common meaſure which is generally uſed in meaſuring all ſorts of cloths, rug, ſilks, moſt linens, tape, and gartering. The ell Engliſh is uſed only for meaſuring ſome linens called hollands: the ell Flemiſh is uſed in meaſuring tapeſtry.

*The Table.*

| | | | | | *Thus marked.* |
|---|---|---|---|---|---|
| $2\frac{1}{4}$ | Inches | make | 1 | Nail, | *nl.* |
| 4 | Nails | —— | 1 | Quarter of a yard | *qr.* |
| 3 | Quarters | —— | 1 | Ell Flemiſh, | *F.E.* |
| 4 | Quarters | —— | 1 | Yard, | *yd.* |
| 5 | Quarters | —— | 1 | Ell Engliſh, | *E.E.* |
| 6 | Quarters | —— | 1 | Ell French, | *Fr.E.* |

Their value and proportion.

Inches.
$2\frac{1}{4}$ = 1 Nail.
9 = 4 = 1 Quarter.
36 = 16 = 4 = 1 Yard.
27 = 12 = 3 = 1 Ell Flemiſh.
45 = 20 = 5 = 1 Ell Engliſh.
54 = 24 = 6 = 1 French Ell.

## LONG MEASURE.

By this kind of meaſure the diſtances of places, one from another, are determined; likewiſe the dimenſions of bodies, when their ſurfaces and ſolid capacities are to to be aſcertained: of which meaſure, according to the ſtatute of 33 EDWARD I. and 25 ELIZABETH, a barley-corn is the leaſt.

*The Table.*

| | | | | |
|---|---|---|---|---|
| 3 | Barley corns | make | 1 | Inch. |
| 3 | Inches | — — | 1 | Palm. |
| 4 | Inches | — — | 1 | Hand uſed in meaſuring horſes. |
| 12 | Inches | — — | 1 | Foot. |

18 Inches make 1 Cubit.
2 Feet 6 Inches — 1 Military Pace.
5 Feet — — 1 Geometrical Pace.
3 Feet — — 1 Yard.
6 Feet — — 1 Fathom, uſed in meaſuring depths.
5½ Yards — — 1 Engliſh Pole or Perch.
7 Yards — — 1 Iriſh Pole or Perch.
40 Perches — — 1 Furlong.
8 Furlongs — — 1 Mile.
1760 Yards — — 1 Engliſh Mile.
2240 Yards — — 1 Iriſh Mile.
2038⅔ Yards — — 1 Sea Mile.
3 Miles — — 1 League.
60 Sea Miles — — 1 Geographical Degree.
$54\frac{17}{58}$ Iriſh Miles — — 1 Degree.
69½ Engliſh Miles — 1 Degree.
360 Degrees (or 25000 miles nearly) make the circumference of the Earth.

Their value and proportion.

Barley-corns.
3= 1 Inch.
36= 12= 1 Foot.
108= 36= 3 = 1 Yard.
594= 198= 16½= 5½= 1 Engliſh Perch.
756= 252= 21 = 7= 1 Iriſh Perch.
23760= 7290= 666 = 220= 40=1 Engliſh Furlong.
30240=10080= 840 = 280= 40=1 Iriſh Furlong.
190080=63360=5280 =1760=320=8=1 Engliſh Mile.
241920=80640=6720 =2240=320=8=1 Iriſh Mile.

Long meaſure is ſuppoſed to have originated from a barley corn, of which three were to make an inch: but as theſe were not always of the ſame length, neceſſity induced an invariable ſtandard, whereby to determine all lengths, which ſtandard, as ratified by Parliament, is now in the Tower of London, from whence aroſe the ſeveral denominations.

## SOLID MEASURE.

Not only the meaſure of every thing that hath length, breadth, and depth, but all meaſures of capacity are to be included under this title; being equally meaſures of bulk and ſolidity, whether liquid or dry, whether of one bulk or collective bulks.

| | | | |
|---|---|---|---|
| 1728 | Solid Inches | make | 1 Solid Foot. |
| 27 | Solid Feet | —— | 1 Solid Yard, in digging. |
| 40 | Solid Feet of unhewn timber | | 1 Load or Ton. |
| 50 | Ditto of hewn timber | | 1 Load or Ton. |
| 40 | Ditto Bales, &c. | —— | 1 Ton of ſhipping. |
| 343 | Solid Yards | —— | 1 Solid Perch. |
| 21952 | Solid Yards | —— | 1 Solid Chain. |
| 11239424000 | Solid Yards | | 1 Solid Mile. |

## SQUARE MEASURE.

A ſurveyor's full chain is in length four perches, and is divided into 100 links, the half chain into 50, and the perch into 25 links: an acre of land contains 100,000 ſquare links; but as the Engliſh ſquare perch contains but $10\frac{89}{100}$ ſquare paces, or $30\frac{1}{4}$ ſquare yards, and the Iriſh 49 ſquare yards; ſo the Engliſh acre in quantity differs from the Iriſh in the ſame proportion.

*Note.* A perch of Iriſh maſons' work is 21 feet long, and 1 foot high; having ſometimes no regard to any certain thickneſs; but the ſtandard thickneſs is 18 inches. *Note, alſo;* carpenters, glaziers, painters, and plaiſterers uſually meaſure by the ſquare foot or ſquare yard.

Where length and breadth only are conſidered, the denominations are as follow:

*The Table.*

| | | | | *Thus marked.* |
|---|---|---|---|---|
| 144 | Square Inches | make | 1 Square Foot. | *ft.* |
| 9 | Square Feet | —— | 1 Square Yard. | *yd.* |
| 100 | Square Feet | —— | 1 Square of carpentry. | |
| 49 | Square Yards | —— | 1 Square Perch. | *p.* |
| 40 | Square Perches | —— | 1 Square Rood. | *r.* |
| 4 | Square Roods | —— | 1 Acre, or 160 Perches. | *acre.* |

Square Inches.

144= 1 Square Foot.
1296= 9= 1 Yard.
63504= 441= 49= 1 Perch.
2540160=17640=1960= 40=1 Rood.
10160640=70560=7840=160=4=1 Acre.

You ſee how many of one denomination are equal to another.

## LIME MEASURE.

The veſſel in which Lime is meaſured is a conical fruſtrum; 21 inches diameter at top, 20 inches diameter at bottom; perpendicular depth 6,6 inches nearly, ſlant height 6,7 inches, and contains 2178 cubic inches, avoiding decimal fractions; and this is called a half-hogſhead.

But to come to the full decimal dimenſions, they are, in diameter, 21 inches at top, 20 at bottom, perpendicular depth 6,5974 inches, ſlant height 6,6727, ſo as the ſolid content may be 2178 cubic inches.

## TIME.

> Time of itſelf is nothing; but from thought
> Receives its riſe, by lab'ring fancy wrought;
> From things conſider'd, whilſt we think on ſome
> As preſent, ſome as paſt, or yet to come;
> No thought can think on time, that's ſtill confeſs'd,
> But thinks on things in motion or at reſt.

The motion of the ſhadow of an object on a plane, not anſwering ſufficiently as an exact meaſure for the uniform flux of time, a more certain method of determining the ſpace elapſed from one period of time to another, has been adopted; a thread, 36½ inches long, with a ſmall weight at one end, and ſuſpended by the other end, being ſet in motion, each vibration or ſwing will be the meaſure of the duration of a ſecond: hence the following diviſions;

| | | |
|---|---|---|
| 60 Seconds | make | 1 Minute. |
| 60 Minutes | —— | 1 Hour. |
| 24 Hours | —— | 1 Day. |
| 7 Days | —— | 1 Week. |
| 52 Weeks | —— | 1 Year. |
| 27 days, 12 h. 43 min. | —— | 1 periodical Month. |
| 29 days, 12 h. 44 min. | —— | 1 Synodical Month. |
| 12 Calendar months | —— | 1 Year. |
| 365 day, 5 hours, 48 min. | 57 ſec. | 1 Solar year. |
| 365 days, 6 hours, 9 min. | 141 ſec. | 1 Siderial year. |
| 365 days, 6 hours, | —— | 1 Civil or common year. |

366 days, 1 leap year, which occurs every fourth year, in which caſe the month of February, which in every common year has but 28 days, hath 29 days.

The twelve months are not equal as to their number of days, ſome having thirty, and ſome thirty-one days; the following diſtich committed to memory, will point out the diviſion:

Thirty days have September,
April, June, and November,
February hath twenty-eight alone,
And all the reſt have thirty-one.

Seconds
60= 1 Minute.
3600= 60= 1 Hour.
86400= 1440= 24= 1 Day.
604800= 10080= 168= 7= 1 Week.
2419200= 40320= 672= 28=4=1 Month.
31557600=525960=8766=$365\frac{1}{4}$=1 Year.

Or, 52 Weeks, 1 Day, 6 Hours = 13 Months, 1 Day, 6 Hours = 1 Year.

## DRY MEASURE,

Is uſed to meaſure corn, and other dry goods; but as they are generally ſold by weight in Ireland, this meaſure is little uſed; however, the denominations are as follow, viz.

2 Pints make 1 Quart.
2 Quarts —— 1 Pottle.
2 Pottles —— 1 Gallon.
2 Gallons —— 1 Peck.
4 Pecks —— 1 Buſhel.
4 Buſhels —— 1 Barrel.
2 Barrels —— 1 Quarter.

Dry Meaſure is different both from Wine and Ale Meaſure, which is, as it were, a mean between both, though not exactly ſo; which, upon examination, will be found to be in proportion to the aforeſaid old ſtandard wine gallon, as Avoirdupois Weight is to Troy Weight, nearly, that is, As one pound Troy is to one pound Avoirdupois Weight: ſo are the cubic inches contained in the old Wine Gallon:

to the cubic inches contained in the dry, or corn gallon, viz. 12 : 14 = : : 224 : 2721, the common content of a corn gallon nearly; although now it is otherwiſe ſettled by an act made and provided, in 1697; the words of that act are theſe: "Every round buſhel, with a plain and "even bottom, being made eighteen inches and an half "wide throughout, and eight inches deep, ſhould be "eſtimated a legal Wincheſter buſhel, according to the "ſtandard in his Majeſty's Exchequer." Now, a veſſel thus made, will contain 2150,42 cubic inches, conſequently the Corn Gallon doth contain but 2681 cubic inches.

# CHAP. VI.

## COMPOUND ADDITION.

### *Of Money.*

The operation of this rule is evident, from what has been ſaid in Simple Addition; for in Addition of Money, as 4 in the place of farthings make one in the place of pence; 12 in the place of pence make one in the place of ſhillings; and 20 in the place of ſhillings, make one in the place of pounds; and 10 in the units' figure of pounds, make one in the place of tens, &c. wherefore

*Rule.* For every four farthings carry one to the pence; for every 12 pence carry one to the ſhillings; and for every 20 in the ſhillings, carry one to the pounds; and the pounds, you muſt carry one for every 10, as in the Simple Addition.

| | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 1479 | 14 | 6½ | (ii) | 3768 | 11 | 8½ | (iii) | 16437 | 19 | 11¾ |
| | 7168 | 17 | 4¾ | | 1313 | 16 | 5¾ | | 94321 | 10 | 5½ |
| | 3133 | 14 | 11¼ | | 1972 | 11 | 10¼ | | 61427 | 17 | 8¼ |
| | 3171 | 19 | 10¾ | | 3168 | 16 | 5½ | | 58654 | 12 | 11½ |
| | 14944 | 0 | 9½ | | 10223 | 16 | 6 | | 230842 | 1 | 1 |
| | 13404 | 12 | 2¾ | | 6455 | 4 | 9½ | | 214404 | 1 | 1¼ |
| (iv) | 1523 | 17 | 2½ | (v) | 1234 | 15 | 6¼ | (vi) | 987654 | 13 | 2¼ |
| | 3456 | 18 | 7¼ | | 7891 | 12 | 3½ | | 113245 | 16 | 7½ |
| | 6543 | 12 | 1¾ | | 4567 | 18 | 9¾ | | 891234 | 15 | 6¾ |
| | 1234 | 15 | 6¼ | | 1234 | 15 | 6¼ | | 789123 | 14 | 5½ |
| | 6543 | 12 | 7¾ | | 7891 | 12 | 3¼ | | 678912 | 13 | 4¾ |
| | 19302 | 16 | 1½ | | 22820 | 14 | 5 | | 3459271 | 13 | 2¼ |
| | 17778 | 18 | 11 | | 21585 | 18 | 10¾ | | 2471617 | 0 | 0½ |
| (vii) | 15747 | 16 | 5¼ | (viii) | 54321 | 16 | 7½ | (ix) | 12345 | 16 | 7½ |
| | 22165 | 16 | 7½ | | 12345 | 14 | 2¼ | | 54321 | 17 | 6¼ |
| | 19472 | 19 | 2¾ | | 67891 | 17 | 8¾ | | 12465 | 14 | 2½ |
| | 16732 | 18 | 11¾ | | 19876 | 18 | 7¼ | | 65421 | 12 | 4¾ |
| | 19721 | 14 | 6½ | | 12657 | 14 | 2¾ | | 95765 | 16 | 7½ |
| | 12345 | 16 | 7¼ | | 75621 | 12 | 3½ | | 56789 | 17 | 6½ |
| (x) | 123456 | 14 | 2½ | (xi) | 529712 | 16 | 4½ | (xii) | 127986 | 17 | 6¼ |
| | 789123 | 16 | 7¾ | | 217925 | 14 | 6¾ | | 689721 | 16 | 7½ |
| | 456789 | 14 | 2¼ | | 122456 | 17 | 8½ | | 123456 | 17 | 8½ |
| | 172436 | 17 | 4½ | | 654321 | 18 | 7¾ | | 654321 | 18 | 7¼ |
| | 124765 | 16 | 4½ | | 987621 | 16 | 2¼ | | 119876 | 16 | 2½ |
| | 117659 | 19 | 10½ | | 126789 | 12 | 6¼ | | 679811 | 12 | 6¾ |
| | 987654 | 13 | 2¼ | | 143212 | 14 | 1¾ | | 123456 | 17 | 8¼ |
| (xiii) | 11142 | 16 | 7½ | (xiv) | 957654 | 17 | 2¼ | (xv) | 576789 | 16 | 4¼ |
| | 24111 | 17 | 6¾ | | 456789 | 12 | 7¾ | | 987657 | 14 | 6¼ |
| | 12345 | 16 | 7¼ | | 123456 | 17 | 8¼ | | 123721 | 11 | 2¾ |
| | 54321 | 17 | 6½ | | 654321 | 18 | 7½ | | 127221 | 12 | 1¼ |

*Note.* Where the columns of the money to be added are long, (as frequently happen in buſineſs,) it is troubleſome and liable to error to collect them into one continued ſum; to remedy which, *practice* has ſuggeſted ſundry contrivances for eaſe and diſpatch: ſome point for every 12 in pence, and every 20 in ſhillings; this is eſteemed inelegant and unclerk-like; but inſtead thereof, we may very conveniently point in pence at 120, or 10s. in ſhillings and pounds, at 100, and then we muſt carry 10 for every point. Some chuſe to point at 60, or 5s. in the pence.

## *Of Troy Weight.*

| | lbs. | oz. | dwt. | gr. | | lbs. | oz. | dwt. | gr. |
|---|---|---|---|---|---|---|---|---|---|
| (i) | 4712 | 11 | 19 | 17 | (ii) | 163 | 10 | 15 | 13 |
| | 3714 | 10 | 17 | 15 | | 971 | 11 | 14 | 16 |
| | 9713 | 11 | 13 | 14 | | 316 | 10 | 13 | 17 |
| | 3174 | 10 | 17 | 12 | | 941 | 10 | 11 | 13 |

| | lbs. | oz. | dwt | gr. |
|---|---|---|---|---|
| (iii) | 7634654 | 11 | 13 | 19 |
| | 9717242 | 11 | 19 | 21 |
| | 5796547 | 10 | 11 | 23 |
| | 9762164 | 11 | 17 | 19 |

*Rule.* For every 24 grains, carry 1 to the pennyweights; for every 20 pennyweights, carry 1 to the ounces; and for every 12 ounces, carry 1 to the pounds; and in pounds, as in Simple Addition.

## *Of Avoirdupois Weight.*

| | tons. | cwt. | qrs. | lbs. | | cwt. | qrs. | lbs. | | lbs. | oz. | dr. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 3746 | 17 | 3 | 14 | (ii) | 134 | 3 | 17 | (iii) | 1376 | 11 | 15 |
| | 1373 | 14 | 1 | 17 | | 131 | 2 | 18 | | 1314 | 10 | 11 |
| | 1468 | 13 | 3 | 15 | | 147 | 1 | 17 | | 3715 | 11 | 14 |
| | 3131 | 11 | 1 | 19 | | 914 | 2 | 17 | | 1123 | 10 | 13 |

*Rule.* For every 16 drachms, carry 1 to the ounces; for every 16 ounces, carry 1 to the pounds; for every 28 lbs. carry 1 to the quarters; for every 4 quarters, carry 1 to the cwts.; for every 20 cwt. carry 1 to the tons; and the tons, as in Simple Addition.

## *Of Apothecaries' Weight.*

|  | lbs. | ℥ | ʒ | ℈ |  | lbs. | ℥ | ʒ | ℈ |  | lbs. | ℥ | ʒ | ℈ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 174 | 10 | 7 | 1 | (ii) | 11 | 11 | 5 | 1 | (iii) | 19 | 11 | 4 | 1 |
|  | 10 | 11 | 5 | 1 |  | 12 | 11 | 4 | 2 |  | 14 | 10 | 5 | 2 |
|  | 11 | 10 | 6 | 2 |  | 17 | 10 | 5 | 1 |  | 15 | 11 | 2 | 1 |
|  | 9 | 6 | 1 | 1 |  | 13 | 11 | 6 | 2 |  | 14 | 10 | 7 | 1 |
|  | 4 | 5 | 4 | 1 |  | 17 | 10 | 5 | 1 |  | 17 | 8 | 5 | 2 |
|  | 14 | 1 | 3 | 2 |  | 12 | 11 | 4 | 1 |  | 18 | 4 | 4 | 1 |

*Rule.* For every 20 grains, carry 1 to the ſcruples; for every 3 ſcruples, carry 1 to the drachms; for every 8 drachms, carry 1 to the ounces; for every 12 ounces, carry 1 to the pounds; and the pounds, as in Simple Addition.

## *Of Liquid Meaſure.*

|  | hhd. | g. | q. |  | tons | hhd. | g. | q. |  | tons | hhd. | gal. | q. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 31 | 57 | 1 | (ii) | 37 | 3 | 27 | 2 | (iii) | 39 | 2 | 14 | 1 |
|  | 31 | 19 | 3 |  | 17 | 2 | 60 | 1 |  | 40 | 1 | 57 | 1 |
|  | 17 | 38 | 2 |  | 39 | 1 | 58 | 1 |  | 99 | 1 | 53 | 1 |

*Rule.* For every 4 quarts, carry 1 to the gallons; for every 63 gallons, carry 1 to the hogſheads; for every 4 hogſheads, carry 1 to the tuns; and the tuns, as in Simple Addition.

## *Of Dry Meaſure.*

|  | qrs. | b. | b. | p. |  | b. | g. | p. | q. | pt. |  | b. | g. | p. | q. | pt. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 37 | 1 | 3 | 3 | (ii) | 27 | 3 | 1 | 1 | 1 | (iii) | 31 | 2 | 1 | 1 | 0 |
|  | 60 | 0 | 1 | 2 |  | 26 | 2 | 0 | 0 | 0 |  | 17 | 1 | 1 | 0 | 1 |
|  | 14 | 1 | 2 | 3 |  | 23 | 3 | 1 | 0 | 1 |  | 20 | 1 | 1 | 1 | 0 |
|  | 15 | 1 | 3 | 1 |  | 13 | 3 | 1 | 1 | 1 |  | 37 | 1 | 1 | 1 | 1 |

*Rule.* For every 2 pints, carry 1 to the quarts; for every 2 quarts, carry 1 to the pottles; for every 2 pottles, carry 1 to the gallons; for every 2 gallons, carry 1 to the pecks; for every 4 pecks, carry 1 to the buſhels; for every 4 buſhels, carry 1 to the barrels; for every 2 barrels, carry 1 to the quarters; and the quarters, as in Simple Addition.

## *Of Cloth Meaſure.*

| yds. | qr. | nai. | yds. | qr. | nai. | ellsE. | qr. | nai. |
|---|---|---|---|---|---|---|---|---|
| 36 | 3 | 1 | 374 | 1 | 2 | 140 | 2 | 2 |
| 37 | 1 | 1 | 314 | 2 | 1 | 371 | 2 | 3 |
| 14 | 1 | 2 | 417 | 3 | 3 | 471 | 1 | 3 |
| 15 | 2 | 3 | 420 | 1 | 2 | 151 | 2 | 3 |

*Rule.* For every 4 nails, carry 1 to the quarters; for every 4 quarters, carry 1 to the yards; for every 5 quarters, carry 1 to the ells Engliſh; and for every 6 quarters, carry 1 to the French ells.

## *Of Long Meaſure.*

| m. | f. | p. | yds. | ft. | in. | miles | f. | yds. | f. | in. | b. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 27 | 5 | 37 | 176 | 1 | 6 | 177 | 7 | 6 | 1 | 10 | 2 |
| 19 | 6 | 36 | 178 | 2 | 8 | 197 | 6 | 5 | 2 | 11 | 1 |
| 14 | 7 | 16 | 197 | 2 | 10 | 189 | 5 | 6 | 1 | 10 | 1 |
| 13 | 5 | 20 | 101 | 1 | 11 | 214 | 1 | 5 | 2 | 11 | 1 |

*Rule.* For every 3 barley-corns, carry 1 to the inches; for every 12 inches, carry 1 to the feet; for every 3 feet, carry 1 to the yards; for every 7 yards, carry 1 to the perches; for every 40 perches, carry 1 to the furlongs; for every 8 furlongs, carry 1 to the miles; for every 3 miles, carry 1 to the leagues; for every 60 miles, carry 1 to the degrees; and the degrees as in Simple Addition.

## *Of Solid Meaſures.*

| perch. | yds. | feet | inches |
|---|---|---|---|
| 374 | 130 | 14 | 150 |
| 371 | 176 | 24 | 140 |
| 914 | 68 | 20 | 13 |

| perch. | yds. | feet | inches |
|---|---|---|---|
| 176 | 126 | 23 | 1718 |
| 314 | 141 | 17 | 1314 |
| 431 | 19 | 25 | 1711 |

*Rule.* For every 1728 ſolid inches, carry 1 to the feet; for every 27 ſolid feet, carry 1 to the yards, and for every 343 ſolid yards, carry 1 to the perches; and perches as in Simple Addition.

## *Of Square Meaſure.*

| Acrs. | Pe. | Yds. | Ft. | Inch. |
|---|---|---|---|---|
| 147 | 27 | 33 | 7 | 78 |
| 192 | 16 | 38 | 4 | 101 |
| 141 | 39 | 16 | 5 | 140 |
| 175 | 28 | 39 | 8 | 132 |

| Acrs. | Pe. | Yds. | Ft. | Inch. |
|---|---|---|---|---|
| 23760 | 16 | 19 | 2 | 144 |
| 19421 | 32 | 17 | 7 | 111 |
| 17236 | 18 | 32 | 6 | 78 |
| 11541 | 19 | 39 | 8 | 132 |

*Rule.* For every 144 ſquare inches, carry 1 to the ſquare feet; for every 9 ſquare feet, carry 1 to the ſquare yards: for every 49 ſquare feet, carry 1 to the ſquare perches; for every 40 ſquare perches, carry 1 to the acres, and the acres as in Simple Addition.

## *Time.*

| Weeks | Days | Hour. | Min. | Secon. |
|---|---|---|---|---|
| 27 | 4 | 18 | 37 | 56 |
| 37 | 6 | 19 | 50 | 51 |
| 31 | 4 | 18 | 51 | 40 |

## *Aſtronomical Time.*

| Degs. | ′ | ″ | ‴ | ⁗ |
|---|---|---|---|---|
| 176 | 30 | 50 | 41 | 25 |
| 196 | 21 | 47 | 46 | 21 |
| 197 | 5 | 43 | 51 | 27 |

*Rule.* For every 60 ſeconds, carry 1 to the minutes; for every 60 minutes, carry 1 to the hours; for every 24 hours, carry 1 to the days; for every 7 days, carry 1 to the weeks;

for every 4 weeks, carry 1 to the months; for every 12 months, carry 1 to the years; and for every 52 weeks, carry 1 to the years; or for every 365¼ days, carry 1 to the years, and years as in Simple Addition.

## *Application of the foregoing Rules in Compound Addition.*

### *Of Money.*

1. A nobleman going out of town, is informed by his ſteward, that his corn-chandler's bill came to 123l. 19s.; his brewer's, to 41l. 10s.; his butcher's, 212l. 12s. 6d.; to his Lordſhip's baker, is owing 24l. 0s. 6d.; to his tallow-chandler, 13l. 8s.; to his taylor, 137l. 9s. 9d.; to his draper, 74l. 13s. 6d.; his coach-maker's demand was 214l. 16s. 6d.: his wine-merchant's, 68l. 12s.; his confectioner's, 16l. 2s.; his rent, 86l. 2s.; and his ſervants' wages, for half a year, came to 46l. 1s.; what money muſt he ſend to his banker for, in caſe he would carry with him 330l. 12s. 1d. to defray his expence on the road? *Anſ.* 1389l. 18s. 10d.

2. A owes to B for tea, 13l. 10s.; for cheeſe, 17l. 13s. 5d.; for cotton, 208l. 17s.; for chintz, 86l. 0s. 9d.; for a former bill, 300l.; for factorage, 15l. 17s. 3d¼.; other charges, 30l. 10s. 4½d.; How much is A indebted to B? *Anſ.* 672l. 8s. 10¼d.

3. A merchant's clerk receives of ſundry perſons for his maſter, viz.—Of A. 13l. and half-a-crown; of B, 2l. 13s.; of C, two piſtoles and half-a-guinea; of D, 1l. 9s. 8½d.; of E, 11s. 6¼d.; of F, a moidore and 13 Engliſh ſhillings; of G, a bank-note of 20l. of H, 15l. and 3 crown-pieces over; I deſire to know what ſum he had in charge? *Anſ.* 68l. 13s. 10¾d.

### *Of Troy Weight.*

4. A nobleman had a ſervice of plate, which conſiſted of 20 diſhes, weighing 203 oz. 8 dwts.; 36 plates, weighing 408 oz. 9 dwts.; 5 dozen of ſpoons, weighing 112 oz. 8 dwts.; 6 ſalts and 6 pepper-boxes, weighing 71 oz. 7 dwts.; knives and forks, weighing 73 oz. 5 dwts.; two large cups, a

tankard and a mug, weighing 121 oz. 7 dwts.; together with ſundry other ſmall articles, weighing 105 oz. 5 dwts. I deſire to know the weight of the whole?

*Anſ.* 91 lb. 3 oz. 9 dwts.

## *Of Avoirdupois Weight.*

5. A hop-merchant buys 5 bags of hops, of which the firſt weighed 2 cwt. 3 qrs. 13 lb.; the ſecond, 2 cwt. 2 qrs. 11 lb.; the third, 2 cwt. 3 qrs. 5 lb.; the fourth, 2 cwt. 3 qrs. 12 lb.; the fifth, 2 cwt. 3 qrs. 15 lb.; I deſire to know the weight of the whole? *Anſ.* 14 cwt.

## *Of Apothecaries' Weight.*

6. An apothecary made a mixture of 5 ingredients, the firſt weighed 13 lb. 7 oz.; the ſecond, 11 oz. 7 drs. 13 grs.; the third 7 lb. 2 ſcru.; the fourth, 11 lb. 3 dr. 1 ſcru.; the fifth, 15 lb. 5 oz. 7 gr. Required the weight of the whole?

*Anſ.* 48 lb. 3 drs. 1 ſcru.

## *Of Liquid Meaſure.*

7. A vintner imports ſeveral quantities of wine as follow: at three different times, by one veſſel, 10 pipes, 14 gallons; by another, 40 pipes, 36 gallons; by a third, 6 pipes, 3 hhds. 59 gals. How much was imported in all?

*Anſ.* 57 pipes and 46 gallons.

## *Of Dry Meaſure.*

8. A corn-merchant ſends over the ſea, of wheat, 20 bar. 3 buſh.; of oats, 96 bar. 3 buſh. 3 pecks; of rye, 100 bar. 2 buſh. 3 pe.; of peas, 9 bar. 3 pe.; of beans, 3 bar. 3 pe. For what has he freight to pay? *Anſ.* 230 b. 3 b.

## *Of Cloth Meaſure.*

9. A woollen-draper bought 4 parcels of cloth, the firſt contains 30 yds. 3 qrs.; the ſecond, 33 yds. 2 nails; the third, 16 yds. 3 qrs. 3 nails; the fourth, 40 yards: How many yards were in them all? *Anſ.* 120 yds. 3 qrs. 1 nail.

### *Of Long Meaſure.*

10. From A to B is 3 miles, 2 fur. 7 perches; from B to C 17 miles, 6 perches; from C to D 7 furlongs; and from D to E 5 miles, 3 furlongs: I deſire the diſtance from A to E? *Anſ.* 27 miles, 2 fur. 7 per.

### *Of Square Meaſure.*

11. There are 5 pieces of ground, the firſt of which meaſures 20 acres, 2 roods, 19 perches; the ſecond, 33 acres, 1 rood, 14 perches; the third, 14 acres, 1 rood, 17 perches; the fourth, 100 acres, 3 roods, 39 perches; and the fifth, 90 acres, 19 perches: What is the content of the whole?
*Anſ.* 259 acres, 1 rood, 28 per.

### *Of Time.*

12. A gentleman was born in the year 1740; when he was 25 years, 1 month, 13 days old, he married; in 1 year, 17 days after which he had a ſon born, who lived to the age of 22 years, 5 months and 26 days, and then married; and after 11 months, 8 days afterwards, he had a daughter, who died when 2 years and 17 days old: How old was the grandfather at the time of her death, ſuppoſing 13 months, of 28 days each, to make a year? *Anſ.* 61 yrs. 6 mo. 25 days.

## CHAP. VI.

## COMPOUND SUBTRACTION.

COMPOUND SUBTRACTION teaches to find the difference between any two numbers of different denominations.

Place the leſſer number under the greater, ſo that thoſe parts which are of the ſame denomination may ſtand directly under each other, and draw a line below them; begin at the right hand, and take each number in the lower line from that above it, and ſet the remainder under it. The method of proof is the ſame as in Simple Subtraction.

## *Of Coin.*

| | | £. | s. | d. | | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (i) | From | 14 | 13 | 6½ | (ii) | 19 | 17 | 4¾ | (iii) | 134 | 17 | 4½ |
| | Take | 11 | 7 | 4¼ | | 11 | 10 | 2¼ | | 94 | 16 | 1¾ |
| | Rem. | | | | | | | | | | | |
| (iv) | From | 1174 | 13 | 10½ | (v) | 100 | 13 | 4 | (vi) | 176 | 13 | 4¼ |
| | Take | 524 | 17 | 11¾ | | 90 | 17 | 8½ | | 99 | 17 | 9¾ |
| | Rem. | | | | | | | | | | | |
| (vii) | From | 1000 | 0 | 0 | (viii) | 109 | 11 | 11 | (ix) | 374 | 11 | 8 |
| | Take | 999 | 19 | 11[illegible] | | 91 | 15 | 11½ | | 199 | 13 | 4½ |
| | Rem. | | | | | | | | | | | |

*Rule.* When you cannot take the lower farths. from the higher, you muſt borrow 4 more to the higher and carry 1 to the pence; when you cannot take the lower pence from the higher, you muſt borrow 12 to the higher and carry 1 to the ſhillings; when you cannot take the lower ſhillings from the higher, you muſt borrow 20 to the higher and carry 1 to the pounds; then the pounds as in Simple Subtraction.

## *Of Troy Weight.*

| | lbs. | oz. | dwt. | gra. | lbs. | oz. | dwt. | gra. | lbs. | oz. | dwt. | gra. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| From | 5 | 6 | 13 | 14 | 374 | 11 | 12 | 6 | 197 | 11 | 13 | 13 |
| Take | 3 | 4 | 6 | 8 | 297 | 10 | 17 | 8 | 178 | 10 | 17 | 10 |
| Rem. | | | | | | | | | | | | |

*Rule.* When you cannot take the lower grs. from the higher you muſt borrow 24 to the higher and carry 1 to the dwts.; when you cannot take the lower from the higher dwts. you muſt borrow 20 to the higher and carry 1 to the ozs.; when you cannot take the lower ozs. from the higher, you muſt borrow 12 to the higher and carry 1 to the lbs.; and the lbs. as in ſimple ſubtraction.

Of

### *Of Avoirdupois Weight.*

| | ct. | qrs. | lbs. | oz. | d. | ton. | ct. | qrs. | lbs. | oz. | dr. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| From | 17 | 3 | 14 | 6 | 14 | 16 | 16 | 2 | 14 | 13 | 14 |
| Take | 12 | 1 | 8 | 4 | 8 | 12 | 17 | 3 | 19 | 15 | 15 |
| Rem. | | | | | | | | | | | |

*Rule.* When you cannot take the lower drs. from the higher, you muſt borrow 16 to the higher and carry 1 to the ounces; when you cannot take the lower ounces from the higher, you muſt borrow 16 to the higher and carry 1 to the lbs.; when you cannot take the lower lbs. from the higher, you muſt borrow 28 to the higher and carry 1 to the quarters; when you cannot take the lower quarters from the higher, you muſt borrow 4 to the higher and carry 1 to the cwts.; when you cannot take the lower cwts. from the higher, you muſt borrow 20 to the higher and carry 1 to the tons; and the tons, as in Simple Subtraction.

### *Of Apothecaries' Weight.*

| | lbs. | ℥ | ʒ | ℈ | gr. | lbs. | ℥ | ʒ | ℈ | gr. |
|---|---|---|---|---|---|---|---|---|---|---|
| From | 19 | 8 | 7 | 2 | 19 | 24 | 6 | 5 | 1 | 16 |
| Take | 15 | 6 | 5 | 1 | 16 | 19 | 8 | 7 | 2 | 19 |
| Rem. | | | | | | | | | | |

*Rule.* When you cannot take the lower grs. from the higher, you muſt borrow 20 to the higher, and carry 1 to the ſcruples; when you cannot take the lower ſcruples from the higher, you muſt borrow 3 to the higher, and carry 1 to the drams; when you cannot take the lower drams from the higher, you muſt borrow 8 to the higher, and carry 1 to the ounces; when you cannot take the lower ounces from the higher, you muſt borrow 12, and carry 1, to the lbs.; and the lbs. as in Simple Subtraction.

### *Of Liquid Meaſure.*

| | tons | hhds. | gal. | q. | p. | tons | hhds. | gal. | q. | p. |
|---|---|---|---|---|---|---|---|---|---|---|
| From | 140 | 3 | 56 | 2 | 1 | 163 | 2 | 56 | 1 | 0 |
| Take | 131 | 2 | 50 | 1 | 1 | 94 | 3 | 61 | 1 | 1 |
| Rem. | | | | | | | | | | |

*Rule.* When you cannot take the lower pts. from the higher, you muſt borrow 4, and carry 1 to the pottles; when you cannot take the lower pottles from the higher, you muſt borrow 2 and carry 1 to the gallons; when you cannot take the lower gallons from the higher; you muſt borrow 63 to the higher, and carry 1 to the hogſheads; when you cannot take the lower hogſheads from the higher, you muſt borrow 4 to the higher, and carry 1 to the tuns; and the tuns as in Simple Subtraction.

### *Of Dry Meaſure.*

| | bar. | buſh. | p. | g. | qt. | bar. | buſh. | p. | g. | qt. |
|---|---|---|---|---|---|---|---|---|---|---|
| From | 136 | 2 | 3 | 2 | 3 | 176 | 2 | 2 | 1 | 2 |
| Take | 79 | 1 | 2 | 1 | 2 | 53 | 3 | 3 | 1 | 3 |
| Rem. | | | | | | | | | | |

*Rule.* When you cannot take the lower qts. from the higher, you muſt borrow 4 to the higher and carry 1 to the gallons; when you cannot take the lower gallons from the higher, you muſt borrow 2 and carry 1 to the peck; when you cannot take the lower pecks from the higher, you muſt borrow 4 and carry 1 to the buſhels; when you cannot take the lower buſhels from the higher, you muſt borrow 4 to the higher and carry 1 to the barrels; and the barrels, as in Simple Subtraction.

### *Of Time.*

| | yrs. | m. | w. | d. | yrs. | m. | w. | d. | h. | m. | ſe. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| From | 1793 | 8 | 2 | 3 | 1803 | 4 | 2 | 5 | 14 | 30 | 40 |
| Take | 1663 | 6 | 1 | 2 | 1800 | 10 | 3 | 6 | 19 | 40 | 50 |
| Rem. | | | | | | | | | | | |

*Rule.* When you cannot take the lower ſecs. from the higher, you muſt borrow 60 to the higher and carry 1 to the mins.; when you cannot take the lower minutes from the higher, you muſt borrow 60 to the higher and carry one to the hours; when you cannot take the lower hours from the higher, you muſt borrow 24 to the higher and carry 1 to the days; when you cannot take the lower days from the higher, you muſt borrow 7 to the higher and carry 1 to the weeks; when you cannot take the lower weeks from the higher, you muſt borrow 4 to the higher and carry 1 to the months; when you cannot take the lower months from the higher, you muſt borrow 12 to the higher and carry 1 to the years; and the years as in Simple Subtraction.

### *Of Square Meaſure.*

| | acres | rd. | per. | yds. | ft. | acres | rd. | per. | yds. | ft. |
|---|---|---|---|---|---|---|---|---|---|---|
| From | 100 | 2 | 36 | 16 | 7 | 114 | 1 | 21 | 15 | 16 |
| Take | 97 | 1 | 30 | 14 | 5 | 74 | 2 | 36 | 16 | 7 |
| Rem. | | | | | | | | | | |

*Rule.* When you cannot take the lower feet from the higher, you muſt borrow 9 to the higher and carry 1 to the yards; when you cannot take the lower yards from the higher, you muſt borrow 49 to the higher and carry 1 to the perches; when you cannot take the lower perches from the higher, you muſt borrow 40 to the higher and carry 1 to the roods; when you cannot take the lower roods from the higher, you muſt borrow 4 to the higher and carry 1 to the acres; and the acres as in Simple Subtraction.

### *Of Cloth Meaſure.*

| | e.E. | yds. | qr. | n. | yds. | qr. | na. | e.E. | yds. | qr. | na. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| From | 73 | 2 | 1 | 2 | 95 | 1 | 1 | 139 | 1 | 2 | 1 |
| Take | 29 | 0 | 1 | 0 | 48 | 2 | 2 | 67 | 2 | 2 | 3 |
| Rem. | | | | | | | | | | | |

*Rule.* When you cannot take the lower nails from the higher, you muſt borrow 4 to the higher and carry 1 to the quarters; when you cannot take the lower quarters from the higher, you muſt borrow 4 to the higher and carry 1 to the yards; and the yards as in Simple Subtraction.

### *Long Meaſure.*

|  | l. | m. | f. | p. | yds. | yds. | f. | in. | b. | m. | fu. | p. | yd. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **From** | 19 | 3 | 3 | 36 | 3 | 222 | 2 | 10 | 2 | 140 | 6 | 5 | 3 |
| **Take** | 12 | 2 | 2 | 34 | 2 | 202 | 2 | 12 | 2 | 132 | 7 | 3 | 6 |
| **Rem.** |  |  |  |  |  |  |  |  |  |  |  |  |  |

*Rule.* When you cannot take the lower barley-corns from the higher, you muſt borrow 3 to the higher and carry 1 to the the inches; when you cannot take the lower inches from the higher, you muſt borrow 12 to the higher and carry 1 to the feet; when you cannot take the lower feet from the higher, you muſt borrow 3 to the higher and carry 1 to the yards; when you cannot take the lower yards from the higher, you muſt borrow 7 to the higher and carry 1 to the perches; when you cannot take the lower perches from the higher, you muſt borrow 40 to the higher and carry 1 to the furlongs; when you cannot take the lower furlongs from the higher, you muſt borrow 8 to the higher and carry 1 to the miles; and miles as in Simple Subtraction.

## *The Uſe of Addition and Subtraction.*

1. A merchant failing in trade, was indebted to A. 71l. 12s. 6d. to B. 34l. 9s. 9d. to C. 16l. 8s. 8d. to D. 44l. to E. 66l. 7s. 6d. to F. 1l. 2s. 3d. to G. 19l. 19s. and to H. 20l. At the time of this diſaſter, he had by him, in caſh, 3l. 13s. 6d. in commodities, 23l. 10s. in houſhold furniture, 13l. 8s. 6d. in plate, 7l. 18s. 5d. in a tenement, 56l. 15s. in recoverable book debts, 87l. 13s. 10d. ſuppoſing theſe things to be faithfully ſurrendered to his creditors, what will they then loſe by him? *Anſ.* 81l. 0s. 5d.

2. A merchant in Vienna owes to a merchant in Dublin, for goods received in January, 1802, the ſum of 103l. 12s. 2d. for goods received in February, 93l. 3s. 4d. for goods received in March, 121l. 17s. for goods received in April, 142l. 15s. 4d. for goods received in May, 171l. 15s. 10d. for goods received in June, 142l. 12s. 6d. My correſpondent informs me, that on the firſt of January, 1803, he had ſhipped, agreeable to my orders, wines to the value of 400l. fruit, to the value of 60l. 10s. 6d. figs, 20l. 10s. 4d. oil,

19l. 10s. 4d. I deſire to know how the account ſtands between us, and who is the debtor? *Anſ.* (due to me) 275l. 5s.

3. A merchant, in ballancing his books, finds he has in ready money, 456l. 17s. in goods, 1749l. 19s. 6d. his ſtock in a company-trade was 199l. 19s. 6½ due to him in open accounts, 2977l. 19s. 7¾d. in bills, 647l. 17s. in ſhips and houſes, 1976l. 14s. 7½d. and in conſignments, 479l. 19s. 7d. He owes to A. 1456l. 18s. 7½d. to B. 90l. 19s. 11d. to C. 497l. 17s. 10d. and to the Bank, 490l. What is his nett ſtock? *Anſ.* 5944l. 10s. 6½d.

4. A merchant hath a bill to pay of 500l. for which he had prepared in caſh 197l. 19s. he hath a bill on Edinburgh for 120l. for how much muſt he draw on the bank to diſcharge the bill of 500l.? *Anſ.* 182l. 1s.

5. A farmer paid of yearly rent for his poſſeſſion, 156l. 17s. 8d. at the expiration of three years, when he was called to ſettle accounts with the landlord, he could produce receipts only for 376l. 19s. 7½d. How much muſt he pay to even the account? *Anſ.* 93l. 13s. 4½d.

6. Bought 8 hogſheads of raiſins, each weighing, groſs, 5 cwt. 1 qr. 11 lbs. upon each hoſhead whereof I am allowed a deduction of 3 quarters and 21 pounds; What will be the nett weight? *Anſ.* 35 cwt. 1 qr. 4 lbs.

7. A merchant ſent his clerk to a fair, when he bought linen to the amount of 105l. 12s. 11d. ſtockings to the amount of 184l. 16s. 11d. he recovered, of accounts due to the merchant, to the amount of 64l. 10s. and got the payment of a bill for 139l. 19s. he paid ſome few demands, amounting in all to 19l. 19s. 11d. his account of petty charges came to 1l. 14s. 6d. and he gave back to his maſter 27l. 11s. 10d. How much money had he got from the merchant before he ſet out? *Anſ.* 135l. 7s. 1d.

8. A caſtle was built in the year 1459: How old was it in the year 1787? *Anſ.* 328 years.

9. Received from Jamaica 103 tons, 13 cwt. of logwood, by the Happy Janet, of which I ſold to A. 15 ton, 10 cwt. 3 qrs. 17 lb. to B. the double of what I ſold to A.; and to C. as much as I had ſold to A. and B. together: How much have I on hand? *Anſ.* 10 ton, 7 cwt. 2 qrs. 10 lb.

10. A father bequeathed his whole fortune to his ſon, A. and his daughter B. in all, 8745l. 18s. 8d. with particular

orders that the ſon ſhould have 1831l. 6s. 8d. more than the daughter: The true dividend is required? *Anſ.* To the ſon, 5283l. 12s. 8d. to the daughter, 3462l. 6s. 0d.

11. A. made a bond for 114l. 10s. the intereſt came to 19l. he then paid off 40 guineas, and gave a freſh bond for what was behind; by this time there were 13l. 4s. 8d. due on the ſecond for intereſt; he paid off 24 moydores, (29s. 3d. each); 2 guineas and 6s. 8d more, took up the old bond, and ſigned a new one ſtill for the reſidue; the principal again ran on until there were 9l. 11s. 3d. more due; then he determined to take it up: How much had his creditors to receive?

*Anſ.* 73l. 1s. 9d.

12. Five noble diſcoveries were made in 215 years, viz.—1ſt, The invention of the compaſs; 2d, Gunpowder; 3d, Printing; 4th, The Diſcovery of America: 5th, Truth, in the Reformation The laſt was brought about *anno*, 1517; the third, 77 years before; the fifth, 42 years after the firſt; and the fourth, 148 years after the ſecond; the queſtion is, in what year of Chriſt did each happen to be found? *Anſ.* Compaſs, 1302; Gunpowder, 1344; Printing, 1440; America, 1492.

13. Three and thirty years before the Reſtoration in 1660, the crown demeſnes to certain uſes, for 210 years then to come. The proprietor, in 1715, procured a reverſionary grant for 99 years, to commence after the expiration of the firſt: In what year will the ſecond term end? *Anſ.* 1936.

14. A merchant taking an inventory of his capital, finds in his vaults, 28 puncheons of brandy, which coſt him 874l. 10s. 6d. Bourdeaux claret, 40 tons, which ſtood him in 754l. 4s. 22 laſts, 4 buſhels of corn, in his granary, worth 675l. 17s. 3d. with 2 laſts of Canary-ſeed, worth 113l. in his ware-houſe were ten caſks of indigo, worth 632l. 12s. a parcel of ſaffron, worth 253l. 5s. W. P. of Limerick, owed him 384l. 10s. in the hands of F. G. at Sligo, he had wines to the amount of 1011l. 10s. pepper in the keeping of S. Q. of the Cuſtom-houſe, value 1252l. 16s. 8d. beſides which, R. O. owes him on bond, 300l. and I. M. on note, 260l. 14s. he has in India bonds, to the value of 459l. and the intereſt of thoſe ſecurities made 25l. 14s. 6d. he had bank ſtock to the value of 2134l. 4s. 6d. there lay in his banker's hands, 1892l. 17s. 6d. he was at that time indebted to D. E. 71[illegible]l. 13s. to

M. F. 352l. 10s. 8d. to L. P. the foot of his account, 172 guineas; to J. B. on ballance, 57l. 12s. 10d. to an inſurance, 190l. The preſent ſtate of this perſon's fortune is required?

*Anſ.* 9530l. 7s. 5d.

15. A merchant, at his outſetting in trade, owed 280l. He had, in caſh, commodities, the ſtocks, and good debts, 11505l. 10s. he cleared, the firſt year, by commerce, 393l. 13s. 1d. What was his nett ballance at the year's end?

*Anſ.* 11619l. 3s. 1d.

16. A. B. and C. open an account with a banker, Jan. 11, 1793, and put into his hands, viz—A. 17 guineas, (17l. 17s.) B. 34l. 11s. 6d, C. 28l. 18s. 10d. On the 21ſt, A. withdrew 9l. 10s. and C. advanced 12l. and a crown; the 24th, B. called for 6l. 10s. the 30th, C. wanted 19l. 8s. 4d. on the 12th February, B. depoſited with him 11 Caroluſes, each 23s. and 3 moydores. On the 19th, A. ſent for 5l. and a noble more, but on the 23d. returned him 42l. On the 2d of March, C. paid in 20 guineas, (21l.) and B. drew for 6 guineas (6l. 6s.); the 14th, B. ſent in 17l. 8s. 8d. and the 17th, A. paid back 12l. 2s. 6d. On the 19th, they ſent for 5 guineas a man, (5l. 5s.) and on the 24th, they returned that ſum, and ten marks a piece more, (13s. 4d.) How much did their ſaid banker owe them jointly and ſeparately, at Lady-day? *Anſ.* To A. 39l. 11s. 2d.—to B. 62l. 11s. 6d.—C. 49l. 8s. 10d.—jointly 151l. 11s. 6d.

17. When the air preſſes with its full weight, in very fair weather, it may be demonſtrated, that there preſſes upon a human body, about 33905 pounds of that fluid matter; and in foul weather, when the air is moſt light, but 30624 pounds: What difference of weight lies upon ſuch a body, in the two greateſt alterations of the weather?

*Anſ.* 3281 pounds.

18. Of the noble family of Cornaro the grandſire's age was 134 years, and he was 93 years older than the ſon, at the time when the ſon and father's ages together made 112 years, diſtinguiſh their ages? *Anſ.* The ſon's age, 41 years; father's 71.

19. Having a piece of ground 127 feet in front; let off to A. 57 feet, to build on at one end; and to B. at firſt, 27½ feet, which he afterwards, by conſent, extended to 42 feet: What ground was left me in the centre? *Anſ.* 28 feet.

20. Hipparchus and Archimedes of Syracuſe, about 200 years before Chriſt; Poſidonius, 50 years before the ſaid grand period; and Ptolemy, 140 years after it, advanced the ſcience of aſtronomy: How long did each of theſe perſons flouriſh before the year of Chriſt, 1793? *Anſ.* Hipparchus and Archimedes flouriſhed 1984; Poſidonius, 1834; and Ptolemy, 1644 years.

21. In the city of Pekin, in China, is a bell, weighing, it is ſaid, 120000 lbs. At Nankin, in the ſame country, is another, weighing 50000 lbs. the firſt exceeds the great bell at Erford, in Upper Saxony, by 94600 lb. How much is the German bell inferior in weight to the ſecond, and what is its actual weight? *Anſ.* Erford bell, 25400, and the difference, 24600 pounds.

22. Your grandfather, if living, is 119 years of age; your father actually 65; you are not ſo old as your grandſire, by 84; What is the difference in years between your father and you? *Anſ.* 29 years.

23. A. is 13 years younger than B. and 17 years older than C. who, in the year 1747, was known to be 24 years of age; How old was each of theſe perſons in 1784?
*Anſ.* A's age, 78; B. 91; and C. 61 years.

24. A public edifice was finiſhed towards the cloſe of the 10th of King John, who began his reign 134 years after the conqueſt in 1066; and it ſtood until within 70 years of the peace of Utrecht, in 1713; Of what duration was it?
*Anſ.* 433 years.

25. A grant was made by the crown, *anno* 1239, which was forfeited 137 years before the revolution in 1688; How long did the ſame ſubſiſt? *Anſ.* 312 years.

26. Moſes was born, *anno mundi*, 2433; Homer, 832 years after him; Julius Cæſar lived 40 years before our Saviour; and Alexander, 312 years before Cæſar; now as Chriſt was incarnate 4000 years after the Creation, the ſum of the intervals between Homer and the three great perſonages laſt mentioned is required? *Anſ.* 1813 years.

27. B. was born 14 years after C. who came into the world 19 years before A. who was 23 years of age 8 years ago; What is the age of D. who is within 22 years of being as old as theſe three together? *Anſ.* 95 years.

28. Sam. was born 28 years before Toby, (who died at 12,) and lived 19 years after him. Rachael came to light when Sam was 16, and died 11 years before him. Joſhua, when Rachael was 7 years, being himſelf then 14, went abroad,

where he continued 9 years, and returning, ſurvived Rachael 4 years; How old was each of theſe, and what is the ſum of their ages? *Anſ.* Sam.'s age 59; Rachael's 32; Joſhua's 43; Toby's age 12; and the ſum of their ages 146 years.

29. From the creation to the flood were 1656 years; thence to the building of Solomon's Temple, 1336 years; thence to Mahomet, who lived 622 years after Chriſt, 1630 years: In what year of the world was Chriſt born?
*Anſ.* In the year 4000.

30. B. born 161 years ago, died when C. was 47 years of age, who, it ſeems, came into the world 180 years ſince, and outlived B. 43 years: The ſum of their ages is required?
*Anſ.* 118.

31. The building of Solomon's Temple was in the year of the world, 3000; Troy was, by computation, built 443 years before the Temple, and 260 years before London; now Cartharge was built 113 years before Rome, founded 744 years before Chriſt, who was born *anno mundi*, 4000; Is London or Carthage the moſt ancient city, and how much?
*Anſ.* London was built before Carthage, 326 years.

32. The Powder Plot was diſcovered 88 years after the Reformation in 1517; the murder of King Charles I. was committed 43 years after that diſcovery; the Acceſſion of the Brunſwick Family to the Crown was in 1714; juſt 54 years after the return of King Charles II. who lived in exile from the death of his Father Charles I. How long was that?
*Anſ.* 12 years.

## CHAP. VII.

## COMPOUND MULTIPLICATION.

COMPOUND MULTIPLICATION teaches to find the amount of any given number, of different denominations, repeated a certain number of times.

RULE.

## RULE.*

Write the multiplier under the loweſt denomination of the multiplicand. 2d, Multiply the number of the loweſt denomination by the multiplier, and divide that product by as many of that denomination as make one of the next higher denomination, in the ſame manner as in Compound Addition; ſet down the remainder under its own place, and add the quotient to the product of the next ſuperior denomination, as you multiply; in this manner proceed with all the other denominations to the higheſt.

*Caſe* 1ſt. When your given number of hundreds, pounds, yards, &c. does not exceed 12.

*Rule.* Multiply the given price by the quantity given, and the product will be the anſwer.

### EXAMPLES.

| | *l.* | *s.* | *d.* | | *l.* | *s.* | *d.* | |
|---|---|---|---|---|---|---|---|---|
| 2 yards, at | 1 | 12 | 6½ | | 13 | 13 | 8¾ | |
| | | | 2 | | | | 9 | |
| 1ſt, | *l.* 3 | 5 | 1 | *Anſ.* | *l.* 123 | 3 | 6¾ | 2d. |

| | | | £. | *s.* | *d.* | | £. | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | *Anſwers.* | | |
| 3. 4 Yards of Cloth, | at | | | 17 | 6¼ | per yard— | 3 | 10 | 2 |
| 4. 5 Hundred of Beef, | — | | 3 | 0 | 6 | —cwt.— | 15 | 2 | 6 |
| 5. 7 Yards of Linen, | — | | | 7 | 10 | —yard— | 2 | 14 | 10 |
| 6. 8 —— of Cloth, | — | | | 18 | 9¼ | —yard— | 7 | 10 | 4 |
| 7. 10 Gallons of Brandy, | — | | | 12 | 8 | —gall.— | 6 | 6 | 8 |
| 8. 11 lbs. of Tea, | — | | | 12 | 7 | —lb. — | 6 | 18 | 5 |
| 9. 12 cwt. of Butter, | — | | 1 | 17 | 6¾ | —cwt.— | 22 | 10 | 9 |

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* The product of a number conſiſting of ſeveral parts or denominations, by any ſimple number whatever, will, evidently, be expreſſed, by taking the product of that ſimple number, and each part by itſelf, as ſo many diſtinct queſtions; thus, 25l. 12s. 6d, multiplied by 9, will be 225l. 108s. 54d. (taking the ſhillings from the pence, and the pounds from the ſhillings, and placing them in the ſhillings and pounds reſpectively)=230l 12s. 6d. which is the ſame as the rule; and this will be true when the multiplicand is any compound number whatever.

*Caſe* 2d. If the given number be a compoſite, *(i. e.)* the product of the multiplication of two numbers.

*Rule.* Multiply the given price by one of the component parts, and that product by the other part, and you will have the Anſwer.

### EXAMPLES.

| | l. | s. | d. | |
|---|---|---|---|---|
| 16 cwt. of butter, at | 1 | 18 | 8 | 1ſt. |
| | | | 4 | |
| | 7 | 14 | 8 | |
| | | | 4 | |
| | *l.* 30 | 18 | 8 | *Anſwer.* |

| | | | l. | s. | d. | | | *Anſwers.* l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|
| 2. | 14 | at | | 6 | $7\frac{3}{4}$ | per | ——— | 4 | 13 | $0\frac{1}{2}$ |
| 3. | 18 | — | | | $10\frac{1}{4}$ | — | ——— | | 15 | $4\frac{1}{2}$ |
| 4. | 27 | — | 2 | 9 | 6 | — | ——— | 66 | 16 | 6 |
| 5. | 30 | — | | 4 | $11\frac{1}{2}$ | — | ——— | 7 | 8 | 9 |
| 6. | 45 | — | | | $2\frac{1}{2}$ | — | ——— | | 9 | $4\frac{1}{2}$ |
| 7. | 50 | — | 1 | 7 | 0 | — | ——— | 67 | 10 | 0 |
| 8. | 56 | — | | 2 | $7\frac{1}{2}$ | — | ——— | 7 | 7 | 0 |
| 9. | 64 | — | 1 | 11 | 0 | — | ——— | 99 | 4 | 0 |
| 10. | 72 | — | | 15 | 9 | — | ——— | 56 | 14 | 0 |
| 11. | 80 | — | | | $11\frac{3}{4}$ | — | ——— | 3 | 18 | 4 |
| 12. | 84 | — | | 9 | 0 | — | ——— | 37 | 16 | 0 |
| 13. | 96 | — | | 1 | $10\frac{3}{4}$ | — | ——— | 9 | 2 | 0 |
| 14. | 99 | — | | 18 | $11\frac{1}{2}$ | — | ——— | 93 | 16 | $10\frac{1}{2}$ |
| 15. | 100 | — | | 10 | 10 | — | ——— | 54 | 3 | 4 |
| 16. | 120 | — | | 1 | 6 | — | ——— | 9 | 0 | 0 |
| 17. | 132 | — | | 1 | 10 | — | ——— | 12 | 2 | 0 |
| 18. | 144 | — | | 1 | $7\frac{1}{2}$ | — | ——— | 11 | 14 | 0 |
| 19. | 121 | — | | 2 | 6 | — | ——— | 15 | 2 | 6 |

*Caſe* 3d. If the given quantity be not a compoſite number, *(i. e.)* a number which cannot be meaſured by any number greater than a unit.

*Rule.* Multiply by the neareſt compoſite number you can find, and if you work for more than the given number, ſubtract ſo many times the price of one from that product; but if you work for leſs than the given quantity, you muſt add ſo many times the price of one to that product, and the total will be the Anſwer.

*Note:* When you are to find the amount of any quantity with odd parts annexed, as ¼, ½, ¾, &c. work for the given number as before directed, and for the odd parts, take ſuch parts of the price of the given quantity as thoſe parts are of the integer.

EXAMPLES.

| | *s.* | *d.* | |
|---|---|---|---|
| If 1 yard of cloth coſt | 6 | 9¾ | what will 75 yards coſt? |

| | | | |
|---|---|---|---|
| | | 9 | |
| 3 | 1 | 3¾ | |
| | | 8 | |
| 24 | 10 | 6 | |
| 1 | 0 | 5¼ | the price of three, added. |
| *l.* 25 | 10 | 11¼ | *Anſwer.* |

*Or thus,*

| | *s.* | *d.* | |
|---|---|---|---|
| | 6 | 9¾ | |
| | | 11 | |
| 3 | 14 | 11¼ | |
| | | 7 | |
| 26 | 14 | 6¾ | |
| | 13 | 7½ | the price of two ſubtracted: |
| *l.* 25 | 10 | 11¼ | *Anſ.* |

| | *lbs.* | | *l.* | *s.* | *d.* | | *l.* | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|
| 1. | 17 | at | 1 | 4 | 10½ | *Anſwer.* | 21 | 2 | 10½ |
| 2. | 19 | — | | 19 | 11¾ | ——— | 18 | 19 | 7½ |
| 3. | 29 | — | | 1 | 7½ | ——— | 2 | 7 | 1½ |
| 4. | 38 | — | 2 | 17 | 6 | ——— | 109 | 5 | 0 |
| 5. | 47 | — | | 5 | 10 | ——— | 13 | 14 | 2 |
| 6. | 23 | — | | 1 | 6½ | ——— | 1 | 15 | 5½ |
| 7. | 46 | — | | 4 | 7¼ | ——— | 10 | 11 | 9½ |
| 8. | 59 | — | | 7 | 10 | ——— | 23 | 2 | 2 |
| 9. | 94 | — | | 12 | 2 | ——— | 57 | 3 | 8 |
| 10. | 117 | — | 1 | 2 | 3 | ——— | 130 | 3 | 3 |

*Caſe* 4th. When the given quantity cannot be produced by the Multiplication of Simple Numbers.

*Rule.* Multiply the given compound number by as many tens as there are places of figures in the given quantity; (leſs, by 1) then multiply the laſt product by the ſignificant figure to the left of the given quantity; the next product, by 10, is then to be multiplied, by the next figure of the given quantity, and ſo on throughout all the figures of the given quantity. Collect the different (or ſeveral) products, thus found, by the reſpective digits of the given quantity, and their ſum will be the anſwer required.

## EXAMPLES.

What is the amount of 648 lbs. of tea, at 4s. 6d. per lb.

| | *s.* | *d.* | |
|---|---|---|---|
| | 4 | 6 | |
| | | 10 | |
| 2 | 5 | 0 | |
| | | 10 | |
| 22 | 10 | 0 | the value of 100 |
| | | 6 | |
| 135 | 0 | 0 | the amount of 600 |
| 9 | 0 | 0 | the amount of 40 |
| 1 | 16 | 0 | the amount of 8 |
| *l.* 145 | 16 | 0 | the amount of 648 |

*Anſwers.*

| | | | *l.* | *s.* | *d.* | | | *l.* | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|---|
| 2. | 355 | at | | 2 | 3½ | per | —— | 40 | 13 | 6½ |
| 3. | 996 | — | | 14 | 5 | — | —— | 717 | 19 | 0 |
| 4. | 8954 | — | | 3 | 5 | — | —— | 1529 | 12 | 10 |
| 5. | 315 | — | 1 | 13 | 8 | — | —— | 530 | 8 | 1½ |
| 6. | 384 | — | | 2 | 4 | — | —— | 44 | 16 | 0 |
| 7. | 1784 | — | | 19 | 6 | — | —— | 1739 | 8 | 0 |
| 8. | 3552 | — | | 4 | 8 | — | —— | 828 | 16 | 0 |
| 9. | 3864 | — | | 1 | 10½ | — | —— | 362 | 5 | 0 |
| 10. | 3782 | — | | 2 | 8½ | — | —— | 512 | 2 | 11 |
| 11. | 1145 | — | | 2 | 9½ | — | —— | 159 | 16 | 5½ |

*Queſtions*

## *Queſtions, ſhewing the uſe of this Multiplication.*

1ſt, In caſting up the price of goods.

1. A merchant receives the following different ſums: I demand how many pounds ſterling they make, viz.

| | | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 467 | at | 5 | 5 | — | — | | | |
| 942 | — | 4 | 9 | — | — | | | |
| 120 | — | 18 | 6 | — | — | | | |
| | | | | | *Anſ.* | *l.* 461 | 4 | 1 |

2. Received of A. B. 582l. 14s. 8d. on account, or in part-payment of 500 caſks of butter, ſold him, viz.

| | | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 709 | at | 5 | 5 | — | — | | | |
| 296 | — | 4 | 9 | — | — | | | |
| 291 | — | 23 | 0 | — | — | | | |
| 96½ | — | 18 | 6 | — | — | | | |
| | | | | | *Anſ.* | *l.* 582 | 14 | 8 |

3. Paid Daniel Raymond in full for John Denham's bill on me, 100l. viz.

| | | l. | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 60 | at | 1 | 2 | 9 | — | — | | | |
| 21 | — | 1 | 9 | 3 | — | — | | | |
| 1 | — | | 19 | 6 | — | — | | | |
| | | | | | Change, | — | | 1 | 3 |
| | | | | | | *Anſ.* | *l.* 100 | 0 | 0 |

4. Received from Abraham Acceptor in full for Richard Drawer's bill on him, 150l. viz.

| | | l. | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 36 | at | 1 | 18 | 10 | — | — | | | |
| 11 | — | 1 | 9 | 1 | — | — | | | |
| 153 | — | | 5 | 5 | — | — | | | |
| 20 | — | 1 | 2 | 9 | — | — | | | |
| | | | | | | | 150 | 1 | 8 |
| | | | | | Change returned, | | 0 | 1 | 8 |
| | | | | | *Anſ.* | | l. 150 | 0 | 0 |

5. Received of D. B. in full, for tobacco ſold him, 27th September, 143l. 12s. 6d. viz.

| | | l. | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 29 | at | 1 | 9 | 3 | — | — | | | |
| 13 | — | 3 | 17 | 8 | — | — | | | |
| 28 | — | | 18 | 3 | — | — | | | |
| 16 | — | | 18 | 1 | — | — | | | |
| 115½ | — | | 1 | 1 | — | — | | | |
| 16½ | — | | 5 | 5 | | | | | |
| | | | | | *Anſ.* | | l. 143 | 12 | 6 |

6. Paid John Hammond 250l. for 250 barrels of beef, viz.

| | | l. | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 19 | at | 3 | 17 | 8 | — | — | | | |
| 25 | — | 3 | 17 | 6 | — | — | | | |
| 14 | — | 1 | 18 | 10 | — | — | | | |
| 20 | — | 1 | 9 | 3 | — | — | | | |
| 83 | — | | 5 | 5 | — | — | | | |
| 8 | — | | 1 | 1 | — | — | | | |
| | | | | | | | 250 | 0 | 1 |
| | | | | | Received back, | | | | 1 |
| | | | | | *Anſ.* | | l. 250 | 0 | 0 |

7. Paid S. I. 200l. 19s. 2d. in part payment of 300 tanned hides, bought of him, viz.

| | | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 100 | at | 5 | 5 | — | — | | | |
| 50 | — | 23 | 0 | — | — | | | |
| 80 | — | 18 | 6 | — | — | | | |
| 70 | — | 6 | 0 | — | — | | | |
| 90 | — | 4 | 9 | — | — | | | |
| | | | | | Anſ. | l. 200 | 19 | 2 |

8. Received of J. L. 207l. for my bill on J. C. of London, for 205l. at 8 per cent. viz.

| | | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 100 | at | 23 | 0 | — | — | | | |
| 95½ | — | 18 | 6 | — | — | | | |
| 75½ | — | 5 | 5 | — | — | | | |
| 94 | — | 5 | 10½ | — | — | | | |
| 300 | — | 1 | 1 | — | — | | | |
| 2 | — | 23 | 0 | — | — | | | |
| In copper, | | 1 | 0½ | — | — | | | |
| | | | | | Anſ. | l. 270 | 0 | 0 |

9. Paid G. F. 261l. 5s. 4½d. in part of 1500 ſtone of wool, bought of him, viz.

| | | s. | d. | | | l. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 300 | at | 5 | 5 | — | — | | | |
| 100 | — | 6 | 0 | — | — | | | |
| 150½ | — | 18 | 6 | — | — | | | |
| 199½ | — | 1 | 1 | — | — | | | |
| | | | | | Anſ. | l. 261 | 5 | 4½ |

10. Received from James Thomas, 327l. for my bill of 300l. Engliſh, which I have drawn on C. B. of London, at 9 per cent.

| | | *l.* | *s.* | *d.* | | *l.* | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|
| 13 | at | 3 | 17 | 8 | —— | | | |
| 25 | — | 3 | 17 | 6 | —— | | | |
| 14 | — | 1 | 18 | 10 | —— | | | |
| 29 | — | | 19 | 6 | —— | | | |
| 27 | — | | 19 | 4 | —— | | | |
| 21 | — | 1 | 2 | 9 | —— | | | |
| 7 | — | 1 | 9 | 3 | —— | | | |
| 9 | — | 1 | 9 | 1 | —— | | | |
| 16 | — | 1 | 8 | 11 | —— | | | |
| 26 | — | | 18 | 3 | —— | | | |
| 3 | — | | 9 | 10 | —— | | | |
| 2 | — | | 9 | 8 | —— | | | |
| 1 | — | | 9 | 2 | —— | | | |
| 1 | — | | 9 | 0 | —— | | | |
| 2 | — | | 4 | 11 | —— | | | |
| 1 | — | | 4 | 9 | —— | | | |
| | | | | | | 327 | 1 | 4 |
| | | | | | Returned, | | 1 | 4 |
| | | | | | *Anſ.* | *l.* 327 | 0 | 0 |

## *Compendiums.*

I ſhall now give you another method for caſting up merchandizes when the given quantity exceeds 144, the higheſt number in the Multiplication Table, which, probably, is preferable to any of the foregoing.

### A GENERAL RULE.

Always divide the given number by 12, then multiply the given price by 12, and the product will be 12, or one dozen, which muſt be multiplied by the number of dozens in your given integer, *(i. e.)* by the quotient of the ſaid diviſion, and you will have the anſwer.

*Note* 1ſt. If any thing remain in the diviſion, there are ſo many times the price of 1s. to be added to the ſaid product.

*Note.*

2d. If your given price conſiſts of pence only, or of pence and farthings, you need not multiply it by 12, but put down the price of 1 dozen, and multiply by the number of dozens contained in your given integers for the anſwer.

As it is plain that 12 farthings are 3 pence; 12 halfpence are 6 pence; 12 three farthings are 9 pence; and as many pence as 1 coſt, ſo many ſhillings will 12 or 1 dozen coſt: ſo if 1 coſt 2¼d. one dozen will coſt 2s. 3d. Again, if 1 coſt 11½d. twelve, or one dozen, will coſt 11s. 6d. and, thirdly, if 1 coſt 19¾d. one dozen will coſt 19s. 9d. &c.

3d. If after the diviſion your quotient happen to be a compoſite number, (*i. e.*) the product of the multiplication of two numbers,

*Rule.* Multiply the price of one dozen by one of thoſe component parts, and that product by the other part for the anſwer.

### EXAMPLES.

*lbs.* *d.*
12)168 of ſugar, at 10½ per lb.
14 multipl$^{r}$. 10 6 the amount of 1 doz.
7
3 13 6
2
*Anſ.* *l.* 7 7 0 the amount of 168lbs.

| | | | | | | | | *Anſwers.* | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | *lbs.* | | *l.* | *s.* | *d.* | | | *l.* | *s.* | *d.* |
| 2. | 216 | at | | | 17¾ | per | — | 15 | 19 | 6 |
| 3. | 576 | — | 1 | 9 | 3 | — | — | 842 | 8 | 0 |
| 4. | 1296 | — | | 4 | 3 | — | — | 275 | 8 | 0 |
| 5. | 120 | — | | 5 | 9 | — | — | 34 | 10 | 0 |
| 6. | 132 | — | | 2 | 4 | — | — | 15 | 8 | 0 |
| 7. | 384 | — | | 2 | 4 | — | — | 44 | 16 | 0 |

*Caſe* 2d. When having divided your given number by 12, your quotient happens not to be a compoſite number.

*Rule.* Multiply by the greateſt compoſite number you can find; if you work for more, ſubtract, or for leſs, add ſo many times the price of 1 dozen for the anſwer.

EXAMPLES.

### EXAMPLES.

1ſt. What is the amount of 228lbs. of any thing at 12¾d. per lb.

12)228 —— 19 the multiplier

| | | s. | d. | |
|---|---|---|---|---|
| | | 12 | 9 | the value of 1 doz. |
| | | | 6 | |
| | 3 | 16 | 6 | |
| | | | 3 | |
| | 11 | 19 | 6 | |
| | | 12 | 9 | add |
| *l.* | 12 | 2 | 3 | *Anſ.* |

| | | s. | d. | |
|---|---|---|---|---|
| *or thus:* | | 12 | 9 | |
| | | | 4 | |
| | 2 | 11 | 0 | |
| | | | 5 | |
| | 12 | 15 | 0 | |
| | | 12 | 9 | ſubtract |
| *Anſ. l.* | 12 | 2 | 3 | as before |

| | | | *l.* | *s.* | *d.* | | | *Anſwers.* *l.* | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|---|
| 2. | 2150 | at | | | 9¾ | per | —— | 87 | 6 | 10½ |
| 3. | 2103 | — | | 4 | 0 | — | —— | 420 | 12 | 0 |
| 4. | 348 | — | 4 | 11 | 11 | — | —— | 1599 | 7 | 0 |
| 5. | 696 | — | | 1 | 5¼ | — | —— | 50 | 0 | 6 |
| 6. | 1356 | — | | 4 | 7½ | — | —— | 313 | 11 | 6 |
| 7. | 708 | — | 17 | 0 | 0 | — | —— | 12036 | 0 | 0 |
| 8. | 623 | — | 3 | 9 | 0 | — | —— | 2149 | 7 | 0 |
| 9. | 318 | — | 4 | 6 | 10 | — | —— | 1380 | 13 | 0 |
| 10. | 941 | — | 7 | 0 | 4 | — | —— | 6602 | 13 | 8 |
| 11. | 532 | — | 2 | 15 | 6 | — | —— | 1476 | 6 | 0 |

*Caſe* 3d. After you have divided your given number, if any remain from that quotient, ſo many times the price of 1 muſt be added to, or ſubtracted from, the product, as you work for either more or leſs than the number of dozens contained.

EXAMPLES.

12)665 at 11¼d.
—
55 remainder 5

| | s. | d. | |
|---|---|---|---|
| | 11 | 3 | the amt. of 1 doz. |
| | | 11 | |
| 6 | 3 | 9 | |
| | | 5 | |
| 30 | 18 | 9 | |
| | 4 | 8¼ | for 5 add. |
| l. 31 | 13 | 5¼ | *Anſ.* |

| | | | s. | d. | | | *Anſwers.* l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 2. | 769 | at | 1 | 3¼ | per | —— | 49 | 13 | 3½ |
| 3. | 1048 | — | 1 | 7¼ | — | —— | 84 | 1 | 2 |
| 4. | 1722 | — | | 14¾ | — | —— | 105 | 16 | 7½ |
| 5. | 416 | — | 10 | 5½ | — | —— | 217 | 10 | 8 |

*Caſe* 4th. If when you have divided your given number, your quotient exceed 144, it may be beſt to proceed with your work, as ſhewn in *Caſe* 4th. foregoing.

EXAMPLES.

12)1824 at 3 1 2 per
—
152

| l. | s. | d. | |
|---|---|---|---|
| 3 | 1 | 2 | per |
| | | 12 | |
| 36 | 14 | 0 | for 1 dozen. |
| | | 10 | |
| 367 | 0 | 0 | |
| | | 10 | |
| 3670 | 0 | 0 | the amount of 100 doz. |
| 1835 | 0 | 0 | the amount of 50 doz. |
| 73 | 8 | 0 | the amount of 2 doz. |
| l. 5578 | 8 | 0 | the anſwer required. |

| | | l. | s. | d. | | | *Anſwers.* l. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 71346 | at | 1 | 9 | 11 | per | —— | 106721 | 14 | 6 |
| 176784 | — | 3 | 6 | 4 | — | —— | 586333 | 12 | 0 |
| 7460844 | — | 1 | 4 | 9 | — | —— | 9232794 | 9 | 0 |

### EXAMPLES IN WEIGHTS AND MEASURES.

| | | | *Anſwers.* |
|---|---|---|---|
| | lbs. oz. dwt. gr. | | lbs. oz. dwt. gr. |
| 1. Multiply | 14 10 9 21 | by 4 —— | 59 4 3 12 |
| | tons cwt. qrs. lb. | | tons cwt. qr. lb. |
| 2. Multiply | 17 17 0 24 | by 2 —— | 35 14 1 20 |
| | cwt. qrs. lb oz. drs. | | cwt. qr. lbs. oz. drs. |
| 3. Multiply | 14 0 21 0 14 | by 7 —— | 99 1 7 6 2 |
| | lbs. oz. drs. ℈ gr. | | lbs. oz. dr. ℈ gr. |
| 4. Multiply | 10 6 4 1 17 | by 9 —— | 94 11 1 1 13 |
| | yds. qrs. nai. | | yds. qr. |
| 5. Multiply | 127 0 3 | by 12 —— | 1526 1 |
| | lea. fur. per. | | lea. m. fur. per. |
| 6. Multiply | 120 7 24 | by 5 —— | 601 1 6 0 |
| | hhds. gall. pts. | | hhds. gall. pts. |
| 7. Multiply | 46 47 7 | by 3 —— | 140 17 5 |
| | days h. m. ſec. | | days h. m. ſe. |
| 8. Multiply | 365 5 48 57 | by 12 —— | 4382 21 47 24 |

*Note.* The method of proof is the ſame as in Multiplication.

---

## CHAP. VIII.

## COMPOUND DIVISION.

COMPOUND DIVISION is the dividing compound numbers into any propoſed number of equal parts.

### RULE.*

1. Place the diviſor and dividend as in Simple Diviſion.

---

* To divide a number conſiſting of ſeveral denominations, by any number whatever, is evidently the ſame as dividing all the parts or members of which that number is compoſed, by the ſame ſimple number, and this will be true when any of the parts are not an exact multiple of the diviſor; for, by conceiving the number, by which it is that multiple, to have its proper value, by being placed in the next lower denomination, the dividend will ſtill be divided into parts, and the true quotient found as before.

2. Begin at the higheſt denomination, and divide each of them by the diviſor, writing the ſeveral quotients in their proper places.

3. But if there be a remainder after dividing any of the denominations except the leaſt, you muſt find how many of the next lower denomination the ſaid remainder is equal to, and add them to the numbers (if any) which are in the dividend of this denomination; then divide the ſum by the diviſor, and ſo proceed till the work is done.

EXAMPLES.

|  | *l.* | *s.* | *d.* |  |  | *l.* | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|
| Divide | 225 | 2 | 4 | by 2 | 2) | 225 | 2 | 4 |
|  |  |  |  |  | *Anſ. l.* | 112 | 11 | 2 the quotient. |

*Anſwers.*

|  |  | *l.* | *s.* | *d.* |  |  |  | *l.* | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|---|
| 2. | Divide | 751 | 14 | 7¾ | by | 3 | —— | 250 | 11 | 6¼ |
| 3. | —— | 821 | 17 | 9¾ | — | 4 | —— | 205 | 9 | 5¼ |
| 4. | —— | 64 | 19 | 0 | — | 36 | —— | 1 | 16 | 1 |
| 5. | —— | 190 | 4 | 6 | — | 42 | —— | 4 | 10 | 7 |
| 6. | —— | 37 | 14 | 8 | — | 48 | —— |  | 15 | 8½ |
| 7. | —— | 4567 | 0 | 10 | — | 55 | —— | 83 | 0 | 8¾ |

*Caſe* 2d. When the diviſor exceeds 12, but is a compoſite number, whoſe component parts are each not greater than 12,

*Rule.* Divide the compound multiplicand continually by the component parts of the diviſor, and the laſt quotient will be the anſwer required.

*Note.* The true remainder of this kind of Diviſion is found by the method, page 39.

### EXAMPLES.

l. s. d.
If 18 pieces coſt 763 11 6 what is the price of 1 piece?

```
   l.   s.  d.
3)763  11   6
 ------------
6)254  10   6
 ------------
 l. 42  8   5 Anſ.
```

2. If 48 hundreds of ſugar coſt 51l. 5s. 9d. what is the price of 1 cwt.? *Anſ.* 1l. 1s. 4$\frac{1}{4}$d.—$\frac{3}{4}$.

3. Suppoſe 6 perſons took a farm, containing 3693 acres, I demand each man's ſhare of ſaid farm? *Anſ.* 615 a. 2 r.

4. Suppoſe 21 merchants bought at auction, ſugar, weighing 38 cwt. 1 qr. 8 lb. To have equal diviſion, what is each man's ſhare? *Anſ.* 1 cwt. 3 qrs. 8 lb. 6 oz. 1$\frac{11}{21}$ dr.

5. Suppoſe a merchant bought 42 bags of wool for 281l. 18s. 6d. What is it per bag? *Anſ.* 6l. 14s. 3d.

6. A merchant bought 36 yards of cloth for 17l. 2s. what did 1 yard ſtand him in? *Anſ.* 9s. 6d.

7. A merchant bought 132 lbs. of tea for 74l. 16s. at what rate is he to ſell it per lb. ſo as to neither gain or loſe by the bargain? *Anſ.* 11s. 4d.

8. A farmer bought a piece of land, containing 121 acres, for 96l. 16s. What does he pay per acre? *Anſ.* 16s.

*Caſe* 3d. To divide divers denominations by a number not compoſite, that is, when the diviſor is ſuch a number which can be meaſured by unity or 1 only; proceed as by *Rule* 1ſt.

### EXAMPLES.

l. s. d.
Divide 74 13 6 by 17.

*l.* *s.* *d.* *l.* *s.* *d.*

17)74 13 6(4 7 10$\frac{1}{4}$ the anſ. required.
68

6
20

133
119

14
12

174
170

4

| | | *l.* | *s.* | *d.* | | | | *Anſwers.* *l.* | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|---|
| 2. | Divide | 199 | 3 | 10 | by | 53 | —— | 3 | 15 | 2 |
| 3. | —— | 315 | 3 | 10$\frac{1}{4}$ | — | 365 | —— | | 17 | 3 |
| 4. | —— | 675 | 12 | 6 | — | 138 | —— | 4 | 17 | 11 |
| 5. | —— | 113 | 13 | 4 | — | 31 | —— | 3 | 13 | 4 |
| 6. | —— | 38 | 17 | 5$\frac{1}{2}$ | — | 47 | —— | | 16 | 6$\frac{1}{2}$ |
| 7. | —— | 132 | 0 | 8 | — | 68 | —— | 1 | 18 | 10 |
| 8. | —— | 85 | 5 | 8 | — | 43 | —— | 1 | 19 | 8 |

### EXAMPLES IN WEIGHTS AND MEASURES.

*Anſwers.*

lb. oz. dwt. gr. — lb. oz. dwt. gr.

1. Divide 8 1 15 8 by 2 —— 4 0 17 16

tons cwt. qrs. lbs. — tons cwt. qrs. lbs.

2. Divide 24 14 0 14 by 3 —— 8 4 2 23$\frac{1}{3}$

lbs. ϵ ʒ ℥ gr. — lbs. ϵ ʒ ℥ gr.

3. Divide 4 11 4 2 12 by 5 —— 0 11 7 1 2$\frac{2}{5}$

cwt. qrs. lbs oz. dr. — cwt. qrs. lbs. oz. dr.

4. Divide 17 2 27 14 15 by 4 —— 4 1 20 15 11$\frac{3}{4}$

yds. qrs. nai. — yds. qrs. na.

5. Divide 214 3 2 by 9 —— 23 3 2

6. Divide

| | *Anſwers.* |
|---|---|
| ells E. qrs. | ells E. qrs. nai. |
| 6. Divide 120 4 by 8 —— | 15 0 2 |
| lea. m. fur. per. | lea. m. fur. per. |
| 7. Divide 12 2 0 26 by 9 —— | 1 1 1 34 |
| yds. ft. in. | yds. ft. in. bar. |
| 8. Divide 147 2 11 by 10 —— | 14 2 4 2$\frac{1}{10}$ |
| hhds. gall. | hhds. gall. pt. qt. pt. |
| 9. Divide 24 51 by 11 —— | 2 16 2 1 0$\frac{1}{11}$ |
| tun p. hhd. gal. qt. | tun p. hhd. gal. qt. |
| 10. Divide 10 1 1 60 3 by 8 —— | 1 0 1 31 0$\frac{7}{8}$ |
| a. r. per. | a. r. per. |
| 11. Divide 140 2 26 by 12 —— | 11 2 35$\frac{1}{2}$ |
| days hrs. m. ſec. | days hrs. m. ſec. |
| 12. Divide 146 23 24 56 by 6 —— | 24 11 54 9$\frac{1}{3}$ |

*Caſe* 4th. Suppoſing it were required to take any fractional part or parts of a number, conſiſting of divers denominations, viz. ſuch as pounds, ſhillings and pence; hundreds, quarters and pounds, &c. &c.

*Rule.* Multiply the given number or quantity by the numerator of the fraction, as in Compound Multiplication, and divide that product by the denominator, as in Compound Diviſion, and that quotient will be the anſwer.

EXAMPLES.

A gentleman purchaſed the $\frac{4}{5}$ part of an eſtate, valued at 2713l. 12s. 8d. I demand the purchaſe-money?

| *l.* | *s.* | *d.* |
|---|---|---|
| 2713 | 12 | 8 |
| | | 4 |
| 5)10854 | 10 | 8 |
| *l.* 2170 | 18 | 1$\frac{1}{2}$—$\frac{2}{5}$ *Anſ.* |

2. Suppoſe A. and B. purchaſed a farm which coſt a [illegible], 486l. 16s. 4d. A. was to pay $\frac{5}{8}$ of the money laid down, and B. $\frac{3}{8}$. Quere the ſum paid by each? *Anſ.* A. paid 304l. 5s. 2$\frac{1}{2}$d. B. 182l. 11s. 1$\frac{1}{2}$d.

3. Suppoſe A. and B. bought 74 cwt. 1 qr. 15 lbs. of tobacco, which coſt 376l. 4s. 9d. A. to have $\frac{7}{12}$ and B. $\frac{5}{12}$: How much tobacco muſt each have, and how much muſt each pay? *Anſ.* A.'s part of the tobacco, 43 cwt. 1 qr. 15lb. 12 oz. and he is to pay 219l. 9s. $5\frac{1}{4}$d. B.'s ſhare of tobacco, 30 cwt. 3 qrs. 27 lb. 4 oz. and he is to pay 156l. 15s. $3\frac{3}{4}$d.

4. Three merchants, viz. A. B. and C. purchaſed a ſhip and cargo, which coſt 586l. 12s. 8d. A. was to pay $\frac{2}{5}$, B. $\frac{3}{8}$, and C. the remainder: What muſt each pay? *Anſ.* A. 234l. 13s. $0\frac{3}{4}$ $\frac{1}{5}$d. B. 219l. 19s. 9d. C. 131l. 19s. $10\frac{4}{5}$d.

5. Three merchants, A. B. and C. have a ſhip in company; A. hath $\frac{5}{8}$, B, $\frac{1}{4}$, and C, $\frac{1}{8}$, and they receive freight, 228l. 16s. 8d. It is required to divide it among the owners according to their reſpective ſhares? *Anſ.* A.'s ſhare, 143l. 0s. 5d. B.'s, 57l. 4s. 2d. and C.'s, 28l. 12s. 1d.

6. A man dying, left his eſtate, worth 1000l. betwixt his wife and three ſons, viz. to his wife, $\frac{1}{3}$, to the eldeſt ſon, $\frac{1}{4}$, and the remainder, to the ſecond and third, ſhare and ſhare alike: What is the ſhare of each? *Anſ.* The wife's, 333l. 6s. 8d. the firſt-ſon, 250l. the ſecond and third ſons, 208l. 6s. 8d. each

## *Compendiums.*

We ſhall now ſhew how to find what number of hundreds, pounds, yards, ells, &c. may be bought for any ſum of money, when the price of one is given, in any even number of ſhillings.

*Rule.* Annex a cypher to the right hand of the given money, and divide by half of the propoſed price.

EXAMPLES.

1. How many yards of cloth can I have for 16l. that is ſold for 8 ſhillings per yard?

*l.*
4)160

*Anſ.* 40 yards.

2. How many groſs of tape can I have for 11l. at 12s. per groſs? *Anſ.* $18\frac{1}{3}$.

3. How many yards of cloth can I have for 36l. at 8s. per yard? *Anſ.* 90.

4. How many yards of linen, at 6s. per yard, can I have for 48l.? *Anſ.* 160.

5. How many dozen of ſtockings, at 22s. per dozen, can I have for 44l.? *Anſ.* 40.

6. How many hundreds of ſugar that is ſold at 36s. per hundred, can I have for 80l.? *Anſ.* 44 cwt. 1 qr. $21\frac{7}{9}$lb.

7. How many hundreds of butter, at 42s. per cwt. can I have for 126l.? *Anſ.* 60.

We have inſiſted the longer on the two foregoing rules, that they might be well underſtood; not only for their elegant, expeditious, and eaſy diſpatch of the ordinary affairs of life, but for their utility in contracting many operations in other rules, as the Rule of Three, &c.

Other authors overlook theſe rules in their Arithmetic, imagining (it may be) the ſcholar not capable of underſtanding them till he has made further advances in the ſcience; but they are only Multiplication and Diviſion: the foregoing methods properly belong to the rules, and may be taught here, eſpecially ſince they are of ſuch excellent uſe in all manner of buſineſs, and may be of the utmoſt ſervice to ſome, whoſe leiſure or ability admit of no longer improvement in Arithmetic.

## *Compendiums in Multiplication and Diviſion*

We ſhall now give 31 Compendiums for caſting up all ſorts of merchandize, with their uſes in ſeveral queſtions in the moſt principal concerns of life:

### *Troy Weight.*

1ſt. By having the price of one penny-weight in farthings, to find what a pound coſt.

*Rule.* Take the $\frac{1}{4}$ of the price of one penny-weight in farthings, and the quotient will be the anſwer in pounds, &c.

1. If 1 dwt. of ſilver coſt $3\frac{1}{4}$d. what will 1 lb. coſt?

4)13

*l.* 3 5 the anſ. requi^rd^.

*Or thus, by the common method:*

| *d.* | *lb.* |
|---|---|
| $3\frac{1}{4}$ | 1 |
| 4 | 12 |
| 13 | 12 oz. |
| | 20 |
| | 240 dwts. |
| | 13 |
| | 720 |
| | 240 |
| | 4)3120 |
| | 12)780 |
| | 2\|0)6\|5 |
| | *l.* 3 5 *the ſame as before.* |

Here is a prodigious difference between one way and the other; one having 35 figures more in the work than the other.

2. If 1 dwt. of ſilver coſt $4\frac{3}{4}$d. what will 1 lb. coſt? *Anſ.* 4l. 15s.

3. If 1 dwt. of ſilver coſt 5d. what will 1 lb. coſt? *Anſ.* 5l.

If the quantity be any number of pounds,

*Rule.* Multiply the price of 1 dwt. in farths. and the given number of pounds together; and then divide that product by 4 and you will have the anſwer.

EXAMPLES.

4. If 1 dwt. of ſilver coſt $4\frac{3}{4}$d. what will 24 lb. coſt?

24
19

4)456

*l.* 114 the anſ. required.

5. If 1 dwt. coſt $6\frac{1}{4}$d. what will 36 lb. coſt? *Anſ.* 225l.

6. If 1 dwt. coſt $7\frac{1}{2}$d. what will 48 lb. coſt? *Anſ.* 360l.

By having the price of an ounce in farthings, to find the price of 100 lb.

*Rule.* To the number of farthings which the oz. coſts, add $\frac{1}{4}$ of itſelf and the ſum will be the anſwer in pounds ſterling.

EXAMPLES.

7. If 1 oz. coſt $7\frac{1}{4}$d. what will 100 coſt?

4)29
7 5

*l.* 36 5 the anſ. required.

8. If 1 oz. coſt $8\frac{1}{2}$d. what coſt 100 lb.? *Anſ.* 42l. 10s.

9. If 1 oz. coſt $11\frac{3}{4}$d. what will 100 lbs. coſt? *Anſ.* 58l. 15s.

The three laſt examples may be performed another way, thus: to the price of 1 oz. in farthings add a cypher; divide by 8, it gives the anſwer in pounds and ſhillings, &c.

100 lbs. at 7¼d. per oz.  
8)290

*Anſ. l.* 36 5 the ſame as the firſt.

And ſo of the remaining 2.

By having the price of 1 oz. in pence or farthings, to find the amount of 112lbs.

*Rule.* To the price of 1 oz. in farthings, add 4 times the ſaid farthings, double the firſt figure for ſhillings, and their ſum will be the anſwer.

EXAMPLES.

10. At 5¾d. the oz. what coſt 112 lb.?

23  
9 4(23×4 per rule.

*l.* 32 4 the anſ. required.

11. At 11¼d. the oz. what coſt 112 lbs.? *Anſ.* 63l.

By having the price of 1 oz. in pence or farthings, to find the value of any number of pounds.

*Rule.* Multiply the number of pounds by the price of the ounce in farthings, double the firſt figure for ſhillings, and divide that product by 8, and you will have the anſwer.

EXAMPLES.

12. At 6¼d. the oz. what coſt 480 lbs.?

480  
25

8)1200

150 the anſwer required.

13. At 7¼d. the oz. what coſt 124 lbs.? *Anſ.* 44l. 19s.

## *Avoirdupois Weight.*

By having the price of one dram in pence or farthings, to find the value of any number of lbs.

*Rule.* Multiply the price of 1 dram by 16, and that product by the number of pounds given; double the firſt figure for ſhillings, then divide that product by 6, and you will have the anſwer.

EXAMPLES.

14. At 3¼d. the dram, what coſt 8 lbs.?

```
    13
    16
   ———
   208
     8
   ———
6)166  8
  ————————
l. 27  14  8 the Anſ.
```

*By this method there are, excluſive of the anſwer and ſtating, 11 fig. in the latter and 56 in the former.*

*Or thus, by the common way:*

```
     8           3¼
    16           4
   ———           —
   128          13
    16           —
   ———       4)26624
   768         ———————
   128        12)6656
   ———          ——————
  2048        2|0)55|4  8  6
    13           ————————————
   ———           27  14  8
  6144   the ſame as before.
 2048
 —————
 26624
 —————
```

15. At 4½d. the dram, what coſt 68lb.? *Anſ.* 326l. 8s.

By having the price of one ounce in farthings, to find the value of a pound.

*Rule.* Divide the price of 1 oz. in farthings by 3, and the quotient will be ſhillings, and what remain are ſo many parts of a ſhilling.

EXAMPLES.

16. At 5¼d. the oz. what coſt 1 lb.?

```
3)21
  ——
s. 7 Anſ.
  ——
```

17. At 9¼d. the oz. what coſt 1 lb.? *Anſ.* 12s. 4d.

18. At 11¼d. the oz. what coſt 1 lb.? *Anſ.* 15s.

By having the price of 1 oz. in pence or farthings, to find the price of 100 lbs.

*Rule.* To the right hand of the farthings which the oz. coſts add a cypher; divide that by 6 and the quotient will be the anſwer.

EXAMPLES.

19. At 2¼d. the ounce, what coſt 100 lbs.?

```
6)90
  ——
l. 15 Anſ.
  ——
```

20. At 3¾d. the ounce, what coſt 100 lbs.? *Anſ.* 25l.

21. At 7½d. the oz. what coſt 100 lbs.? *Anſ.* 50l.

By having the price of one ounce in farthings, to find the price of 112 lbs.

*Rule.* Multiply 112 by the price of 1 oz. in farthings; double the firſt figure for ſhillings and divide that product by 6, and you will have the anſwer.

EXAMPLES.

22. At 4¼d. the oz. what coſt 112 lbs. of tea?

```
 112        d.
  17        4¼
 ———        4
6)190  8    —
 ———        17
l. 31 14 8 the Anſ.
```

*or thus, by the common way:*

```
      112
       16
     ————
      672
      112
     ————
     1792
       17
     ————
    12544
     1792
    —————
  4)30464
  ———————
 12)7616
 ————————
 2|0)63|4  8
     ———————
Anſ. l. 31  14  8
```

You ſee, by the firſt method, it has in the work but 15 figures; in the latter 50, excluding the anſwer in both.

23. At 8½d. the ounce, what coſt 224 lbs.? *Anſ.* 126l. 18s. 8d.

24. At 7¾d. the ounce, what coſt 346 lbs.? *Anſ.* 178l. 15s. 4d.

By having the price of 112 lbs. to find the value of one ounce.

*Rule.* Multiply the price of 112 lbs. by 5, and divide that product by 7, and ¼ of that quotient ſubtracted from itſelf is the anſwer in farthings.

EXAMPLES.

25. If 112 lbs. of tea coſt 31l. 14s. 8d. what will 1 oz. coſt?

```
                  £.  s.  d.
                  31  14   8
                           5
               ——————————————
               7)158  13   4
               ——————————————
               4)22   13   4
                  5   13   4
                 ————————————
Anſ. in farthˢ.  17    0   0
  =4¼d.          ————————————
```

*or thus, by the common way:*

```
               £.  s.  d.
  112          31  14  8
   16          20
 ————         ————
  672          634
  112           12
 ————         ————
 1792         7616( 4¼d.
 ————         7168
              ————
               448
                 4
              ————
              1792( ¼
              1792
              ————
```

The latter method amounts to upwards of 50 figures.

26. If 1 cwt. of tobacco coſt 36l. 6s. 6d. what is the value of 1 oz. at that rate?

*Anſ.* 4¾d.

By knowing the price of 120 lbs. to find the value of one ounce.

*Rule.* Divide the price of 120 by 2, and the quotient will be the anſwer in farthings.

EXAMPLES.

27. If 120 lbs. coſt 60l. what will 1 oz. coſt?

2)60
— *d.*
the anſ. in farthings. 30=7½ the oz.
—

By this method there is but 1 figure uſed beſides the anſwer; in the common method, 50.

28. If 120 lbs. coſt 80l. what will 1 oz. coſt?

*Anſ.* 10d.

*Note.* The £. which 60 lbs. coſt, will be the farthings, which 1 ounce coſts.

By knowing the price of 1 lb. in farthings to find the price of 112 lbs.

*Rule.* Set down the farthings which 1 lb. coſts; double the unit's figure for ſhillings, to which add ⅕ of itſelf for the anſwer.

EXAMPLES.

29. If 1 lb. of ſugar coſt 4½d. what coſt 112 lbs.?

£. *s.*
6)1 16 the farthings
6 doubled which
—— 1 lb. coſt.
£. 2 2 *Anſ.*
——

The common method will take nearly 30 figures.

Or thus: take 2 ſhillings and 1 groat for every penny in the price of 1 lb. If 1 lb. coſt 4½d. what will 112 lbs. come to? thus, 18 farthings, the price of 1 cwt. will be 1l. 16s. and 18 groats=6s. therefore 1l. 16s.+6s.=2l. 2s. as before.

If one ſpend any number of pence per day, the yearly expence will readily be found by taking for every penny a pound, half-a-pound, a groat, and 1 penny, their ſum will be the required yearly expence.

30. Thus, if one ſpend 15d. per day, how much will it come to in one year?

| | £. | *s.* | *d.* |
|---|---|---|---|
| | 15 | 0 | 0 |
| 15 half £. | 7 | 10 | 0 |
| 15 groats, | | 5 | 0 |
| 15 pence, | | 1 | 3 |
| | £. 22 | 16 | 3 *Anſ.* |

31. Wool, at 9d. per lb. what will 112 lbs. come to?

*Anſ.* 4l. 4s.

By knowing the price of 1

in farthings, to find the amount of 120.

*Rule.* Take the $\frac{1}{8}$ part of the price of 1 in farthings for the anſwer.

EXAMPLES.

32. Lemons, at 3¼d. what will 120 come to?

8)13

£. 1 12 6 *Anſ.*

*By the common method.*

3¼ 120
4 13
13 360
120
4)1560
12)390
2|0)3|2 6
£. 1 12 6 *Anſ.*

*You ſee by the firſt method there is but one figure uſed beſides the anſwer and ſtating; in the latter* 36.

33. At 9d per yard, what will 120 yards come to?
*Anſ.* 4l. 10s.

34. Linen. at 11½d. what is the amount of 120?
*Ans.* 5l. 15s.

By having the amount of 1 cwt. of any merchandizes, (ſubject to waſte) to find the price of 1 lb.

*Rule.* To twice the price of 112 add $\frac{1}{7}$ of itſelf, and the pounds of that ſum will be the pence; in the price of 1, and for every 5 ſhillings, allow a farthing.

EXAMPLES.

35. If 112 lbs. coſt 4l. 4s. what is that per lb.?

£. s.
4 4
2
8 8
12
the anſwer 9 0 in pence 9d.

*the common method will be upwards of 20 fig.*

36. 112 lbs. at 6l. 6s. what is that per lb.? *Anſ.* 13½d.

Of goods ſold by the thouſand, by knowing the price of 1 in farthings, to find the amount of 1000.

*Rule.* To the price of 1 in farthings, counted as pounds, add the ſame number of tenpences, ſo ſhall the ſum be the anſwer.

EXAMPLES.

37. If 1 lemon coſt 1¾d. what will 1000 coſt?

7 farthings.
0 5 10
£. 7 5 10 the *Anſ.*

38. If one dozen of any thing coſt 4¾d. what will 1000 dozen coſt? *Anſ.* 19l. 15s. 10d.

*Caſe* 2d. If the price be given in pence, to 4 times the price of 1 in pence, add ⅙ of the ſaid price for the anſwer.

EXAMPLES.

39. If 1 coſt 5d. what will 1000 coſt?

6)5
4
20
0 16 8

£.20 16 8 *Anſ.*

40. If 1 cwt. coſt 7¾d. what will 1000 coſt?
*Anſ.* £. 32l. 5s. 10d.

41. If 1 cwt. coſt 11¼d. what will 1000 coſt?
*Anſ.* 46l. 17s. 6d.

By knowing the price of 1000, to find the price of 1.

*Rule.* From the price of a thouſand ſubtract ⅕ of ⅕ of ſaid price, and the remainder will be the anſwer in farthings.

EXAMPLES.

42. If 1000 coſt 19l. 15s. 10d. what coſt 1?

| | £. | d. | s. |
|---|---|---|---|
| ⅕— | 19 | 15 | 10 |
| | 3 | 19 | 2 |
| ⅓— | 0 | 15 | 10 |
| *Anſ.* in | £.19 | 0 | 0=4¾ farthings. |

43. If 1000 coſt 25l. what will 1 coſt? *Anſ.* 6s.

Of goods ſold by five ſcore to the hundred:

44. If 1 coſt 2¼d. what will 100 coſt? *Anſ.* 18s. 9d.

*Rule.* To the right hand of the price of 1 in farthings, annex a cypher; then take ⅛ of $\frac{1}{12}$ thereof, for the anſwer.

12)90
8)7 10
£.0 18 9 *Anſ.*

*the common way, thus:*

| d. | |
|---|---|
| 2¼ | 100 |
| 4 | 9 |
| 9 | 4)900 |
| | 12)225 |
| | *Anſ.* £.0 18 9 |

45. If 1 coſt 7½d. what coſt 100? *Anſ.* 3l. 2s. 6d.

By knowing the amount of 100, to find the price of 1.

*Rule.* To 8 times the price add ⅕ of itſelf, and that ſum is the anſwer in farthings.

EXAMPLES.

46. If 1 cwt. coſt 2l. 18s. 4d. what coſt 1?

£. 2

£. s. d.
2 18 4
8

5)23 6 8
4 13 4

£. 28 0 0 *Anſ.* in farthings=7d.

47. If 100 coſt 5l. 16s. 8d. what will 1 coſt? *Anſ.* 14d.

48. If 100 coſt 11l. 13s. 4d. what will 1 coſt?
*Anſ.* 2s. 4d.

Of goods ſold by the groſs, containing 24 pieces, and each piece 36 yards, viz. 864 yards to the groſs.

By having the price of one yard, to find the price of 864 yards.

*Rule.* Multiply the price of 1 yard in farthings by 9, double the firſt figure of that product for ſhillings and you have the anſwer.

EXAMPLES.

48. If 1 yard of incle coſt 5 farthings, what will one groſs coſt, viz. 864 yards?

5
9

£. 4 10 the *Anſ.* required.

*or thus, by the common method:*

yds.
864 at 5 farthings per.
5

4)4320

12)1080

2|0)9|0

£. 4 10 as before.

49. If 1 yard of tape coſt ¾d. what will one groſs coſt?
*Anſ.* 2l. 14s.

By knowing the price of 1 groſs of 864 yards, to find the price of 1 yard.

*Rule.* To the price of the groſs add ⅑ of itſelf, and that ſum is the anſwer in farthings.

EXAMPLES.

50. If one groſs of 864 yards coſt 4l. 10s. what will 1 yard coſt?

£. s. d.
9)4 10 0
10 0

the *Anſ.* required in farths.=1¼d. £. 5 0 0 *or thus, by the common method:*

£. s.
4 10
20
—
90
12
——
864)1080(1$\frac{1}{4}$d.
864
——
216
4
——
)864($\frac{1}{4}$
864

*In the former method are but 10 figures; in the latter, 34.*

51. If a groſs coſt 2l. 5s. what will 1 yard coſt?
*Anſ.* 3d. and 3 farth.

Of goods ſold by the groſs of 144 yards.

By having the price of 1 yard, to find the amount of 144 yards.

*Rule.* If the price of 1 conſiſt but of one place of figures, multiply it by 3, and the unit's place of that product will be the ſhillings, and the tens' place will be the pounds; or thus, if the price conſiſt of 2 places of figures, double the firſt place for ſhillings, and the other figures will be pounds; to theſe pounds and ſhillings add half of themſelves, and the ſum will be the anſwer; or ſet down the given price of 1 yard in farthings, and multiply them by 3, and you will have the price of a groſs of 144, in ſhillings.

52. If 1 yard of tape coſt 1$\frac{1}{4}$d. what will 1 groſs coſt?

5
3
—
*Anſ.* 15s.
—

*the common way:*

*yd.*
144
5
——
4)720
——
12)180
——

*d.*
1$\frac{1}{4}$
4
—
5
—

the *Anſ.* 15s. as before.

*In the former, there is but one, excluſive of the anſwer and ſtating, in the latter, 20 fig.*

53. If 1 yard of tape coſt 1$\frac{3}{4}$d. what will 144 yards coſt?
*Anſ.* 1l. 1s.

54. If 1 yard of tape coſt 4d. what will 1 groſs come to?
*Anſ.* 2l. 8s.

By knowing the price of 144 yards, or 1 groſs, to find the price of 1 yard.

*Rule.* Multiply the price of the groſs by 10, and from that product ſubtract $\frac{1}{3}$ of itſelf for the anſwer in farthings.

EXAMPLES.

55. If 1 groſs coſt 1l. 1s. what will 1 yard coſt?

£. s. *by the common way.*
1 1 £. s.
10 1 1
20

3)10 10
3 10 21
12
7 0
144)252(1¾d.
the *Anſ* in 144
farths.=1¾d.
108
4
)432(¾
432

*In the former,* 16, *in the latter,* 32 *fig.*

56. If 1 groſs coſt 2l. 2s. what will 1 yard coſt?
*Anſ.* 3½d.

Of goods ſold by the great groſs, conſiſting of 12 ſmall groſs, of 144 or 1728.

By knowing the price of 1 in farthings, to find the price of a groſs, or 1728 yards.

*Rule.* Multiply the price of 1 yard in farthings, by 18, doubling the firſt figure of the product for the ſhillings, and you will have the anſwer.

EXAMPLES.

57. If 1 yard coſt 7¼ what will 1728, or 1 groſs coſt?

29
18
£.52 4 *Anſ.*

*or thus, by the common way:* *d.*
1728 7¼
29 4
15552 29
4356
4)50112
12)12524
2|0)104|4
£. 52 4 *Anſ.*

*In the former way, only* 7. *figures, in the latter,* 53.

If the price be given in ſhillings and pence,

*Rule.* Multiply the price in pence by 6, doubling the firſt figure for ſhillings, and you will have the anſwer.

EXAMPLES.

58. If 1 dozen of ſilver buttons coſt 5s. 9d. what will 1 groſs of 144 dozen coſt?

69
6
£. 41 8 the *Anſ.*

*Or thus, by the common way:*

144
69
——
1296
864
——
12)9936
2|0)82|8
——
£.41 8 *Anſ.*
——

*s. d.*
5 9
12
—
69
—

*In the firſt, only 1 figure, excluſive of the anſwer and ſtating; in the latter,* 34.

59. If 1 dozen coſt 3s. 11d. what will 144 coſt?

47
6
——
£.28 4 *Anſ.*
——

*or thus, by the common way:*

144
47
——
1008
576
——
12)6768
——
2|0)56|4
——
£.28 4 the *Anſ.*
——

*s. d.*
3 11
12
—
47
—

*In the former there is but one figure used, excluſive of the anſwer and ſtating; in the latter,* 28.

60. If 1 dozen coſt 2s. 6d. what will 144 dozen come to?
*Anſ.* 18l.

By having the price of 1 groſs, containing 144 dozen, or 1728, to find the price of one dozen.

*Rule.* Multiply the price of the groſs by 5, and divide that product by 3, and the quotient will be the price of 1 dozen, the anſ. in pence.

EXAMPLES.

60. If 1 groſs coſt 41l. 8s. what will 1 dozen coſt?

£. *s.*
41 8
5
——
3)207 0
——
69 0
the *Ans.* in
pence=5s. 9d.

*or thus, by the common method:*

£. *s.*
41 8
20
—— *Anſ.*
144)828(5*s.*9*d.*
720
——
108
12
——
144)1296(9d.
1296
——

*In the firſt method there are but 5 figures uſed, excluſive of the anſwer and ſtating; in the ſecond,* 27.

61. If 1 groſs coſt 28l. 4s. what will 1 dozen coſt?
*Ans.* 3s. 11d.

## Of Liquid Meaſure.

By knowing the price of a gallon of wine, to find the price of a tun.

*Rule.* Firſt, to the price of a gallon in pence add $\frac{1}{28}$ of itſelf, and that ſum is the anſwer.

*Rule* 2d. Multipiy 1l. 1s. by the price of the gallon in pence, and that product is the anſwer.

EXAMPLES.

62. If 1 gallon coſt 6s. 3d. what will 1 tun coſt?

d.
20)75
3 15
*Anſ.* £.78 15

*or thus, by the common way:*

s. d.
6 3
12
75

1
4
4
63
252
75
1260
1764
12)18900
2|0)157|5.
£. 78 15 *Anſ.*

*In the firſt, there are but 5 fig. uſed, (remarking as before) in the 2d, 40.*

63. At 6s. 9d. the gallon of rum, what will 1 tun come to? *Anſ.* 85l. 1s.

By knowing the price of a tun, to find the price of a gallon.

*Rule.* From the price of the tun, ſubtract $\frac{1}{3}$ of $\frac{1}{7}$, or $\frac{1}{7}$ of $\frac{1}{3}$ of the ſaid price, and the remainder is the price of 1 gallon in pence.

EXAMPLES.

64. If 1 tun of ſpirits coſt 78l. 15s. what will 1 gallon coſt?

£. s.
7)78 15
3)11 5
3 15
£.75 0 the *Anſ.* in pence=6s. 3d.

*the common method.*

1
4
4
63
252

£. s.
78 15
20
s. d.
252)1575(6 3 the *Anſ.*
1512
63
12
)756(3d.
756

*The firſt method has but 8 figures, excluſive of the anſwer and ſtating; in the ſecond, 32.*

65. If 1 tun of whiſkey, or any other X waters, coſt 85l. 1s. what is it per gallon?
*Ans.* 6s. 9d.

*Of goods ſold by the ton weight:*

By knowing the price of 1 lb. in farthings, to find the price of a ton weight.

*Rule.* Multiply the price of 1 lb. in farthings by 7, and divide the product by 3 for the anſwer.

Or, to 7 times the price of a lb. in pence, add $\frac{1}{3}$ of itſelf, and that ſum will be the anſwer.

EXAMPLES.

66. If 1 lb. of iron coſt 1¾d. what coſt 1 ton?

```
   7
   7
   —
 3)49
 ————
£. 16 6 8 the Ans.
```

*the common way:*

*In the firſt method, excluſive of the anſwer and ſtating, only 4 figures; in the ſecond, 43.*

```
          l      d.
         20     1¾
         —      4
         20     —
          4     7
         —      —
         80
         28
        ———
        640
        160
        ———
       2240
          7
      —————
    4)15680
      —————
   12)3920
      ————
   2|0)32|6 8
      ——————
Ans. £. 16 6 8
```

67. If 1 lb. of iron coſt 2½d. what will 1 ton coſt?
*Ans.* 23l. 6s. 8d.

By knowing the price of 1 ton, to find the price of a lb.

*Rule.* Multiply the price of a ton by 3, and divide that product by 7, the quotient will be the anſwer in farths.

EXAMPLES.

If a ton of iron coſt 16l. 6s. 8d. what is that a lb?

```
  £.  s.  d.
  16  6   8
          3
 ----------
7)49  0   0
 ----------
Ans. 7  0  0 in far-   d.
   ---------- things=1¾.
```

*by the common way:*

```
              £.  s.  d.
 1            16  6   8
 20           20
 --           ---
 20           326
  4           12
 --           ---   d.
 80   2240)3920(1¾ Anſ.
 28         2240
 ---        ----
 640        1680
 160           4
 ----       ----
 2240      )6720(¾
 ----       6720
            ----
```

*In the firſt there are but 6 figures, excluſive of the anſwer and ſtating; in the ſecond,* 54.

69. If 1 tun weight of any thing coſt 23l. 6s. 8d. what is it per lb.? *Anſ.* 2½d.

*Of goods ſold by the pack, of* 12 *ſcore.*

By having the price of 1 lb. in farthings, to find the price of a pack.

*Rule.* Divide the farthings the lb. coſts by 4, and the quotient is pounds; and if any thing remain, they are ſo many 5 ſhillings.

EXAMPLES.

70. If 1 lb of wool coſt 1s. 2d. what will a pack, containing 240 lbs. come to?

```
4)56 the farths. in 1s. 2d.
  --
£.14 the Ans.
  --
```

71. If 1 lb. of wool coſt 1s. 4½d. what coſt a pack containing 240 lb.

```
4)66 the farths. in 1s. 4½d.
  --
£.16  10 the Anſ.
```

*the common way:*

```
s.  d.          lbs.
1  4½           240
12               33
--             ----
16              720
 2             720
--             ----
33           2)7920
--           ------
            12)3960
             ------
            2|0)33|0
             -------
          £. 16  10 Anſ.
             -------
```

*In the firſt is but* 1 *figure, excluſive of the anſwer and ſtating; in the ſecond,* 58.

By knowing the price of a pack, to find the price of 1lb.

*Rule.* For every pound which the pack coſts, reckon 1d. in the price of the lb.; and for every 5 ſhillings, reckon 1 farthing.

72. If 1 pack of wool coſt 18l 5s. what will 1 lb. come to? *Anſ.* 1s. 6¾d.

You ſee, by inſpection of the above queſtion, and the 6 following, how all queſtions of the ſame kind are performed.

*The common way:*

£. s.
18 15
20
s. d.
240)375(1 6¾
240

135
12

)1620(6
1440

180
4

)720(¾

73. If 1 pack of wool coſt 3l. 10s. what will 1 lb. coſt?

*Anſ.* 3½d.

74. If 1 pack coſt 11l. 5s. what coſt 1 lb.

*Ans.* 11¼d.

Before we diſmiſs this excellent rule, it may not be amiſs to give a few Compendiums how to find the gain of any propoſed ſum, when the gain of one ſhilling only is given.

*Rule.* Reduce the given gain and 1 ſhilling into one denomination or name, if need be, then multiply the ſum propoſed, by the gain ſo reduced, as is taught in page 86, and you have the anſwer.

1. If 1 ſhilling gain 2d. what will 100l. gain? *Anſ.* 16l. 13s.

£.
6)100 *The common way will take* 28 *figures.*

*Anſ.* £. 16 13 4

*Demonſtration.*—As 1 ſhilling gains ⅙ of itſelf, 100l. muſt gain ⅙ of itſelf. Q. E. D.

*Anſwers.*

2. If by laying out 1 ſhilling, I gain 1¾d. what will I gain in laying out 100l. at that rate? 14l. 11s. 8d.

3. If 1 ſhilling gain 1¼d. what will 100l. gain? 10l. 8s. 4d.

4. If 1s. gain 2¼d. what will be the gain of 100l.? 18l. 14s. 2d.

5. If

| | *Anſwers.* |
|---|---|
| 5. If 1s. gain 3d. what will 85l. 16s. 6d. gain ? | 21l. 9s. 2½d. |
| 6. If 1s. gain 2¾d. what will 96l. gain ? | 22l. 0s. 0d. |
| 7. If 1s. gain 6d. what will 100 guineas gain ? | 56l. 17s. 6d. |
| 8. If 1s. gain 8d. what will 34l. 2s 6d. gain ? | 22l. 15s. 0d. |

# CHAP. IX.

## REDUCTION.*

REDUCTION is the bringing or reducing numbers of one denomination, into other numbers of another denomination, retaining the ſame value, and is performed by Multiplication or Diviſion.

1ſt. All great names are brought into ſmall ones, by multiplying with ſo many of the leſſer as make one of the greater.

2d. All ſmall names are brought into greater, by dividing with ſo many of the leſs, as make one of the greater.

---

* When greater denominations are to be brought into leſſer ones, as pounds into ſhillings; tons into hundreds, &c. &c. this is called Reduction Deſcending, and is performed by Multiplication.

When ſmall numbers are to be brought into great, as ſhillings into pounds, hundreds into tons, then it is called (perhaps improperly)) Reduction Aſcending, and is performed by Diviſion.

*Note.* When you multiply or divide, it muſt always be as many of the leſſer as make one of the greater.

Reduction by Multiplication, is performed by this general rule : Reduce the greateſt denomination to the next leſs, adding in the numbers that are of that leſs denomination ; reduce this ſum to the next lower denomination, adding in the numbers belonging thereto ; and ſo proceed till all is done.

Reduction by Diviſion, is the converſe of that by Multiplication, for all ſmall names are brought into great, by dividing the ſmall, by ſo many of theſe as make one of the greater ; the ſeveral diviſors are known from the Tables of Coin, Weights, and Meaſures.

*General Rule.* Reduce the given denomination into the next greater, and the quotient into the next greater ; and ſo on, till you come to the denomination required.

By this rule, we change Money, Weights, and Meaſures, from one denomination to another, and know how many of one are equal to any number of another.

### GENERAL RULE.

All great names are made leſs by Multiplication.
All leſs names are made greater by Diviſion.

## *Of Coin.*

Pounds multiplied by 20, are ſhillings; ſhillings multiplied by 12 are pence; and pence multiplied by 4, are farthings.

### EXAMPLES.

1. In 360l. how many farthings?

£.
360
20

7200 ſhillings.
12

86400 pence.
4

3456000 farthings; the *Anſ.* required.

2. In 345600 farthings, how many pounds, ſhillings, and pence?

4)345600 farthings.

12)86400 pence.

2|0)720|0 ſhillings.

*Anſ.* 360 pounds.

*Note.* The operation of this queſtion is the preceding one reverſed.

3. In 5l. how many pence? *Anſ.* 1200.

4. In 1200 pence, how many pounds? *Anſ.* 5l.

5. In 7l. 14s. 9¼d. how many ſhillings, pence, and farthings? *Anſ.* 154 ſhillings, 1857 pence, and 7429 farthings.

6. In 7429 farthings, how many pence, ſhillings, and pounds? *Anſ.* 7 pounds, 154 ſhillings, 1857 pence.

7. How many ſhillings and pence, in 15 crowns (each 65 pence)? *Anſ.* 81s. 3d. and 975 pence.

8. How many crowns, of 65 pence, are contained in 975 pence? *Anſ.* 15 crowns.

9. How many ſhillings and pence in 23l.? *Anſ.* 460s. and 5520d.

10. In 5520 pence, how many ſhillings and pounds? *Anſ.* 460 ſhill. and 23 pounds.

11. Reduce 351l. 13s. 0¾d. to farthings. *Anſ.* 337587

12. In 337587 farthings, how many pounds? *Anſ.* 351l. 13s. 0¾d.

13. In 51l. 16s. 9¾d. how many ſhillings, pence, and farthings? *Ans.* 1036 ſhillings, 12441 pence, and 49767 farthings.

14. In 49767 farthings, how many pence, ſhillings and pounds? *Ans.* 12441 pence, 1036 ſhillings, and 51 pounds.

15. In 19 guineas, how many pence? *Ans.* 5187.

16. How many guineas in 5187 pence? *Ans.* 19 guin.

17. In 24 guineas, how many farthings? *Ans.* 26208

18. In 26208 farthings, how many guineas? *Ans.* 24.

19. In 34 moydores (each 27s.) how many farthings? *Ans.* 108864.

20. In 108864 farthings, how many moydores, (each 27s.)? *Ans.* 84.

21. In 84 moidores, (each 29s. 3d.) how many farthings? *Ans.* 117936.

22. In 117936 farthings, how many moidores *Ans.* 84.

23. How many ſhillings in 20 guineas? *Ans.* 455.

24. How many guineas are there in 455 ſhillings? *Ans.* 20.

25. Reduce 25 half-guineas into farthings? *Ans.* 13650.

26. In 13650 farthings, how many half-guineas? *Ans.* 25.

To reduce one kind of coin into another kind of coin.

*Rule.* Bring both coins into the ſame denomination, and divide by the coin into which it is to be reduced, and you have the anſwer.

EXAMPLES.

27. How many crowns, of 5s. 5d. per, are contained in 19045 cobs, of 4s. 9d. per piece?

```
            s.  d.                 s.  d.
19045 at    4   9                  5   5
   57      12                     12
 ------    --                     --
133315     57                     65
 95225     --                     --
 ------
65)1085565(16701 Anſ.
   65
   --
   435
   390
   ---
    455
    455
    ---
     65
     65
     --
```

28. How many piſtoles of 18s. 6d. per piece, are contained in 1001 crowns, of 5s. 5d. per piece?

*Anſ.* 293 piſtoles and 1s. 7d.

29. In 16701 crowns, of 5s. 5d. each, how many cobs of 4s. 9d. each? *Anſ.* 19045

30. How many crowns, of 5s. 5d. each, are in 976 piſtoles, of 18s. 6d. each? *Anſ.* 3333 crowns and 2s. 3d.

31. How many guineas, of 23s. per piece, are worth 1984 moidores, of 30s. each? *Anſ.* 2587 guineas and 19s.

32. In 10000 pieces, of 13d. how many crowns of 5s. 5d.?

*Anſ.* 2000.

33. How many crowns, of 5s. 5d. are equal to 20000 pieces of 6½d.? *Anſ.* 2000.

34. How many cobs, of 4s. 7½d. per piece, are worth 564 ducatoons, of 5s. 10½d. per piece? *Anſ.* 716 cobs and 2s.

35. A merchant has a certain quantity of crowns, of 5s. 5d. each, which he changes with a caſhier for guineas, at 23s. per piece, and receives of him 1000 guineas, I demand how many crowns the merchant had? *Anſ.* 4246 crowns, and 10d.

## EXAMPLES IN WEIGHTS AND MEASURES.

### *Of Troy Weight.*

*Note.* Pounds multiplied by 12, are ounces; ounces multiplied by 20, are pennyweights; pennyweights multiplied by 24, are grains.

36. In 24 lb. Troy, how many ounces, pennyweights, and grains? *Ans.* 288 oz. 5760 dwts and 138240 gr.

37. How many lbs. Troy in 138240 grains? *Ans.* 24 lbs.

38. In 14 lb. 11 oz. 1 dwt. 16 gr. how many gr.? *Ans.* 85960 gr.

39. How many lbs. Troy Weight in 85960 grains? *Ans.* 14 lb. 11 oz. 1 dwt. 6 gr.

40. In 75 lb. 11 oz. 19 dwt. 23 gr. how many grains? *Ans.* 437759 gr.

41. In 437759 grains, how many lbs. Troy Weight? *Ans.* 75 lb. 11 oz. 19 dwt. 23 gr.

42. In 16 lb. 0 oz. 14 dwt. 21 gr, how many grains? *Ans.* 92417 grains.

43. In 92417 grains, how many lbs. Troy Weight? *Ans.* 16 lb. 0 oz. 14 dwt. 21 gr.

### *Of Avoirdupois Weight.*

*Note.* Tons multiplied by 20, are hundreds; hundreds multiplied by 4, are quarters; quarters multiplied by 28 are pounds; pounds multiplied by 16 are ounces; ounces multiplied by 16, are drams.

### *A compendious Method of reducing Hundreds, Quarters, and Pounds, into Pounds.*

*Rule.* Multiply the cwt. by 12, and to the product mentally add the lbs. of the odd weight, which ſum is to be ſo placed under the cwts. that the place of hundreds in this may fall under the units of that, then the ſum of theſe two lines will give the anſwer required.

44. In

## EXAMPLES.

44. In 123 cwt. 3 qrs. 10 lbs. how many pounds?

cwt. qrs. lb. lb.
123 3 10=94
1570

13870 *Ans.*

*or thus, the method generally used in schools:*

cwt. qrs. lb.
123 3 10
4

495
28

3960
991

13870 *Ans.*

*The firſt has but 4 figures, excluſive of the anſwer and ſtating: the ſecond, 14.*

The firſt will be of great uſe in the Rule of Three, Trett and Tare, &c. as it is much ſhorter and leſs ſubject to error, having but one line of Multiplication, viz. by 12.

45. In 13870 lb. how many hundred weight?
*Ans.* 123 cwt. 3 qrs. 10 lb.

46. In 264 cwt. 3 qrs. 12 lb. 11 oz. how many ounces? *Ans.* 474635.

47. In 474635 oz. how many hundreds?
*Ans.* 264 cwt. 3 qrs. 12 lb. 11 oz.

48. In 139 cwt. 1 qr. 22 lb. 13 oz. how many oz.?
*Ans.* 249901.

49. How many hundreds in 249901 oz.?
*Ans.* 139 cwt. 1 qr. 22 lb. 13 oz.

50. In 135 cwt. (ſhort cwt. each 112 lb.) how many long cwt. (each 120 lb.)?
*Ans.* 126.

51. In 976 cwt. 3 qrs. 27 lb. how many pounds?
*Ans.* 109423.

## *Of Apothecaries' Weight.*

Pounds multiplied by 12, are ounces; ounces multiplied by 8; are drams; drams multiplied by 3, are ſcruples; ſcruples multiplied by 20, are grains.

EXAMPLES.

52. In 17 lb. how many ounces, drams, and ſcruples? *Ans.* 204 ounces, 1632 drams, 4896 ſcruples.

53. How many pounds in 4896 ſcruples? *Ans.* 17 lb.

54. How many grains are in 231 lb. 3 oz. 5 gr. *Ans.* 1332005

55. In 1332005 grains, how many lbs.? *Ans.* 231 lb. 3 oz. 5 gr.

## *Of Long Meaſure.*

*Note.* Miles multiplied by 8, are furlongs; furlongs multiplied by 40, are perches; perches by 16½, are feet Engliſh, or 21, Iriſh; feet multiplied by 12, are inches; inches multiplied by 3, are barley-corns.

EXAMPLES.

56. In 273 miles, how many inches? *Ans.* 17297280.

57. In 17297280 inches, how many miles? *Ans.* 273 Engliſh.

58. In 273 Iriſh miles, how many inches? *Aus.* 22014720.

59. How many miles are there in 22014720 inches? *Ans.* 273.

60. In 45 miles, 3 furlongs, 4 yards, 3 feet, how many feet? *Ans.* 304935.

61. How many miles are there in 304935 feet? *Ans.* 45 miles, 8 furlongs, 4 yds. 3 feet.

## *Of Cloth Meaſure.*

*Note.* Yards multiplied by 4, are quarters; quarters multiplied by 4, are nails; yards multiplied by 3, are quarters Flemiſh; yards multiplied by 5, are quarters Engliſh; yards multiplied by 6, are quarters French.

EXAMPLES.

62. How many nails in 40 yards? *Anſ.* 640.

63. In 640 nails, how many yards? *Anſ.* 40 yards.

64. In 20 yards, 3 qrs. 1 nail, how many nails? *Anſ.* 333.

65. How many yards are in 333 nails? *Anſ.* 20 yards, 3 qrs. 1 nail.

66. How many quarters in 30 yards, 3 qrs.? *Anſ.* 123.

67. In 123 quarters, how many yards? *Anſ.* 30 yards, 3 qrs.

68. How many quarters in 40 ells Engliſh? *Anſ.* 200.

69. In 200 quarters, how many yards Engliſh? *Anſ.* 40.

## *Of Liquid Meaſure.*

*Note.* Tuns multiplied by 4, are hogſheads; hogſheads multiplied by 63, are gallons; gallons multiplied by 4, are quarts; quarts multiplied by 2, are pottles; pottles multiplied by 2, are pints; pints multiplied by 4, are naggins.

EXAMPLES.

70. In 20 tuns, 3 hogſheads, 50 gallons, how many gallons? *Anſ.* 5279.

71. In 5279 gallons, how many tuns? *Anſ.* 20 tuns, 3 hogſheads, 50 gallons.

72. How many pints are there in 3 hogſheads, 20 gallons, 1 quart? *Anſ.* 1674.

73. In 1674 pints, how many hogſheads? *Anſ.* 3 hogſheads, 20 gallons, 1 quart.

74. In 21 gallons, 2 quarts, 1 naggin, how many naggins? *Anſ.* 689.

75. In 689 naggins, how many gallons? *Anſ.* 21 gallons, 2 quarts, 1 naggin.

## *Of Dry Meaſure.*

*Note.* Barrels multiplied by 4, are buſhels; buſhels multiplied by 4, are pecks; pecks multiplied by 2, are gallons; gallons multiplied by 2, are pottles.

EXAMPLES.

76. How many pottles in 40 barrels, 3 buſhels, 1 peck of meal? *Ans.* 2612.

77. I demand how many barrels are in 2612 pottles of meal? *Anſ.* 40 barrels, 3 buſh. 1 peck.

78. How many packs in 17 barrels, 3 buſhels, 1 peck? *Anſ.* 285.

79. In 285 pecks, how many barrels? *Anſ.* 17 bar. 3 buſh. 1 peck.

## *Of Time.*

*Note.* Years multiplied by 365, are days; days multiplied by 24, are hours; hours multiplied by 60, are minutes; minutes multiplied by 60, are ſeconds.

EXAMPLES.

80. In 72015 hours, how many weeks? *Anſ.* 428 weeks, 4 days, 15 hours.

81. How many hours are there in 428 weeks, 4 hours, 15 days? *Anſ.* 72015.

82. In 1440 minutes, how many ſeconds? *Anſ.* 86400.

83. How many minutes are there in 86400 ſeconds? *Anſ.* 1440.

84. How many inches would reach round the globe of the earth, its circumference being 360 degrees, and each degree, 60 miles? *Anſ.* 1368576000.

85. If the circumference of the globe of the earth in inches, be equal to 1368576000 I want to know its circumference in miles? *Anſ.* 21600.

# CHAP. X.

## THE RULE OF THREE DIRECT.

THE RULE OF THREE DIRECT, teacheth, by three numbers, given to find a fourth, in ſuch proportion to the third, as the ſecond is to the firſt; for which reaſon it is called, or termed, The Rule of Three, from its having three numbers given.

It is alſo called The Golden Rule, by way of eminence, becauſe a perfect knowledge of it is of ſuch great moment in all tranſactions of trade and commerce, that indeed they could not be carried on well without it; and therefore, as it is ſo neceſſary and ſerviceable to mankind, no wonder it has been, from time immemorial, called The golden Rule.

## *A general Rule for ſtating and performing all Queſtions in this Rule.*

1ſt. State, or place the numbers in ſuch order, that the firſt and third numbers be of the ſame name or kind; and the ſecond, of the ſame name or kind with the number required.

2d. If your firſt and third terms conſiſt of divers denominations, reduce them to one, that is the loweſt denomination in either, and the ſecond into the loweſt name mentioned therein.

3d. Multiply the ſecond and third terms together, and divide that product by the firſt, the quotient will be the anſwer to the queſtion, in the ſame name or denomination the ſecond term was left in.

4th. If there happen to be a remainder after the diviſion, reduce it into the next denomination below the laſt quotient, and divide by the ſame diviſor, the quotient will be ſo many of the ſaid next name; proceed in this manner to the leaſt name, and all the quotients together will be the anſwer.

The firſt number is always known by the words *if*, *at*, or *per*; the ſecond is eaſily known by the word *coſt*, (for it depends on the firſt); the third number is always known by theſe, (or ſuch like) words, *How much? How many? What will? How long? How far? &c. &c.*

### EXAMPLES.

*Caſe* 1ſt. If 8 yards of cloth coſt 1l. 4s. 8d. what will 26 yards come to?

```
yds.  £.  s.  d.     yds.
If 8 : 1   4   8 : : 26
       20
       ——
       24
       12
      ————
      296
       26
      ————
     1776
     592
      ————
   8)7696
      ————
  12)962
      ———— d.
   2|0)8|0  2
        ———
Ans. £.4 0 2
        ————
```

Note. *When it can be done, multiply and divide as in Compound Multiplication and Diviſion* (page 84 and 91) *which will ſhorten the operation.*

*Example.* If 8 yards of cloth come to 1l. 4s. 8d. what will 24 yards tome to?

```
yds. £.  s.  d.     yds.
If 8 : 1   4   8 : : 24
                3
     ————————————
       3  14   0
               8
     ————————————
   8)29  12   0
     ————————————
   £.3  14   0 Ans.
     ————————————
```

☞ There are other methods of contractions, which will be ſhewn in their proper places.

| | Anſwers. £. | s. | d. |
|---|---|---|---|
| 2. If 13 yards of linen coſt 1l. 7s. 6d. what will 486 yards, at the ſame rate coſt? nearly | 51 | 8 | 1 |
| 3. How much will 35 yards of ſuperfine cloth come to, at 1l. 17s. 10d. per? | 66 | 4 | 2 |
| 4 If 1 barrel of barley coſt 12s. 8d. what will 110 barrels come to? | 69 | 13 | 4 |
| 5. What will 240 barrels of oats come to, at 13s. per barrel? | 156 | 0 | 0 |
| 6. If 1 ſtone of wool coſt 17s. what will 89[illegible] ſtone come to? | 762 | 9 | 0 |
| 7. What is the amount of 437 yards of broad cloth, at 16s. per yard? | 349 | 12 | 0 |
| 8. What will 379 yards of cambrick come to, at 8s. per yard? | 151 | 12 | 0 |
| 9. What will 3210 yards of cloth come to, at 15s. 7½d. per yard? | 2507 | 16 | 3 |
| 10. If 1 cwt. of butter coſt 1l. 17s. what will 3215 cwt. come to? | 5947 | 15 | 0 |

*Caſe* 2d. When your firſt and third numbers differ in name, as alſo if the middle number conſiſt of divers denominations

*Rule.* 1ſt, Reduce your third and firſt number into one name.

2d. Reduce the middle number to its loweſt mentioned; then proceed by the general rule, and the quotient ariſing will be of the ſame name as the middle number was reduced to, which muſt be reduced back by Reduction Aſcending.

11. If 5 cwt. of beef coſt 17l. 10s. what will 12 tons, 4 cwt. come to?

| | *cwt.* | | *£.* | *s.* | | *tun.* | *cwt.* |
|---|---|---|---|---|---|---|---|
| If | 5 | : | 17 | 10 | : : | 12 | 4 |
| | | | 20 | | | 20 | |

350 | 244
350
12200
732
5)85400
2|0)1708|0
*£.* 854 the *Anſ.*

12. A farm, of 456 acres, 3 roods, 36 perches, is let at the rate of 1l. 16s. 6d. per acre, I demand the yearly rent? *Anſ.* nearly 833l. 19s. 7d.

13. How many yards of cloth can be bought for 44l 16s. if for 9 yards I pay 5l. 12s.? *Anſ.* 72 yards.

14. If 1 cwt. of ſugar coſt 1l. 6s. 8d. what will 17 cwt. 2 qrs. 14 lb. come to? *Anſ.* 23l. 10s.

15. What will the coſt of 1 lb. be, if for 17 cwt. 3 qrs. 11 lb. there is paid 6l. 4s. 11¼d.? *Anſ.* 3½d.

16. What muſt be given for 7 cwt. 3 qrs. 14 lb. of beef, at 1l. 14s. 2d. per cwt.? *Anſ.* 13l. 9s. 0¼d.

17. What will 53 cwt. 1 qr. 21 lb. of cheeſe come to, at 9s. 9d. per cwt.? *Anſ.* nearly 26l. 1s.

18. At 12s. the yard, how many yards can I buy for 36l.? *Anſ.* 60 yards.

19. What is the amount of 39 cwt. 0 qrs. 10 lb. at 1l. 17s. 10d. per cwt.? *Anſ.* 73l. 18s. 10½d.

20. What is the amount of 3 caſks of merchandize, each weighing 2 cwt. at 7d. per lb.? *Anſ.* 19l. 12s.

21. If for 7s. 3d. I buy 3 quarters of a yard of cloth, how many yards can I buy for 13l. 15s. 6d.? *Anſ.* 28 yards, 2 qrs.

22. If one pair of ſhoes coſt 4s. 6d. what will one dozen come to? *Anſ.* 32l. 8s.

23. If 1 cwt. of cheeſe coſt 1l. 14s. 8d. what will 3½ lb. come to? *Anſ.* 1s. 1d.

24. Bought 7 yards of cloth for 17s. 8d. what muſt be given for 5 pieces, each containing 27½ yards

*Anſ.* 17l. 7s. 0¼d.—rem. 2.

25. If 27 yards of Holland coſt 5l. 12s. 6d. how many yards can I buy for 100l.? *Anſ.* 480 yards.

26. If 1 cwt. coſt 12l. 12s. 6d. what muſt I give for 14 cwt. 1 qr. 19 lb.? *Anſ.* 182l. 0s. 11¼d.—remr. 4.

27. If 7 oz. 11 dwt. of gold be worth 35l. what is the value of 14 lb. 9 oz. 11 dwts. 16 gr. at the ſame rate?

*Anſ.* 823l. 4s. 8d.—remr. 576.

28. A draper bought 420 yards of broad cloth, at the rate of 14s. 10¾d. per ell Engliſh, how much did he pay for the whole? *Anſ.* 250l. 5s.

29. A gentleman bought a wedge of gold, which weighed 14 lb. 3 oz. 8 dwt. for the ſum of 514l. 4s. at what rate did he pay for it per oz.? *Anſ.* 3l.

30. A grocer bought 4 hogſheads of ſugar, each weighing neat 6 cwt. 2 qrs. 14 lb. which coſt him 2l. 8s. 6d. per cwt. what is the value of 4 hhds.? *Anſ.* 64l. 5s. 3d.

31. A draper bought 8 packs of cloth, each containing 4 parcels, each parcel, 10 pieces, and each piece 26 yards, and gave at the rate of 4l. 16s. for 6 yards, I deſire to know what the 8 packs ſtood him in? *Anſ.* 6656 l.

32. If 1 oz. of ſilver coſt 5s. 9d. what muſt be paid for 73 lb. 5 oz. 15 dwts.? *Anſ.* 253l. 10s. 0¾d.

33. What is the amount of 9 pieces of cloth, each containing 21 yards, 1 quarter, if for 5 yards I pay 14s. 2d.?

*Anſ.* 27l. 1s. 10½d.

34. A bankrupt owes 1490l. 5s. 10d. and has in money and recoverable debts, 784l. 17s. 4d. how much can he pay his creditors in the pound? *Anſ.* 10s. 6¼d nearly.

35. A ſervant embezzled a hogſhead of his maſter's wine; to make reſtitution, he is to ſerve 1¼ year, at 3s. 3d. per week, what was the wine per gallon? *Anſ.* 3s. 4d. nearly.

36. What quantity of cloth can I buy for 61l. 12s. 11½d. if for 1 yard I pay 9s. 8½d.? *Anſ.* 127 yards.

*Caſe* 3d. If the firſt and third numbers are compoſite numbers, whoſe component parts are each not greater than 12,

*Rule.* Multiply the middle number by one of the component parts of the third number, and then that product by another, and ſo continually till all the parts of the third are uſed, by Compound Multiplication; then divide by the component parts of the firſt, by Compound Diviſion, and the laſt quotient will be the anſwer.

37. What is the commiſſion on 412l. 16s. 8d. at 1¼?

*Anſ.* 5l. 3s. 2d.

```
       £.   s.      £.  s.  d.
    If 100 : 25 : : 412 16  8
                             5
                    ----------
                   2064  3  4
                             5
                    ----------
              10)10320 16   8
                    ----------
               10)1032  1   8
                    ----------
                 £.103  4   2 Anſ.
                    ----------
```

38. A merchant failing in trade, owes his creditors 248l. 12s. 6d. but is willing to come to a compoſition, by paying 10s. 6d. to the pound; what ſum is he to pay?

*Anſ.* 130l. 10s. 6¾d.

If

| ſixpenc. | ſixpenc. | £. | s. | d. |
|---|---|---|---|---|
| If 40 : | 21 : : | 248 | 12 | 6 |
| | | | | 7 |
| | | 1740 | 7 | 6 |
| | | | | 3 |
| | | 8)5221 | 2 | 6 |
| | | 5)652 | 12 | 9¾ |
| | | £. 130 | 10 | 6¾ *Anſ.* |

39. A merchant in Limerick ordered his factor in Dublin to ſend him 14 cwt. of tobacco, for the carriage of which he agreed to pay 5l. 7s. 8d. What muſt be paid to have 42 cwt. carried the ſame diſtance at that rate? *Anſ.* 16l. 3s.

40. A merchant in Dublin buys 64 tuns of French wine, which ſtood him on the quay, firſt coſt, charges, &c. 963l. the queſtion is, how muſt he ſell 24 tuns of the ſame wine, ſo as to neither gain or loſe by the bargain? *Anſ.* 361l. 2s. 6d.

41. If for 35s. I can buy 7 yards of cloth, what quantity can be bought for 40l. ſterling? *Anſ.* 160 yards.

42. If 28 lb. of raiſins coſt 6s. 6d. what will 18 caſks, containing each, 3 qrs. 14 lb. come to? *Anſ.* 20l. 9s. 6d.

*Caſe* 4. Here follow 4 rules of operation, which, when they are uſed, will perform the work in a much ſhorter manner than the common one generally does.

1ſt. Divide the ſecond term by the firſt, and multiply the quotient by the third, and the product will be the anſwer of the ſame name with the middle number.

### EXAMPLES.

43. What is the amount of 40 cwt. of hops, if for 4 cwt. I pay 24l.?

*cwt.* £. *cwt.*
If 4 : 24 : : 40
6
6
£. 240 the *Anſ.*

*the common way.*

*cwt.* £. *cwt.*
If 4 : 24 : : 40
24
160
80
4)960
£. 240 *Anſ.*

*Here you see, by the firſt method, there is but* 1 *figure used* (6) *; in the ſecond,* 11, *beſides the anſwer and stating.*

44. If 12 hhds. of brandy coſt 1296l. what will 19 hhds. come to? *Anſ.* 2052l.

45. How much will 96 yards of cloth come to, if for 8 yards I pay 24s.? *Anſ.* 14l. 8s.

46. How many yards of cloth can be bought for 14l. 8s. when 8 yards coſt 24s.? *Anſ.* 96 yards.

47. If in one year, of 365 days, I ſpend 18l. 5s. what is my weekly and daily expence? *Ans.* 7s. weekly, and 1s. daily.

48. If 26 cwt. of madder coſt 68l. 18s. what is the coſt of 8 cwt. of the ſame? *Anſ.* 21l. 4s.

*Rule* 2d. Divide the third term by the firſt, multiply the quotient by the ſecond, and the product will be the anſwer of the ſame name with the ſecond number.

EXAMPLES.

49. Suppoſe I bought 25 lb. of any thing, for 1l. 12s. 6d. what will 100 lb. come to?

If

```
    lb.  £. s.  d.       25
If  25 : 1  12  6 : : 100lb.   or thus, by the common method:
         ————————      ———        lb.  £. s.  d.     lb.
     £.6 10   0 Anſ.  4       If 25 : 1  12  6 : : 100
                                         20
                                         —
                                         32
                                         12
                                        ————
                                        390
                                         100
                                        —————
                                   25)39000( 12)1560
                                      25       ————
                                     ———     2|0)13|0
                                     140      ——————
                                     125     £.6 10
                                     ———     the Anſ.
                                      15
                                     ———
```

*The first has but one figure, exclusive of the answer and stating; in the second*, 45.

50. If half a hundred of Indigo coſt 11l. 4s. what will 9 cwt. coſt? *Anſ.* 201l. 12s.

51. If half a hundred of Indigo coſt 11l. 4s. what will 10 cwt. coſt? *Anſ.* 224l.

52. What is the amount of 365 yards of rateen, if for 5 yards I pay 2l. 10s.? *Anſ.* 182l. 10s.

53. If 7 yards of linen coſt 19s. 6d. what will 112 yards come to? *Anſ.* 15l. 12s. 0d.

54. What will 600 ſheep come to, at 13l. 7s. 6d. per ſcore? *Anſ.* 401l. 5s.

*Caſe* 3d. Divide the firſt by the ſecond; divide the third by the quotient, and the laſt quotient will be the anſwer.

### EXAMPLES.

55. How many yards of cloth can be bought for 28l. 16s. when 16 yards coſt 2l. 8s.? *Anſ.* 192 yards.

If

```
    s.  yds.     £.  s.
If 48 : 16 : : 28  16
                   20
16)48           ———
   ——         3)576
    3           ———
                192 the Anſ.
```

*or thus:*

```
    s.  yds.     £.  s.
If 48 : 16 : : 28  16
                   20
                  ———
                  576
                   16
                  ———
                 3456
                 576
                 ——— yds.
              48)9216(192 Anſ.
                 48
                 ——
                 441
                 432
                 ———
                   96
                   96
                   ——
```

*In the first,* 14; *in the second,* 32 *figures.*

56. If 7 lb. of figs coſt 21d. what quantity can be bought for 63d.? *Anſ.* 21 lb.

57. If 12 tuns of wine coſt 1296l. how many tuns can be bought for 2052l.? *Anſ.* 19 tun.

58. If 9 hhds. of wine coſt 72l. how many gallons can be bought for 100l.? *Anſ.* 787 gall. and 2 qts.

59. If for 14s. 2d. I have 5 lb. of tea, what quantity can I have for 27l. 1s. 10½d.? *Anſ.* 1 cwt. 2 qr. 23 lb. 4 oz.

60. A gentleman has an eſtate of 1000l. per annum, and is rated at 2s. 6d. per pound, income tax, what is his yearly income? *Anſ* 875l.

61. Suppoſe a farmer brings to market 14 cwt. 3 qrs. 16 lb. of butter, and ſells the ſame for 28s. per cwt. what is he to receive? *Anſ* 20l. 17s.

62. What will 35 lb. of any thing come to, if 840 lb. coſt 10l. 10s.? *Anſ.* 8s. 9d.

63. Suppoſe a perſon owes 400l. 19s. 6d. but not being able to pay the whole, compounds with his creditors, to pay them 5s. in the pound, how much money muſt he pay his creditors, and how much has he ſaved? *Ans.* He paid 100l. 4s. 10½d. by this compoſition; and ſaved 300l. 14s. 7½d.

*Caſe* 5th. Divide the firſt term by the third, and the ſecond, by that quotient, and the laſt quotient will be the anſwer.

### EXAMPLES.

64. If one pound, fifteen ſhillings, gain fifteen ſhillings, what will 7 ſhillings gain? *Ans.* 3s.

```
If )35 : 15 : : 7
  7—  5)15(3s. the Anſ.
   5   —
```

65. What quantity of tallow can be bought for 64l. 8s. if for 5 cwt. I pay 3l. 10s.? *Anſ* 92 cwt.

66. What muſt I give for a caſk of rum, containing 9 gallons, if for a hogſhead I pay 15l. 13s. 10d.? *Anſ.* 2l. 4s. 10d.

67. What quantity of whiſkey can I buy for 2l. 11s. if 3 hogſheads ſtand me in 68l. 17s.? *Ans.* 7 gallons.

68. If for 56 hogſheads of cyder I pay 15l. 1s. what do 28 hogſheads ſtand me in? *Anſ.* 7l. 10s. 6d.

69. If by buying goods I find in laying out 104l. I gain 14l. how much will I gain in laying out 13l.? *Ans.* 1l. 15s.

70. If for 273 guineas I can buy 68 cwt. 1 qr. what quantity can I have for 3l. 19s.? *Ans.* 3 qrs. 13$\frac{21}{91}$ lb.

71. What quantity of cloth can I buy for 6s. if for 10 pieces (each 42 yards) I pay 126l.? *Anſ* 1 yard.

72. What will 14 oz. 8 dwt. of gold come to, at 3l. 19s. 11d. per oz.? *Anſ.* 57l. 10s. 9½d.—$\frac{2}{5}$ q.

```
 oz.  £. s.  d.     oz.dwt.          oz.  £. s.  d.      oz.dt.
If 1 : 3  19  11 : : 14 8 or thus, If 1 : 3  19  11 : : 14 8
  20          8      20               20  20              20
  —  ————————————   ————               —   —            ————
4)20 31  19   4    4)288               20  79             288
  —           9    ————                    12
  5 ————————————     72                  ————
 5)287  14    0                           959 Carried over.
    ————————————
 £.57  10   9½-⅔ Anſ.
```

*Brought over,* 959
288

7672
7672
1918

2|0)27619|2

12)13809 $\frac{12}{20}$ or 2$\frac{2}{5}$ q.

2|0)115|0 9$\frac{1}{2}$-$\frac{2}{5}$

£. 57 10 9$\frac{1}{2}$-$\frac{2}{5}$ q.

*In the 1ſt, thère are but 27 figures used, exclusive of the answer and stating; in the 2d,* 66.

*Note.* As ſeveral of the Contractions require a method of finding the greateſt common diviſor, or common meaſure of two of the terms, we ſhall therefore make it,

*Caſe* 6th. To find the greateſt common meaſure of two given numbers;

*Rule.** Divide the greater by the leſſer, if nothing remain, that leſſer number is the common meaſure required; but if there

---

* The truth of this will appear, from the firſt example; for ſince 16 meaſures 48, it meaſured 48, or 96, and ſince 16 meaſures 96 and 16, it alſo meaſures 96+16, or 112, therefore 16 meaſures 48 and 112; and it is alſo the greateſt common meaſure; for ſuppoſe there be a greater, ſince it will meaſure 48 and 112, it will alſo meaſure the remainder 16; that is, the greater meaſures the leſs, which is abſurd; therefore 16 is the greateſt common meaſure of 48 and 112.

Hence the following corollaries:

1ſt, If the two right hand figures of any number are divided by 4, the whole of that number is diviſible by 4.

2d, If the three right-hand figures of any number are diviſible by 8, the whole of that number is diviſible by 8.

3d, If the ſum of the digits of any number is diviſible by 3 or 9, the whole of that number is diviſible by 3 or 9, as was before demonſtrated.

4th, If the right-hand figure of any number be a cypher, or 5, that number is diviſible by 5.

5th, All even numbers are diviſible by 2.

6th, When one, two, &c. cyphers are on the right hand of the ſignificant figures of any number, then that number is accordingly diviſible by 10, 100, 1000, &c.

there be a remainder, divide the laſt diviſor by that remainder, and if there ſtill be a remainder, divide the laſt diviſor by the laſt remainder; and ſo continually, till you come to a diviſion without a remainder, then the laſt diviſor will be the common meaſure required.

### EXAMPLES.

1. Find the greateſt common meaſure of 48 and 112.

48)112(2
96
—
16)48(3
48

Hence 16 is the common meaſure required.

2. Find the greateſt common meaſure of 26 and 62.

26)62(2
52
—
10)26(2
20
—
6)10(1
6
—
4)6(1
4
—
2)4(2
4
—

Therefore 2, the laſt diviſor, is the greateſt common meaſure.

| | *Anſwers.* |
|---|---|
| 3. What is the greateſt common meaſ. of 117 and 45? | 9. |
| 4. Find the greateſt common meaſure of 279 and 403. | 31. |
| 5. What is the greateſt com. meaſ. of 114 and 256? | 16. |
| 6. What is the greateſt com. meaſ. of 48 and 56? | 8. |
| 7. Find the greateſt common meaſure of 28 and 112? | 28. |

---

*Note.* If the learner is taught to find the leaſt whole numbers poſſible, which have to one another that proportion which ſubſiſts between the firſt and ſecond, or the firſt and third terms of a Rule of Three ſtating (which is effected by finding the greateſt common meaſure) it will ſave a vaſt deal of trouble and labour, and render the operations in general more elegant.

73. What will 135 acres of land come to, if for 27 acres I pay 17l. 10s.?

*A.* £. *s.* *A.*

If 27 : 17 10 : : 135(5

5 135

87 10 the *Ans.* required.

*The firſt method has but 5 fig. excluſive of the anſwer and ſtating; the ſecond,* 47.

Thus, by one of theſe ſhort methods, may moſt queſtions in this rule be anſwered, and much time and many figures be ſaved.

*or thus, the common way:*

*A.* £. *s.* *A.*

If 27 : 17 10 : : 135

20

350

135

1750

1050

350

27)47250( 2|0)175|0

27

87 10

202 the ſame as

189 before.

135

135

74. How many dozen of ſtockings, at 11 groats per pair, can I have for 190l. 12s.? *Anſ.* 86 doz. 7 p. $\frac{28}{44}$.

75. If 9 tun of wine coſt 72l. how many gallons can I have for 100l.? *Anſ.* 3150 gallons.

76. A merchant bought 13 packs of goods for 326l. what will 39 packs of the ſame goods come to at that rate? *Anſ.* 978.

77. What will 144 yards of cloth come to, if for 12 yards I pay 18l.? *Anſ.* 216l.

78. If for 64 yards of cloth I pay 38l. 8s. what muſt I pay for 5 yards at that rate? *Anſ.* 3l.

79. If one pound Engliſh, be worth 1l. 13s. 4d. Flemiſh, how many pounds Flemiſh, are in 279l. 8s.? *Anſ.* 465l. 13s. 4d.

80. If 240 gallons of Madeira coſt 58l. what is the coſt of 126 gallons of the ſame, at that rate? *Anſ.* 30l. 9s.

81. If 7 lb. of ſugar coſt 5s. how many hundreds can I have for 127l.? *Anſ.* 31 cwt. 3 qrs.

82. If 135 cwt. of merchandize ſtand me in 121l. 10s. what is that per cwt.? *Anſ.* 18s.

83. If for 320l. I have 60 cwt. of hops, what quantity can I have for 4601l.? *Anſ.* 862 cwt. 2 qrs. 21 lb.

84. A merchant bought 9 cwt. 1 qr. 26 lb. of ſtarch for 77l. 8s. 9d. I demand what does it ſtand him in per lb.?

*Ans.* 1s. 5$\frac{1}{4}$d.

*Caſe* 7th. When any number of barrels, bales or packages are given, each containing an equal quantity, let the content of one, reduced to its loweſt name, be multiplied by the quantity, packages, or pieces, and then find the amount, as before.

EXAMPLES.

85. A merchant bought 12 caſks of copperas, each weighing 1 cwt. 2 qrs. 17 lb. what is the amount at 2l. 0s. 8d. per cwt.? *Ans.* 37l.

```
cwt. qr. lb.          lb.     d.      cwt.  qr.  lb.
 1   2   17       If  112  :  448  ::  19    3    8
         12                           320
 ----------                           ----
19   3    8                           2220
                                       448
                                      ----   £.   s.  d.
                              112)994560(   37    0   0
                                  ------
```

86. What will 3 caſks of raiſins, each weighing 2 cwt. 2 qrs. 25 lb. come to, at 2l. 1s. 8d. per cwt?

*Ans.* 17l. 0s. 4d. $\frac{92}{112}=\frac{2}{7}$.

87. What will 1 yard of cloth come to, if for 10 pieces (each 42 yards) I pay 126l.? *Ans.* 6s.

88. How many ingots of ſilver (each 4 lb. 1 oz. 14 dwts.) can I buy for 102l. 6s. 6d. ſuppoſing I gave at the rate of 5s. 10d. per oz.? *Ans.* 7$\frac{410}{6958}$.

89. A woollen-draper bought 10 packs of cloth, every pack had 10 parcels, and every parcel 10 pieces, and every piece 40 yards, and he gave at the rate of 2l. 10s. for every 6 yards: I deſire to know what the 10 packs coſt him, and how much per yard? *Ans.* 16666l. 13s. 4d. and 8s. 4d. per yard.

90. A grocer bought 10 hogſheads of ſugar, each weighing 10 cwt. 1 qr. 6 lb. and gave after the rate of 8d. per lb. I demand what the 10 hogſheads came to? *Ans.* 384l. 13s. 4d.

91. If 5 yards of linen coſt 11s. 2d. what muſt be given for 9 pieces, each containing 21 yards, 1 qr.

*Ans.* 21l. 7s. 1½d.

92. What will 30 boxes of raiſins, each weighing 1 cwt. 0 qr. 12 lb. come to, at the rate of 16s. 4d. per cwt.

*Ans.* 27l. 2s. 6d.

*Case* 8th. **If the given pieces, barrels, bales, &c. be unequal; ſet the contents of each, orderly under each other, and then add them together, and proceed as before.**

93. Shipped off 350 caſks of butter, wt. 546 cwt. 2 qrs. 14 lb. which coſt 22s. 6d. per cwt.—Paid duty, 6d. per cwt. —For porterage, 2l. 3s. 7d.—Cellarage, 3l. 4s. 7d.—Cooperage, 2l. 16s.—Boat-hire, 18s.—I demand what 1 cwt. ſtands me in on board?

| *cwt. qr. lb.* | | *£. s. d.* | | *s. d.* |
|---|---|---|---|---|
| 546 2 14 | Cooperage, | 2 16 0½ | Firſt coſt per cwt. | 22 6 |
| 4 | Boat-hire, | 0 18 0 | Duty do. ——— | 0 6 |
| —— | Porterage, | 2 3 7 | Petty charges, pr. cwt. | 0 4 |
| 2186 | Cellarage, | 3 4 7 | | —— |
| 28 | | ——— | the *Ans.* required. | 23 4 |
| —— | If 61222 : | 9 2 2½ | : : 112 | —— |
| 17492 | | 20 | | |
| 4373 | 24 | —— | | |
| —— | | 182 | | |
| 61222 | | 12 | | |
| | | —— | 14)112 | |
| | | 2186 | —— | |
| | | 2 | 2)8 | |
| | | —— | — | |
| | | 4373 | 4 | |

94. Bought 4 pieces of linen, containing 23, 24, 25, 27 yards, at 5s. 5d. per yard, what does the whole amount to?

*Ans.* 26l. 16s. 3d.

95. Bought 4 parcels of butter, the firſt weighed 10 cwt. 3 qr. 27 lb.; the ſecond, 13 cwt. 0 qrs. 1 lb.; the third, 23 cwt. 2 qrs.; the fourth, 19 cwt. 3 qrs. 14 lb. at 16s. 8d. the cwt. what do they amount to? *Ans.* 56l. 2s. 11d.

96. Received 4 pipes of oil, containing 480 gallons, at 5s. $5\frac{1}{2}$d. per gallon; paid for freight, 4s. per pipe; for duty, 6d. per gallon; porterage, 1s. per pipe: I demand the whole coſt, and what the oil ſtands me in per gallon?

*Ans.* 144l. and is 6s. per gallon.

97. Bought 200 carcaſes of beef, wt. nt. 850 cwt. 2 qrs. at 6s. 6d. per cwt. and ſalted them up in 400 barrels, at 2s. 9d. each; paid for 50 barrels of ſalt, at 20s. per barrel; paid for ſalting and labourers, 4l. 0s. 1d. I demand to know what 1 barrel ſtands me in? *Ans.* 19s. $3\frac{1}{4}$d.

98. Bought 3 hogſheads of ſugar; No. 1, wt. nt. 7 cwt. 2 qrs. 11 lb.; No. 2, 8 cwt. 1 qr. 10 lb.; No. 3, 9 cwt. 1 qr. 8 lb. at 37s. 4d. per cwt. I demand how much they came to, and what they coſt per lb.? *Anſ.* 47l. 3s. and 1 lb. coſt 4d.

99. A merchant bought 4 ſorts of butter for 400l. viz. at 4l. 10s., 5l. 15s., 5l. 19s. and 6l. the cwt. and had of each ſort, an equal quantity: I demand how much of each ſort he bought? *Anſ.* 18 cwt. 0 qr. $2\frac{2}{111}$ lb.

100. Bought 3 ſorts of tobacco, and of each ſort an equal quantity, the prices at 1l. 8s., 1l. 12s. and 1l. 18s. the cwt. and the whole amounts to 980l. how many cwt. of each ſort had I? *Ans.* 200 cwt.

101. A merchant bought certain merchandizes for 1900l. with liberty, if the debt was paid within a certain time to be allowed 2 per cent for ſuch payment, I demand how much ready money he is to pay? *Anſ.* 1862l.

102. How much is 1825l. deducting 2 per cent. for immediate payment? *Ans.* 1788l. 10s.

103. How much ready money muſt be paid for 1400l. 19s. 6d. deducting $2\frac{1}{2}$ per cent for commiſſion?

*Ans.* 1365l. 19s. $0\frac{3}{20}$d.

## *Queſtions for Exerciſe.*

104. If 1 lb. of pepper coſt 13d. one lb. of ſugar, 14d. and 1 lb. of tea, 22d. how many lbs. of each can be bought for 6l. 5s. 4d.? *Ans.* 30 lbs. and 2s. 10d. over.

105. Footman John ſets out from a certain place and travels 4 miles an hour; and 7 hours after Joſeph Speedwell ſets out at the rate of 6 miles an hour and purſues him: I

want to know how many miles he muſt run to overtake John, and how many hours he muſt take?

*Ans.* 84 miles, and run 14 hours.

106. Bought 7 tuns of butter for 245l. which being ſold out at 10 per cent profit, I demand the whole amount and profit on the 7 tuns? *Ans.* 269l. 10s. and profit 24l. 10s.

107. At what rate muſt the aforeſaid butter be ſold, ſo as to gain as much by every tun as 2 cwt of the ſame coſt me?

*Ans.* 38l. 10s.

108. There is an iſland 72 miles in circumference, two footmen ſet out together, and travel round it the ſame way, A. travelling 9 miles a day, and B. 7; now ſuppoſing they continue travelling till A. overtakes B. and that they come together again, pray what time will it take, and how often muſt A. move round the iſland and how often B.?

*Ans.* A. overtakes B. in 36 days, after going $4\frac{1}{2}$ times, and B. $3\frac{1}{2}$ times.

109. There is a ciſtern having 2 cocks; the firſt running alone, would empty it in $4\frac{1}{2}$ hours, the other in $6\frac{1}{2}$; the queſtion is, if both cocks were ſet running together, how ſoon would the ciſtern be emptied? *Ans.* 5 hours, 35 min.

110. Bought 420 yards of broad cloth, at the rate of 14s. $10\frac{3}{4}$d. per ell Engliſh: How much does the whole amount to? *Ans.* 250l. 5s.

111. If I buy 100 yards of ribbon, at 3 yards for a ſhilling, and a hundred at 2 for a ſhilling, and ſell out the whole at the rate of 5 yards for 2 ſhillings, how much is the gain or loſs on the whole? *Anſ.* 3s. 4d. loſs.

112. How many dozen of hats, at 18s. 6d. a piece, can be bought for 364l.? *Anſ.* 32 dozen and 9 hats.

113. Whereas, a noble and a mark, juſt 15 yards did buy,
How many ells of the ſame cloth, for 50l. had I?
*Anſ.* 600.

114. If 15 pounds coſt 16d. of bread, that's made of rye;
How many loaves, of 6 pounds, each, for four ſcore
pounds had I? *Anſ.* 3000.

115. If 15 cwt. weight of lead coſt 16 pounds in gold,
How much of that Saturnian ore, for 40l. was ſold?
*Anſ.* 37 cwt. 2 qrs.

116. If 30 pence and 40 groats buy 50 pints of wine,
What is the coſt of 60 quarts, in current Engliſh coin?
*Ans.* 1l. 18s.

117. Of Iriſh wool, for 9 full pounds, if 90d. I gave;
The value of one hundred weight, I inſtant wiſh to have?
*Anſ.* 4l. 13s. 4d.

118. If the Sun moves every day one degree, and the Moon thirteen, and at a certain time the Sun is at the beginning of Cancer, and in three days after, the Moon in the beginning of Aries; the place of their next following conjunction is required? *Anſ.* 10¾ degrees of Cancer.

119. If 16s. and a crown, for 13 weeks ſupply
With meat and drink;
In good ſound chink,
How much, each day, gave I? *Anſ.* 2¾d.

120. As I was beating on the foreſt grounds,
Up ſtarts a hare before my two greyhounds;
The dogs being light of foot did fairly run,
Unto her fifteen rods, juſt twenty-one.
The diſtance that ſhe ſtarted up before,
Was fourſcore, ſixteen rods juſt, and no more;
Now this I'd have you unto me declare,
How far they ran before they caught the hare?
*Anſ.* 336 rods.

121. If the circumference of the earth at the equator be 25020 miles, at what rate per hour, muſt a body be carried to paſs completely round it in 23 hours, 56 min. which is the length of a ſiderial day?

*Anſ.* The body muſt move 1045 miles, 3 furlongs, $9\frac{89}{359}$ perches. per hour.

122. A may-pole, 50 feet, 11 inches high, when the ſun is on the meridian, will caſt a ſhadow 98 feet, 6 inches long, I would hereby find the breadth of a river running due E. and W. within 20 feet, 6 inches on the north ſide of the foot of a ſteeple, 300 feet, 8 inches, which at the ſame time throws the extremity of its ſhadow 30 feet, 9 inches beyond the ſtream? *Anſ.* 530 feet, 5 inches, nearly.

123. Bought 45 barrels of beef, at 21s. per barrel, among which are 16 barrels, which, being damaged, I take upon being allowed 4 inſtead of three; I demand what I muſt pay for them? *Anſ.* 43l. 1s.

124. Admit a tax, or rate, of 39l. is laid on a town for the building of a bridge, and the value of the town-rent is 900l. per annum; each man for his portion paying, according to his rent; if one man in the town be worth 100l. a year, what ſhall he pay to the ſaid rate of 39l.? *Anſ.* 4l. 6s. 8d.

125. A gentleman, whoſe yearly income is 265l. 19s. 2d. would regulate his expence in ſuch a manner as to lay up 60 guineas a year: how much may he then ſpend per day?

*Anſ.* 10s. 10d.

126. Shipped for Jamaica, 550 pair of ſilk ſtockings, at 11s. 6d. per pair, and 460 yards of ſtuff, at 14d. per yard; and in return of which, I have had 46 cwt. 3 qrs. of ſugar at 24s. 6d. per cwt.; 1570 lb. of Indigo, at 2s. 4d. per lb.: what remains due to me of my adventure?

*Ans.* 102l. 12s. 11½d.

127. How many dozen of gloves, at 16d. per pair, will pay for 36 dozen and 8 pair of ſtockings, at 4s. 6d. per pair?

*Ans.* 123 doz. 9 pair.

128. Bought a parcel of cloth, at 2s. 6d. for every 2 yards, of which I ſold a certain quantity, at the rate of 18s. 9d. for 5 yards, and gained thereby as much as 180 yards coſt: now I demand how many yards I ſold? *Ans.* 1170 yards.

The following queſtions are taken from Clare's Recreations, which were judiciouſly collected by Mr. Telfair, and added to *Gough's Arithmetic;* but their ſolution not appearing in that edition, we will give them here for the improvement of the learner and eaſe to the maſter.

## *Of Sound.*

Sound, if not interrupted, is found by experiments, uniformly to move about 1150 feet in one ſecond of time.

129. How long after firing a cannon off Dunleary, may the ſame be heard at Dublin, taking the diſtance at $5\frac{2}{3}$ miles?

*Solution.* $5\frac{2}{3}$ miles=29920 feet, then as 1150 feet : 1 ſec. :: 29920 : *Ans.* $26\frac{1}{23}$.

130. If I ſee the flaſh of a piece of ordnance, fired by a veſſel in diſtreſs at ſea, which happens, we will ſuppoſe, nearly at the inſtant of its going off, and hear the report a minute and three ſeconds afterwards; how far was ſhe off, reckoning for the paſſage of ſound as before?

*min. sec. sec. s. ft. s.*

*Solution.* 1 3=63 As 1 : 1150 :: 63 : *Ans.* 13 miles 3 fur. 3 per. 10 yds. Engliſh. or 10 miles, 6 fur. 3 per. Iriſh meaſure.

## *Of Levers.*

There being three orders of Levers, or three varieties, wherein the weights, prop, or moving powers, may be differently applied to the vectis, or inflexible bar, in order to effect mechanical operations in a convenient manner.—*Of the firſt order, ſee* page 137.

## *Of the ſecond and third Order of Levers.*

In mechanics, a Lever of the ſecond order, is where the power acts at one end, the prop fixt directly at the other, and the weight ſomewhere between them.

In this order of Levers, their force is in a contra proportion to their length.

In a Lever of the third order, the prop is planted at one end of the bar, the weight at the other end, and the moving force ſomewhere between.

131. If a lever be 100 inches long, what weight, lying 7½ inches from the end, reſting on a pavement, may be moved with the force of 168 lifting at the other end of the Lever.

*Solution.* From 100 take 7½=92½; then As 7½ : 168 :: 92½ : *Ans.* 2072.

132. A water-wheel turns a crank, working 3 pump-rods, fixed juſt 6 feet from the joint or pin, by which their ſeveral levers, each 9 feet in length, are faſtened for the ſake of the intended motion; at one end, the ſuckers of the pumps being wrought by the other, ſhew them to be levers of the third order; now I would know what the length of the ſtroke in each of the barrels will be, if the crank be made to play juſt 9 inches round its centre? *Ans.* 27 inches.

*Solution.* $9 \times 2 = 18$ inches, the diameter of the crank; then, As 6 : 18 : : 9 : 27 inches,

133. With what force ought that water-wheel to be driven, which, circumſtanced as in the laſt queſtion, raiſes three cubic feet of water at every revolution of the wheel, each experimentally weighing $62\frac{1}{2}$ lb. Avoirdupois Wt. the friction of the machine rejected? *Ans.* $281\frac{1}{4}$ lb.

*Solution.* Firſt, $62\frac{1}{2} \times 3 = 187\frac{1}{2}$ lb. weight of 3 ſolid feet of water, As 6 : $187\frac{1}{2}$ : : 9 : *Anſ.* $281\frac{1}{4}$.

## *Motions of Bodies, with their Velocities.*

1ſt, If the quantities of matter, in any two or more bodies, put in motion, be equal, the forces wherewith they are moved, will be in proportion to their velocieties.

2d, If the velocities of theſe bodies be equal, their forces will be directly as the quantity of matter contained in them.

3d, If both the quantities and velocities be unequal, the forces with which the bodies are moved, will be in a proportion compounded of the quantities of matter they contain, and of the velocities wherewith they move.

134. There are two bodies, the one contains 25 times the matter of the other, (or 25 times heavier) but the leſſer moves with 1000 times the ſwiftneſs of the greater; in what proportion are the forces by which they are moved? *Anſ.* leſſer, 4 to 1.

*Solution.* As 25 : 1000 : : 1 : 40; ſo the leſſer is moved with a force 40 times greater than the other.

135. There are two bodies, one of which weighs 100 lb. the other 60 lb. but the leſſer body is impelled by a force 8 times greater than the other; the proportion of the velocities wherewith theſe bodies move is required? *Anſ.* $13\frac{1}{3}$ to 1.

*Solution.* 60 : 8 : : 100 to $13\frac{1}{3}$ to 1. the *Anſ.*

1ſt. In comparing the motion of bodies, if their velocities be equal, the ſpaces deſcribed by them are in direct proportion of the times in which they are deſcribed.

2d, If the times be equal, then the ſpaces deſcribed will be as their velocities.

3. If the times and velocities be unequal, the ſpaces will be in a proportion compounded of the times and velocities.

136. There are two bodies, one of which moves 40 times ſwifter than the other; but the ſwifter body has moved but 1 minute, whereas, the other has been in motion 2 hours; the ratio of the ſpaces deſcribed by theſe two bodies is required? *Anſ.* the ſwifter to the ſlower, as 1 to 3.

*Solution.* Firſt, 2 hours×60=120 minutes; then, as 40 : 120 :: 1 : *Anſ.* 3; the ſwifter to the ſlower follows, as 1 to 3.

137. Suppoſe one body to move thirty times ſwifter than another, as alſo the ſwifter to move 12 minutes, the other only 1; what difference will there be between the ſpaces by them deſcribed, ſuppoſing the laſt moved, 60 inches?

*Anſ.* 1795 feet

*Solution.* Firſt, 30×12=360, and 60 inches=5 feet; then, As 1 : 5 :: 360 : 1800; conſequently, 1800−5 =1795 the *Ans.*

138. There are two bodies, one of which has deſcribed 50 miles, the other only 5, but the firſt hath moved with 5 times the velocity of the ſecond; what is the ratio then of the times they have been deſcribing thoſe ſpaces?

*Anſ.* The firſt body hath been in motion double the time of the latter.

*Observation.* We have been the more particular in this rule, becauſe, writers on Arithmetic think, that if a pupil be well verſed therein, all other rules of Arithmetic will appear perfectly eaſy to him; and boys put apprentices to Mechanics, Merchants, &c. &c. knowing this rule well, and our Compendiums in Compound Diviſion, &c. (*page* 88 to 99,) will have, perhaps, but ſmall occaſion for more; which will ſave at leaſt, probably, many years of the time, uſually ſpent at ſchool.

# CHAP. XI.

## THE RULE OF THREE INVERSE.

THE RULE OF THREE INVERSE teaches, from three numbers given, (as in the Rule Direct) to find a fourth between which and one of the terms of ſuppoſition, there ſhall be the ſame proportion as between the other and the term of the demand.

In ſtating queſtions in this rule, the ſame directions are to be obſerved, as have already been given in the Rule of Three Direct; which, when you have done, and reduced your terms (if need be, as there remarked;) to find the anſwer, obſerve the following

### RULE.*

Multiply the firſt and ſecond terms together, and divide the product by the third term, and the quotient will give the fourth term, or anſwer required. The method of proof is by reverſing the queſtion.

### EXAMPLES.

1. If 8 men can do a piece of work in 12 days, how many days can 16 men perform the ſame in? *Ans.* 6 days.

```
   men  days   men
If  8  :  12  ::  16
           8
          --
       16)96( 6 days.—the Ans.
          96
          --
```

---

* The reaſon of this Rule may briefly be explained as follows: If a man perform a piece of work in 12 days, in what time will 6 men perform the ſame? Here more requires leſs; for common reaſon tells us, that if one man can perform it in 12 days, 6 men will perform it in the 6th part of the time; therefore, we can here ſay, as in the Direct proportion, As 1 : 12 :: 6 to a fourth

2. If 54 men can build a houſe in 90 days, how many men can do the ſame in 50 days? *Ans.* 97 men, nearly.

3. If, when a peck of wheat is ſold for 2s. the penny loaf weigh 8 oz. how much muſt it weigh when the peck is worth but 1s. 6d. *Ans.* 10 oz. $10\frac{2}{3}$ dr.

4. How many pieces of money, of 20s. value, are equal to 240 pieces, of 12s. each? *Ans.* 144.

5. If 4 cwt. 2 qrs. be carried 36 miles for 35s. how many lb. can I have carried 20 miles for the ſame money?
*Ans.* 907 lb. 3 oz. $3\frac{1}{5}$ dr.

6. How many yards, of 3 quarters wide, are equal in meaſure to 30 yards, of 5 quarters wide? *Ans.* 50.

7. Borrowed of my friend 64l. for 8 months, and he hath occaſion another time to borrow of me for 12 months, how much muſt I lend him to requite his former kindneſs?
*Ans.* 42l. 13s. 4d.

8. Suppoſe 800 ſoldiers in a garriſon, and their proviſion is computed ſufficient for 2 months, how many ſoldiers muſt depart, that the proviſion may ſerve them 5 months?
*Ans.* 480.

9. If 14 pioneers make a trench in 18 days, how many days will 34 men take to do the ſame?
*Ans.* 7 days, 9 hours, $52\frac{16}{17}$ min. at 12 hours per day.

10. There is a veſſel having a cock, which will empty it in 6 hours; I demand how many cocks, of the ſame capacity, there muſt be to empty the ſaid veſſel in fifteen minutes?
*Anſ.* 24 cocks.

The following are Telfair's additional queſtions to *Gough's Arithmetic*, taken from *Clare's Recreations*; we will give them a place here with their ſolutions.

*A Lever*

---

a fourth number, which ought, in that caſe, to be 6 times as much as 12; but contrariwiſe, by an inverted proportion; As 1 : 12 :: 6 to a fourth number, which, (in this caſe) muſt be 6 times leſs by the ſame rule; in the following queſtion, leſs requires more; if 6 men can do a piece of work in 2 days, in what time will one man perform the ſame? for it is evident that 1 man will take 6 times the time that 6 men would do, to perform the ſame work; hence it follows, that in all queſtions in The Rule of Three Inverſe, the product of the third term multiplied by the fourth, will be equal to the product of the 1ſt multiplied by the ſecond.

## *A Lever of the firſt Order.*

A Lever of the firſt order, hath the power at one of its ends, the weight to be raiſed is put at the other, and the fulcrum or prop ſomewhat between them. In this order, the power applied at one end will be reciprocally proportional to the diſtance of thoſe ends from the fulcrum or point ſupported; or in the ſteel-yards, as the diſtance of the weight from the point of ſuſpenſion.

10. What weight will a man be able to raiſe, who preſſes with the force of a hundred and a half, on the end of an equipoiſed handſpike, 100 inches long, which is to meet with a convenient prop, exactly $7\frac{1}{2}$ inches above the other end of the machine? *Anſ.* $18\frac{1}{2}$ cwt. or 2072 lb.

*Solution.* Then $100-7\frac{1}{2}=92\frac{1}{2}$ As $92\frac{1}{2}$ : $1\frac{1}{2}$ or 168 lb. : : $7\frac{1}{2}$ to $18\frac{1}{2}$ cwt. or 2072 lb. the *Anſ.*

11. What weight, hung at 70 inches diſtance from the fulcrum of a ſteel-yard, will equipoiſe a hogſhead of tobacco, weighing $9\frac{1}{2}$ cwt. freely ſuſpended at 2 inches diſtance on the contrary ſide? *Anſ.* 30 lb. 6 oz. $6\frac{28}{70}$ dr.

*Solution.* As 2 inch. : $9\frac{1}{2}$ cwt. : : 70 inch. or as 2 inch. : 1064 lb. : : 70 inch. : 30 lb. 6 oz. $6\frac{28}{70}$ dr.

## *Motion of Bodies, with their Velocities.*

In comparing the motion of bodies, the ratio or proportion between their velocities, will be compounded of the direct ratio of the forces wherewith they are moved, and the reciprocal of their quantities of matter they contain.

12. The battering ram of Veſpaſian weighed, ſuppoſe 100000 lb. and was moved, let us admit, with ſuch a force or velocity, by ſtrength of hands, as to paſs through 20 feet in one ſecond of time, and this was found ſufficient to demoliſh the walls of Jeruſalem. With what velocity muſt a bullet that weighs but 30 lb. be moved, in order to do the ſame execution?

*Solution.* As 100000 : 20 : : 30 : 66666 feet, 8 inch. per ſecond, the *Anſ.* required.

13. A body, weighing 200 lb. is impelled by ſuch a force as to ſend it 100 feet in a ſecond; with what velocity would a body of 8 lb. move, if it were impelled by the ſame force?

*Anſ.* 2500 feet.

*Solution.* As 200 lb. : 100 feet :: 8 lb. : 2500 feet per ſecond, the *Anſ.* required.

# CHAP. XII.

## THE DOUBLE RULE OF THREE.

THE DOUBLE RULE OF THREE is ſo called, from its having 5 numbers or terms given to find a ſixth, which, if the proportion is direct, the ſixth term muſt bear the ſame proportion to the 4th and 5th compounded, as the third bears to the firſt and ſecond compounded; but if the proportion have one ſtating direct and the other inverſe, then the ſixth term muſt bear ſuch proportion to the fourth and fifth compounded, as the firſt bears to the ſecond and third compounded, or as the ſecond bears to the firſt and third. The three firſt terms form a ſuppoſition; the two laſt a demand: one of the terms of the ſuppoſition being of the ſame name with the number required, and the other two of the ſame kind as the demanding terms.

It is ſometimes uſual to make two ſtatings in queſtions of compound proportion, and to diſtinguiſh them into direct and reciprocal proportion; we ſhall give three methods of working all queſtions in this Rule, and leave the reader to choſe that which he thinks beſt, but for our part, we always prefer the firſt.

1. *Rule.* Let the principal cauſe of loſs or gain, intereſt or decreaſe, action or paſſion, be put in the firſt place.

2. Let that which betokeneth time, diſtance of place, and the like, be in the ſecond place, and the remaining one of the ſuppoſition in the third.

3. Place the other two terms, forming the demand, under their like in the ſuppoſition.

4. If the blank fall under the third term, the queſtion belongs to the Compound Rule of Three Direct. Then multiply the firſt and ſecond terms together, for a diviſor, and the other three for a dividend, the quotient is the anſwer.

5. But if the blank fall under the firſt and ſecond terms, the queſtion belongs to the Double or Compound Rule of Three Inverſe. Then multiply the third and fourth for a diviſor, and the other three for a dividend, and the quotient will be the anſwer.

### EXAMPLES.

1. If 240l. in 16 months gain 64l. how much will 60l. gain in 6 months? *Anſ.* 6l.

*prin. c. time. gain.*
240 : 16 :: 64
60 : 6 *Blank.*

* Here the blank falls under the 3d term the queſtion is direct.

240 × 16 = 3840 divſr.
64
360
3840
192
3840)23040(£.6 the *Anſ.*

*or thus, by 2 ſtatings:*

If 240 : 64 :: 60
4 )64(16 1

*m. l. m.*
If 16 : 16 :: 6 *Anſ.* 6.

*or thus, by a double ſtating:*

$$\text{If }\left.\begin{matrix}240\\16\end{matrix}\right> 64 \left<\begin{matrix}60\\6\end{matrix}\right.$$

Diviſor 3840
360
64
)23040(6
23040

This Rule is alſo performed by one continued ſtating.

*Rule.* The third number from the left hand muſt always be of the ſame name or denomination with that which is ſought for. Thoſe two numbers on which the third depends muſt be the firſt and ſecond; then place the other two numbers in ſuch order, that the fourth may be of the ſame name or denomination with the firſt, and the fifth of the ſame name with the ſecond. Then multiply the firſt and ſecond for a diviſor, and the other three for a dividend, and the quotient will be the 6th or anſwer ſought, when direct.

£. *m.* £. £. *m.*
The foregoing queſtion 240—16—64—60—6
16

$\overline{3840}$ $64\times60\times6=23040$
$\frac{23040}{3840}=6$ *Anſ.*

2. If a quantity of proviſion ſerve 1400 men for 5 months, at the rate of 14 oz. per day, each man, how many men will the ſame proviſion maintain for 8 months, at the rate of 8 oz. per day each? *Anſ.* 500.

*prin.c. time. gain.*
1400—5——14 $14\times8=112$ a diviſor.
*Blank.** 8—— 8
$1400\times5\times8=56000$ for a dividend;
therefore, $\frac{5600}{112}=50$.

*Note.* Here the blank falling under the firſt term, ſhews the queſtion to be inverſe.

The firſt method is ſo general as to comprehend all caſes that come under the Compound Rule, whether Direct or Inverſe, Integers or Fractions; ſo that queſtions in this Rule, any how propoſed, may be readily ſolved thereby.

3. Suppoſe 140l. would defray the expences of 5 men, for 24 weeks and 4 days; how long would 12 men be in ſpending 200l. at the ſame rate

*Anſ.* 3 months, 2 weeks, $4\frac{64}{168}$ days.

4. If 14 horſes eat 56 buſhels of oats in 16 days, how many buſhels will be ſufficient for 20 horſes for 24 days? *Anſ.* 120.

5. If 8 men in 14 days can mow 112 acres of graſs, how many men muſt there be to mow 2000 acres in 10 days? *Anſ.* 200.

6. If 100l. in 12 months gain 6l. intereſt, how much will 75l. gain in 9 months? *Anſ.* 3l. 7s. 6d.

7. If a carrier receive 2l. 2s. for the carriage of 3 cwt. 150 miles, how much ought he to receive for the carriage of 7 cwt. 3 qrs. 14 lb. for 50 miles. *Anſ.* 1l. 16s. 9d.

8. An uſurer put 75l. out to receive intereſt for the ſame, and when it had continued for 9 months, he received for principal and intereſt, 78l. 7s. 6d. I demand at what rate per cent. per annum he received intereſt? *Anſ.* 6 per cent.

9. If a regiment of ſoldiers, conſiſting of 136 men, conſumed 351 quarters of wheat in 108 days, how many quarters of wheat will 11232 ſoldiers conſume in 56 days?
*Anſ.* $15031\frac{1}{17}$.

10. If 40s. will pay 8 men for 5 days' work, how much will pay 32 men for 24 days? *Anſ* 38l. 8s.

11. If 100l. in 12 months gain 7l. intereſt, what is the intereſt of 571l. for 6 years? *Anſ.* 239l. 16s. $4\frac{3}{4}$d.

12. If I pay 10s. for the carriage of 2 tons, 6 miles, what muſt I pay for the carriage of 12 tons, 17 cwt. 17 miles?
*Anſ.* 9l. 2s. $0\frac{1}{2}$d.

13. If 80000 cwt. of ammunition was to be removed from a place in 9 days, and that in 6 days time I find 4500 cwt. is carried away by 18 horſes, how many will be wanted to carry away the remainder in 3 days. *Anſ.* 604.

14. If 8 cannon in 1 day, conſume 48 barrels of powder, How many barrels will ſerve 24 cannon for 12 days at that rate? *Anſ.* 1728.

15. If a perſon travel 300 miles in 10 days, when the day is 12 hours long, in how many days may he travel 600 miles when the days are 16 hours long? *Anſ.* 15.

16. If 248 men in 5 days, of 11 hours each, can dig a trench 230 yards long, 3 feet wide and 2 feet, deep; in how many days, of 9 hours long, will 24 men dig a trench of 420 yards long, and 3 deep? *Anſ.* $288\frac{59}{207}$

### *The following Queſtions are taken from Clare's Recreations:*

17. A weight of $1\frac{1}{2}$ lb. laid on the ſhoulder of a man, is no greater burthen to him than its abſolute weight; what difference will he feel between the ſaid weight, applied near his elbow, at 12 inches from the ſhoulder, and in the palm of his hand, 28 inches therefrom; and how much more muſt his muſcles then draw to ſupport it at right angles; that is, have his arm extended right out? *Anſ.* 24 lb.

$$\left.\begin{array}{l}\qquad\qquad\quad \textit{inch.}\quad \textit{lb.}\quad \textit{inch.}\quad \textit{lb.}\\ \textit{Solution.}\ \text{As } 1 : 1\frac{1}{2} :: 12 : \text{to } 18 \\ \text{Again, As } 1 : 1\frac{1}{2} :: 28 : \text{to } 42 \text{ lb. wt.}\end{array}\right\} \begin{array}{c}\\12\\28\end{array} \left\{\begin{array}{l}\\\text{From the}\\\text{shoulders.}\end{array}\right.$$

Conſequently, $42-18=24$ lb. the *Anſ.* required.

18. A. and B. are on oppoſite ſides of a wood, 134 fathoms about; they begin to go round it both the ſame way, at the ſame inſtant of time: A. goes 11 fathoms in 2 minutes, and B. 17 in 3; the queſtion is, how many times will they ſurround this wood, before the nimbler overtakes the ſlower?

*Anſ.* 17 times by A. 16 times by B.

*Solution.* As 2 : 11 :: 3 : 16½ A. goes, while B. goes 17. then 17—16½ B. gained of A. in going 17 fathoms.

Again, As ½ : 17 :: ½ to 17 times round, gone by A. and 16½ by B. the anſwer.

19. If a lever, 40 effective inches long, will, by a certain power thrown ſucceſſively thereon, in 13 hours raiſe a weight 104 feet, in what time will two other levers, each 18 effective inches long, raiſe an equal weight 73 feet; the force of ſtraight levers being in a direct proportion of their lengths?

*Anſ.* 10 hr. 8 min. 20 ſec.

# CHAP. XIII.

## VULGAR FRACTIONS.

A FRACTION is that which expreſſeth a part or parts of a quantity, and conſiſts of two terms, called its Denominator and Numerator; the firſt denotes the number of parts into which the Integer is divided, and the other, the number of ſuch parts as are contained in the Fraction.

Fractions are *proper* or *improper*, *ſimple* or *compound*. Proper Fractions have their numerators leſs than their denominators; thus $\frac{2}{3}$ and $\frac{3}{4}$ are proper Fractions.

Compound Fractions are Fractions of Fractions; thus, $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ are Compound Fractions.

Improper Fractions have their numerators equal to, or greater than their denominators: thus, $\frac{3}{2}$, $\frac{4}{3}$ and $\frac{7}{7}$ are improper Fractions.

A number is called mixed when it is integral and fractional, as $2\frac{5}{6}$; that is, when it conſiſts of a whole number and fraction, as $6\frac{3}{5}$, &c.

The common meaſure of two or more numbers, is a number which divides or meaſures them both without a remainder; thus, 3 is a common meaſure of 9, 12 and 21.

And how the greateſt common meaſure may be found, ſee *Euclid*, VII. Pro. 1, 2, 3.

* *Prob* 1. To find the greateſt common meaſure of any two numbers.

*Rule.* Divide the greater by the leſs, and the diviſor by the remainder, and ſo on, till nothing remain, and the laſt diviſor will be the greateſt common meaſure required.

*Note.* If the laſt diviſor happen to be unity or 1, then are thoſe numbers prime, and are already in their loweſt terms.

☞ This rule has been largely inſiſted on; *page* 123.

| | | | *Anſwers.* |
|---|---|---|---|
| 1. | Find the greateſt com. meaſ. of | 30950 & 120086. | 1238. |
| 2. | —— —— | 10138 & 10686. | 274. |
| 3. | —— —— | 194928 & 215364. | 1572. |
| 4. | —— —— | 4845 & 8265. | 285. |
| 5. | —— —— | 4628 & 9968. | 356. |
| 6. | —— —— | 10539 & 18736. | 1171. |
| 7. | —— —— | 122755 & 227580. | 18965. |

Prob.

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* PARTICULAR RULES.

1. Any number ending with an even number, or a cypher, may be divided by 2.

2. Any number ending with 5 or 0, is diviſible by 5.

3. Any number is diviſible by 3, if the ſum of its digits be ſo: thus, 417 is diviſible by 3; becauſe 12, which is the ſum of 4, 1 and 7, is ſo.

EXAMPLES.

$$\frac{45}{120}=\frac{15}{40}=\frac{3}{8}$$

$$\frac{126}{441}=\frac{42}{147}$$

$$\frac{534}{672}=\frac{267}{336}$$

4. If there be any cyphers at the end of each, cut off as many as are common to both.

EXAMPLES.

*Prob.* 2. To reduce a Fraction to the loweſt denomination,

*Rule.* Find the greateſt common meaſure, by which divide the numerator and denominator, the quotients are the Fractions in their loweſt denomination.

| | | | | *Com. meaſure.* | *Anſwers.* |
|---|---|---|---|---|---|
| 8. | Reduce | $\frac{16623}{20317}$ | to the loweſt terms. | 1847 | $\frac{9}{11}$ |
| 9. | ——— | $\frac{370}{555}$ | ——— | 185 | $\frac{2}{3}$. |
| 10. | ——— | $\frac{9144}{10287}$ | ——— | 1143 | $\frac{8}{9}$. |
| 11. | ——— | $\frac{1890}{2646}$ | ——— | 378 | $\frac{5}{7}$. |
| 12. | ——— | $\frac{10539}{18736}$ | ——— | 1171 | $\frac{9}{16}$. |
| 13. | ——— | $\frac{287937}{391968}$ | ——— | 1 | $\frac{287937}{391968}$. |
| 14. | ——— | $\frac{1284324}{1454100}$ | ——— | 1572 | $\frac{817}{925}$. |
| 15. | ——— | $\frac{44233}{62125}$ | ——— | 497 | $\frac{89}{125}$. |

When the numerator and denominator do each of them end with cyphers, ſtrike off an equal number of cyphers in both, and the remaining will be a Fraction of the ſame value, which may be reduced to its loweſt terms.

### EXAMPLES.

| | | | | *Anſwers.* |
|---|---|---|---|---|
| 16. | Reduce | $\frac{19000}{27000}$ | to its loweſt terms. | $\frac{19}{27}$. |
| 17. | ——— | $\frac{420}{5400}$ | ——— | $\frac{7}{90}$. |

When

### EXAMPLES.

$$\frac{200}{340}=\frac{20}{34}=\frac{10}{17}$$

$$\frac{1200}{18000}=\frac{12}{18}=\frac{2}{3}$$

5. When any number which is expreſſed by ſeveral others, with the ſigns of Addition or Subtraction between them, is to be divided by any number; then all the parts of it muſt be divided by this number:

$$\text{Thus, } \frac{4+6-8}{2}=2+3-4=5-4=1$$

6. But if the given number be expreſſed by orders, with the ſign of Multiplication between them, their product only muſt be divided; ſo

$$\frac{7\times3\times8\times10}{7\times2\times6}=\frac{3\times4\times10}{1\times6}=\frac{1\times4\times10}{1\times2}=\frac{1\times2\times10}{1\times1}=\frac{20}{1}=10$$

And in this caſe, when the ſame number is in both the numerator and denominator, it may be left out of them.

N. B.

When you diſcern any number will equally divide both numerator and denominator, you may abbreviate the Fractions thereby.

*Anſwers.*

18. Reduce $\frac{24}{112}$ to its loweſt terms. $\frac{3}{14}$.
19. ——— $\frac{96}{324}$ ——— $\frac{8}{27}$.

*Prob.* 3. To reduce compound Fractions to ſimple ones,

*Rule.* Multiply the numerators together for a new numerator, and the denominators for a new denominator.

*To reduce a Fraction to its loweſt terms, by the laſt Prob.*

*Rule.* When it can be done, you may cancel the Fractions, by dividing the numerator and denominator of any two terms by the ſame number, and uſe the quotients inſtead thereof.

**EXAMPLES.**

*Anſwers.*

20. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{2}{7}$ of $\frac{2}{3}$ to a ſimple Fraction. $\frac{2}{21}$.
21. ——— $\frac{1}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ of $\frac{2}{7}$ ——— $\frac{1}{28}$.
22. ——— $\frac{1}{7}$ of $\frac{7}{8}$ of $\frac{8}{5}$ of $\frac{5}{3}$ of $\frac{2}{7}$ ——— $\frac{2}{21}$.
23. ——— $\frac{5}{3}$ of $\frac{3}{5}$ of $\frac{2}{9}$ of $\frac{9}{14}$ ——— $\frac{1}{7}$.
24. ——— $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{8}{15}$ of 15 $\frac{168}{81} = \frac{56}{27}$.

---

N. B. But inſtead of writing down the whole Fraction anew, after every diviſion in the abbreviation, as is here done in the laſt example in 5 different values, one after another, in the whole length of the line; the ſhorteſt method, and which is conſtantly practiſed by thoſe who beſt know the value and uſe of this part of the abbreviation, is thus: whatever numbers are abbreviated, or which cancel one another, draw a ſmall line through them with the pen, and place the quotients of the numbers above them, or below them, according as they are in the numerator or denominator. So the foregoing Fraction being here again written down, daſh or draw a ſmall line through the two ſevens, becauſe they cancel each other as being equal; then daſh the 3 and the 6, and write the quotient 2 below the 6; next daſh the 8 and the 2, writing the quotient 4 above the 8; laſtly, daſh the 4 and the other 2, writing the quotient 2 above the 4; there being then no more figures to divide by; multiply together the numbers 2 and 10, which are not daſhed, and the product 20, is the value of the Fraction required; and in this manner let the pupil be exerciſed in many examples in this Rule till he is perfect in

$$\frac{\cancel{7}\times\cancel{3}\times\overset{\overset{2}{\cancel{4}}}{\cancel{8}}\times 10}{\cancel{7}\times\cancel{2}\times\underset{2}{\cancel{6}}}=20$$

*Prob.* 4. To reduce mixed numbers to improper Fractions.

*Rule.* 1. If the whole number has no aſſigned denominator, an unit ſubſcribed underneath, muſt be the denominator.

2. If the whole number has an aſſigned denominator, then multiply the whole number by the aſſigned denominator, and the product will be the numerator to the aſſigned denominator.

3. If the whole number has a Fraction annexed, multiply the whole number by the denominator of the Fraction, and to the product add the numerator, which place over the denominator.

EXAMPLES.

| | *Anſwers.* |
|---|---|
| 25. Reduce 12, 14 and 120 to Fractions. | $\frac{12}{1}$ $\frac{14}{1}$ and $\frac{120}{1}$ |
| 26. ——— 30 to a Fraction, whoſe denominator ſhall be 15. | $\frac{450}{15}$ |
| 27. Reduce $2\frac{3}{7}$ to an improper Fraction. | $\frac{17}{7}$ |
| 28. ——— $5\frac{2}{7}$ ——— | $\frac{37}{7}$ |
| 29. ——— $8\frac{1}{7}$ ——— | $\frac{57}{7}$ |
| 30. ——— $12\frac{1}{7}$ ——— | $\frac{85}{7}$ |

*Prob.* 5. To reduce an improper Fraction into its equivalent whole number and proper one, that is, into a whole or mixt number.

*Rule.* Divide the numerator by the denominator, you have the equivalent; under the remainder (if any) ſubſcribe the denominator.

EXAMPLES.

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in it, becauſe it is of the greateſt uſe of any, by abbreviating all Arithmetical operations, wherever Multiplication and Diviſion are concerned. Where this method is deviated from in any of the following pages of this book, it is done only to avoid the trouble of expreſſing theſe operations properly in print.

### EXAMPLES.

| | | *Anſwers.* |
|---|---|---|
| 31. | Reduce $\frac{25}{3}$ to a mixed or proper Fraction. | $8\frac{1}{3}$ |
| 32. | ——— $\frac{738}{18}$ ——— | $41$ |
| 33. | ——— $\frac{697}{17}$ ——— | $41$ |
| 34. | ——— $\frac{30}{7}$ ——— | $4\frac{2}{7}$ |
| 35. | ——— $\frac{33}{2}$ ——— | $16\frac{1}{2}$ |
| 36. | ——— $\frac{3283}{83}$ ——— | $39\frac{46}{83}$ |

*Prob.* 6. To reduce Fractions of different denominators, to Fractions of equal value, that ſhall have one common denominator.

*Rule.* Multiply each numerator into all the denominators but its own, for new numerators, and all the denominators continually for a common denominator.

### EXAMPLES.

37. Reduce $\frac{3}{4}$, $\frac{11}{12}$ and $\frac{7}{10}$ to a common denominator.

$$\frac{3}{4},\ \frac{11}{12}\ \text{and}\ \frac{7}{10}$$

$$\text{Firſt,}\quad \left\{\begin{array}{l} 3\times12\times10=360 \\ 11\times4\times10=440 \\ 7\times12\times4=336 \end{array}\right.$$

And $4\times12\times10=480$ common denominator.

*Anſ.* $\frac{360}{480}$ $\frac{440}{480}$ $\frac{336}{480}$.

| | | *Anſwers.* |
|---|---|---|
| 38. | Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{8}$ to Fractions having one common denominator. | $\frac{144}{288}$ $\frac{192}{288}$ $\frac{240}{288}$ $\frac{252}{288}$ |
| 39. | Reduce $\frac{1}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{8}{12}$ to a com. den. | $\frac{240}{720}$ $\frac{540}{720}$ $\frac{576}{720}$ $\frac{420}{720}$ |
| 40. | Reduce $\frac{5}{6}$, $\frac{7}{10}$, $\frac{3}{4}$ and $\frac{3}{4}$ ——— | $\frac{800}{960}$ $\frac{670}{960}$ $\frac{720}{960}$ $\frac{720}{960}$ |

*Prob.* 7. To reduce Fractions of one denomination to thoſe of another, retaining the ſame value.

*Rule.* If the Fraction given is to be brought from a leſs to a greater denomination, multiply the denominator by the parts contained in the ſeveral denominations between it and that you would reduce it to, for a new denominator, which placed under the given numerator, will give the new Fraction, which reduce to its loweſt terms.

2. If the Fraction given is to be brought from a greater to a leſs denomination, then multiply the numerator in the ſame manner as you did before the denominator, and place it over the given denominator, and it will give the new Fraction, which alſo reduce to its loweſt terms.

## EXAMPLES.

41. *(c)* Reduce $\frac{7}{8}$ of a penny to the Fraction of a £. ſterling. *Anſ.* $\frac{7}{1920}$.

$\frac{7}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}=\frac{7}{1920}$ the *Anſ.* required.

| | | *Anſwers.* |
|---|---|---|
| 42. | Reduce $\frac{1}{4}$ of a penny to the Fraction of a £. ſterling. | $\frac{1}{960}$ |
| 43. | Reduce $\frac{4}{5}$ of a dwt. to the Fraction of a lb. Troy | $\frac{4}{1200}=\frac{1}{300}$ |
| 44. | Reduce $\frac{4}{7}$ of a lb. Avoirdupois Wt. to the Fraction of a cwt. | $\frac{4}{784}=\frac{1}{196}$ |
| 45. | Reduce $\frac{5}{7}$ of a penny to the Fraction of a £. | $\frac{1}{336}$ |
| 46. *(b)* | ——— $\frac{3}{4}$ of a dram ——— cwt. | $\frac{33}{114688}$. |
| 47. | ——— $\frac{11}{12}$ of a gallon ——— hhd. | $\frac{11}{756}$. |
| 48. | ——— $\frac{10}{11}$ of a minute ——— day. | $\frac{10}{1584}$. |
| 49.* | ——— 7s. 3d. ——— £. ſterl. | $\frac{29}{80}$. |
| 50. | ——— 17s. 6d. ——— £. | $\frac{7}{8}$. |
| 51. | ——— 13s. 4d. ——— — | $\frac{2}{3}$. |
| 52. | ——— 6d. ——— crown. | $\frac{1}{10}$. |
| 53. | ——— 8d. ——— guinea. | $\frac{8}{273}$. |
| 54. | ——— 3s. 9½d. ——— —— | $\frac{1}{6}$. |

55. Reduce

---

*(c)* Thus 7s. 3d.=87 and 1l.=240 ∴ $\frac{87}{240}=\frac{29}{80}$ the frac. req'd.

*(b)* $\frac{3}{4}\ \frac{}{16}\times\frac{}{16}\times\frac{}{112}=\frac{3}{114688}$ the anſwer required.

* Thus, $\frac{7}{\overline{8\times20\times12}}=\frac{7}{1920}$ the anſwer required.

*Anſwers.*

55. Reduce 1 qr. 14lb. to the Fraction of a cwt. $\frac{2}{7}$.
56. ——— 36 gallons ——— hhd. $\frac{4}{7}$.
57. ——— 2 feet, 6 inches ——— yard. $\frac{5}{6}$.
58. ——— $2\frac{3}{4}$d. ——— ſhilling. $\frac{11}{48}$.
59. ——— 3 qrs. 2 nails ——— ell E. $\frac{7}{10}$.
60. ——— 15s. 2d. ——— guinea. $\frac{2}{3}$.

*Here follow examples anſwering the ſecond part of the Rule:*

*Anſwers.*

61. ——— $\frac{3}{22}$ of a pound to the Fraction of a ſhilling. $\frac{30}{11}$.
62. ——— $\frac{4}{25}$ ——— ——— penny. $\frac{192}{5}$.
63. ——— $\frac{3}{8}$ ——— ——— ſhilling. $\frac{30}{4}$.
64. ——— $\frac{1}{300}$ of a lb. ——— cwt. $\frac{28}{75}$.
65. ——— $\frac{7}{20}$ of a £. ——— ſhilling. 7s.
66. ——— $\frac{1}{296}$ of a cwt. ——— lb. $\frac{14}{37}$.
67. ——— $\frac{3}{114688}$ — ——— dram. $\frac{3}{4}$.
68. ——— $\frac{11}{756}$ of an hhd. ——— gallon. $\frac{11}{12}$.
69. ——— $\frac{1}{1584}$ of a day ——— minute. $\frac{10}{11}$.

*Prob.* 8. To find the proper quantity or value of a Fraction, in Money, Weights and Meaſures.

*Rule.* Multiply the numerator of the given Fraction, by the parts contained in the integer to which it belongs; then divide that product by the denominator, and if any thing remain, reduce it to the next denomination leſs, and divide again by the denominator; thus proceed to the leaſt denomination, and their quotients placed in their order, will be the anſwer.

EXAMPLES.

70. Find the value of $\frac{3}{8}$ of a £. ſterling, in numbers of inferior denominations.

```
   3
  20
  — s. d.
8)60(7 6
  56
  —
   4
  12
  — d.
8)48( 6
  48
  —
```

*or thus;*

$\frac{3}{8}$ of $\frac{20}{1}=\frac{60}{8}=\frac{15}{2}=$7s. 6d. *Ans.*

| | | *Anſwers.* |
|---|---|---|
| 71. | Reduce $\frac{5}{8}$ of a ſhilling to its proper value. | $7\frac{1}{2}$. |
| 72. | ——— $\frac{5}{7}$ of £. ſterling ——— | 14s. $3\frac{3}{7}$. |
| 73. | ——— $\frac{4}{5}$ of 3l. 12s. ——— | 2l. 17s. $7\frac{1}{5}$d. |
| 74. | ——— $\frac{3}{4}$ of a £. ——— | 15s. |
| 75. | ——— $\frac{3}{5}$ of lb. Troy to its proper quantity | 7 oz. 4 dr. |
| 76. | ——— $\frac{4}{7}$ —— Avoirdupois Wt. —— | 9 oz. $2\frac{2}{7}$dr. |
| 77. | ——— $\frac{7}{8}$ of a cwt. —— | 3 qrs. 14lb. |
| 78. | ——— $\frac{5}{9}$ of an ell E. —— | 2 qrs. $3\frac{1}{9}$ nai. |
| 79. | ——— $\frac{4}{5}$ of a mile —— | 6 fur. 16 per. |
| 80. | ——— $\frac{5}{8}$ of an acre —— | 2 r. 20 per. |
| 81. | ——— $\frac{6}{7}$ of an hhd of wine —— | 54 gall. |
| 82. | ——— $\frac{3}{5}$ of a month —— | 2w. 2d. 19h. 12m. |

*Prob.* 9. To reduce Money, Weights and Meaſures into Fractions.

*Rule.* Reduce the given quantity to the loweſt name mentioned, for a numerator; under which put the number of thoſe parts contained in an unity of the integer, for a denominator; then reduce the Fraction to its loweſt terms.

EXAMPLES.

83. Reduce 6s. 8½d. to the Fraction of a £. ſterling.

6s. 8½d.

| *s.* | *d.* | |
|---|---|---|
| 6 | $8\frac{1}{2}$ | 1 |
| 12 | | 20 |
| 80 | | 20 |
| 2 | | 12 |
| 161 | | 240 |
| | | 2 |
| | | 480 |

*Anſ.* $\frac{161}{480}$ is the Fraction reqd.

| | | | | *Anſwers:* |
|---|---|---|---|---|
| 84. | Reduce $4\frac{4}{5}$ to the Fraction of a ſhilling. | | | $\frac{2}{5}$. |
| 85. | —— 9 oz. $2\frac{2}{7}$dr. to the Fraction of a lb. Avoirdupois Wt. | | | $\frac{4}{7}$. |
| 86. | —— 3 qrs. 14 lb. | —— | cwt. | $\frac{7}{8}$. |
| 87. | —— 7 oz. 4 dwt. | —— | lb. Troy. | $\frac{3}{5}$. |
| 88. | —— 2 qrs. $3\frac{1}{5}$ nails | —— | E. ell. | $\frac{5}{9}$. |
| 89. | —— 6 fur. 16 per. | —— | mile. | $\frac{4}{5}$. |
| 90. | —— 2 roods, 20 per. | —— | acre. | $\frac{5}{8}$. |
| 91. | —— 54 gallons | —— | hhd. wine | $\frac{6}{7}$. |
| 92. | —— 2 w. 2 d. 19h. 12m. | —— | month. | $\frac{3}{5}$. |

## CHAP. XIV.

### ADDITION OF FRACTIONS.

*Prob.* 1. To add Fractions together having different denominations.

* *Rule.* 1. Reduce the given Fractions to a common denominator, by *Prob.* 6. in Reduction.

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* Fractions, before they are reduced to a common denominator, are entirely diſſimilar, and therefore cannot be incorporated with one another; but when they are reduced to a common denominator, and made parts of the ſame thing, their ſum or difference may then be as properly expreſſed by the ſum or difference of their numerators, as the ſum or difference of any two quantities, whatever, by the ſum or difference of their individuals; whence the reaſon of Addition and Subtraction of Vulgar Fractions is manifeſt.

2. Add all the numerators together for a new numerator, under which ſubſcribe the common denominator; and if it is an improper Fraction, reduce it to its proper terms, by *Prob.* 7. in Reduction, and you have the ſum of all the Fractions.

### EXAMPLES.

1. Add $\frac{2}{3}$ and $\frac{5}{7}$ together $\frac{14}{21}+\frac{15}{21}=\frac{29}{21}$ or $1\frac{8}{21}$. *Anſ.*

| | *Anſwers.* | | *Anſwers.* |
|---|---|---|---|
| 2. Add $\frac{3}{4}$, $\frac{2}{7}$, $\frac{5}{6}$. | $1\frac{146}{168}$. | 6. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ of $\frac{5}{6}$ | $1\frac{1}{4}$. |
| 3. —— $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{3}{8}$ | $1\frac{19}{24}$. | 7. —— $\frac{5}{12}$, $\frac{3}{64}$, and $\frac{5}{128}$ | $\frac{205}{384}$. |
| 4. —— $\frac{5}{8}$, $\frac{1}{7}$, $\frac{1}{9}$ and $\frac{1}{3}$ | $1\frac{107}{504}$. | 8. —— $\frac{1}{8}$, $\frac{5}{6}$, and $\frac{4}{7}$ | $1\frac{69}{168}$. |
| 5. —— $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{3}{8}$ and $\frac{8}{11}$ | $3\frac{5477}{9240}$. | 9. —— $\frac{1}{7}$, $\frac{5}{6}$, $\frac{7}{12}$ and $\frac{13}{14}$ | $2\frac{41}{84}$ |

*Prob.* 2. To add mixed numbers,

*Rule.* Reduce the Fractions to a common denominator, if they have not one, and add them together (by *Prob.* 1.) and annex their ſum to the ſum of the integers.

### EXAMPLES.

10. Add $4\frac{1}{2}$ and $17\frac{3}{4}$ together. $\frac{1}{2}+\frac{3}{4}=\frac{4}{8}+\frac{6}{8}=\frac{10}{8}=1\frac{1}{4}$, then, $4+17+1\frac{1}{4}=22\frac{1}{4}$ the *Anſ.* required.

| | *Anſwers.* |
|---|---|
| 11. Add $5\frac{1}{2}$, $7\frac{7}{8}$ and 15 together. | $28\frac{3}{8}$. |
| 12. —— $5\frac{2}{3}$, $6\frac{7}{8}$ and $4\frac{1}{2}$. | $17\frac{1}{2}$. |

*Prob.* 3. When the given Fractions are of ſeveral denominations,

*Rule.* Reduce them to one denomination, (by *Prob.* 9. of Reduction) then add them together as before directed, and you have the anſwer.

### EXAMPLES.

13. Add $\frac{3}{4}$ of a £. to $\frac{5}{6}$ of a ſhilling.

$\frac{3}{4}$ of a £.=15s. and $\frac{5}{6}$=10d. therefore 15s. 10d. the *Anſ.*

| | *Anſwers.* |
|---|---|
| 14. Add $\frac{1}{2}$ of a penny and $\frac{2}{3}$ of a pound. | 13s. $4\frac{1}{2}$d. |
| 15. —— $\frac{3}{4}$ of a lb. Troy and $\frac{1}{6}$ of an oz. | 9 oz. 3 dwt. 8 gr. |
| 16. —— $\frac{4}{5}$ of a ton and $\frac{5}{6}$ of a lb. | 16 cwt. 0 qr. 0 lb. 13 oz. $5\frac{1}{3}$d. |
| 17. —— $\frac{1}{6}$ of a yard and $\frac{2}{3}$ of an inch. | 6 inch. 2 b. c. |
| 18. —— $\frac{5}{6}$ of a mile and $\frac{2}{3}$ of a yd Eng. | 6 fur. 26 pe. 4 yd. 1 f. |
| 19. —— $\frac{3}{4}$ of a week and $\frac{5}{6}$ of a month. | 1 m. 6 h. |
| 20. —— $\frac{3}{4}$ of an hour and $\frac{5}{8}$ of a week. | 4 d. 9 h. 45 m. |
| 21. —— $\frac{2}{3}$ of 12l. $+4\frac{3}{7}+\frac{1}{3}$ of $\frac{9}{20}$ of a £.$+\frac{3}{8}$. | 9l. 8s. $8\frac{2}{35}$d. |

## CHAP. XV.

## SUBTRACTION OF FRACTIONS.

RULE. Reduce the given Fractions to a common denominator, if they have not one already; then ſubtract the leſſer numerator from the greater, and place the remainder over the common denominator.

* When the lower Fraction is greater than the upper, ſubtract the numerator of the lower Fraction from the denominator, to that difference add the upper numerator, carrying 1 to the units' place of the lower whole number.

### EXAMPLES.

### *Caſe.* I.

1. From $\frac{3}{4}$ take $\frac{5}{7}$. $3\times7=21$. $5\times4=20$ then from $\frac{21}{28}$ take $\frac{20}{28}=\frac{1}{28}$ and $4\times7=28$ com. den. the *Anſ.*

---

* *Note.* If you cannot diſtinguiſh the greater of the two Fractions, that the numerator of which being multiplied by the denominator of the other Fraction makes the greater number, is the greater fraction.

| | *Anſwers.* | | *Anſwers.* |
|---|---|---|---|
| 2. From $\frac{5}{6}$ take $\frac{3}{5}$ of $\frac{5}{8}$. | $\frac{11}{24}$ | 5. From $\frac{19}{20}$ take $\frac{1}{7}$ of $\frac{2}{3}$. | $\frac{359}{420}$ |
| 3. —— $5\frac{2}{3}$ —— $\frac{9}{10}$. | $4\frac{23}{30}$ | 6. —— $\frac{14}{17}$ —— $\frac{7}{11}$. | $\frac{35}{187}$ |
| 4. —— $\frac{33}{47}$ —— $\frac{3}{5}$. | $\frac{24}{235}$ | 7. — 12 — $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{3}{4}$. | $11\frac{29}{64}$ |

3. When the Fractions are of ſeveral denominations, reduce them to their proper quantities, (by *Prob.* 7. of Reduction,) and ſubtract as before.

### EXAMPLES.

#### *Caſe* II.

| | *Anſwers.* |
|---|---|
| 8. From $\frac{2}{3}$ of a ſhilling take $\frac{3}{4}$ of a penny. | $7\frac{1}{4}$d. |
| 9. —— $\frac{1}{3}$ of a cwt. take $\frac{3}{4}$ of an ounce. | 1 qr. 9lb. 4oz. $9\frac{1}{3}$dr. |
| 10. —— $\frac{3}{4}$ of a yard take $\frac{3}{8}$ of an inch. | 2 feet, $2\frac{3}{8}$ inch. |
| 11. —— $\frac{2}{6}$ of a day take $\frac{3}{4}$ of an hour. | 7 h. 15 min. |
| 12. —— $\frac{4}{5}$ of a ton take $\frac{5}{6}$ of a lb. | 15cwt. 3qrs. 27lb. 2oz. $10\frac{2}{3}$d. |

# CHAP. XVI.

## MULTIPLICATION OF FRACTIONS.

* RULE. 1. Reduce compound Fractions to ſimple ones, (per *Prob.* 2. Reduction).

2. Bring mixed numbers into improper Fractions, (per *Prob.* 3.)

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* Multiplication, by a Fraction, implies the taking ſome part or parts of the multiplicand, and therefore may be truly expreſſed by a compound Fraction, thus:

$\frac{3}{4}$ multiplied by $\frac{5}{8}$ is the ſame as $\frac{3}{4}$ of $\frac{5}{8}$

and as the directions of the Rule agree with the method already given, to reduce theſe Fractions to ſimple ones, it is ſhewn to be right, ſee more of this, *page* 145.

3. Multiply the numerators together for a new numerator, and the denominators for a new denominator, and reduce them to their loweſt terms, and it is done.

### EXAMPLES.

1. Multiply $\frac{3}{4}$ by $\frac{3}{5}$. $3\times3=9$ num. $4\times5=20$ den.

*Anſ.* $\frac{9}{20}$.

| Multiply | | *Anſwers.* | Multiply | | *Anſwers.* |
|---|---|---|---|---|---|
| 2. $\frac{7}{9}$ | by $\frac{2}{3}$. | $\frac{14}{27}$ | 11. $5\frac{1}{2}$ | by $6\frac{3}{4}$. | $37\frac{1}{8}$ |
| 3. $48\frac{2}{5}$ | by $13\frac{5}{6}$. | $669\frac{8}{15}$ | 12. $37\frac{1}{8}$ | by $3\frac{5}{6}$. | $142\frac{5}{16}$ |
| 4. $430\frac{6}{10}$ | by $18\frac{3}{7}$. | $7935\frac{12}{35}$ | 13. $19\frac{4}{15}$ | by $17\frac{7}{12}$. | $338\frac{139}{180}$ |
| 5. $\frac{16}{21}$ | by $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{7}$. | $\frac{16}{49}$ | 14. $35\frac{13}{16}$ | by $11\frac{25}{27}$. | $446\frac{41}{68}$ |
| 6 $\frac{9}{10}$ | by $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$. | $\frac{3}{8}$ | 15. $2\frac{16}{17}$ | by $31\frac{11}{19}$. | $92\frac{284}{323}$ |
| 7. $\frac{3}{4}$ of $\frac{2}{3}$ | by $\frac{2}{3}$ of $\frac{1}{3}$. | $\frac{1}{9}$ | 16. $\frac{1}{2}$ of $\frac{1}{4}$ | by 16. | 2 |
| 8. $\frac{1}{4}$ of $\frac{3}{8}$ | by $\frac{5}{7}$. | $\frac{15}{224}$ | 17. $\frac{7}{16}$ of $3\frac{4}{7}$ | by 8. | $12\frac{1}{2}$ |
| 9. $5\frac{6}{7}$ | by $\frac{5}{6}$. | $4\frac{37}{42}$ | 18. $\frac{7}{8}$ of $\frac{1}{3}$ | by $\frac{2}{5}$. | $\frac{7}{6}$ |
| 10. 24 | by $\frac{2}{3}$. | 16 | 19. $\frac{3}{4}$ of $\frac{1}{3}$ | by $11\frac{7}{8}$. | $2\frac{31}{32}$ |

# CHAP. XVII.

## DIVISION OF FRACTIONS.

PROB. I. To divide a whole, mixed, or fractional number, by a Fraction or mixed number.

*Rule.* Prepare the terms as in Multiplication, (if they require it) by the rules in Reduction.

2. Multiply the numerator of the dividend into the denominator of the dividing Fraction, for a new numerator, and multiply the other numerator and denominator together, for

a new denominator; or, invert the diviſor, and proceed as in Multiplication.*

### EXAMPLES.

1. Divide $\frac{9}{20}$ by $\frac{3}{5}$ $5\times9=45$ num. $3\times20=60$ den. then $\frac{45}{60}=\frac{3}{4}$ the *Ans.*

*Or thus:* by inverting the diviſor $\frac{9}{20}\times\frac{5}{3}=\frac{45}{60}=\frac{3}{4}$ the ſame as before.

| | Divide | | *Anſwers.* | | Divide | | *Anſwers.* |
|---|---|---|---|---|---|---|---|
| 1. | $\frac{5}{9}$ | by $\frac{2}{15}$. | $4\frac{1}{6}$ | 11. | 7 | by $8\frac{5}{8}$. | $\frac{56}{69}$ |
| 2. | $\frac{16}{75}$ | by $\frac{4}{5}$. | $\frac{4}{5}$ | 12. | $25\frac{1}{2}$ | by 5. | $5\frac{1}{10}$ |
| 3. | $\frac{7}{18}$ | by $\frac{2}{3}$. | $\frac{7}{12}$ | 13. | 34 | by $3\frac{7}{9}$. | 9 |
| 4. | $\frac{14}{9}$ | by $\frac{7}{6}$. | $1\frac{1}{3}$ | 14. | $1\frac{1}{9}$ | by 15. | $\frac{2}{27}$ |
| 5. | $\frac{5}{6}$ | by $\frac{15}{7}$. | $\frac{7}{18}$ | 15. | 108 | by $6\frac{3}{4}$. | 16 |
| 6. | $\frac{12}{35}$ | by $\frac{3}{5}$. | $\frac{4}{7}$ | 16. | $119\frac{1}{4}$ | by 8. | $14\frac{29}{32}$ |
| 7. | $\frac{2}{7}$ | by $\frac{3}{5}$. | $\frac{10}{21}$ | 17. | 25 | by $3\frac{1}{5}$. | $7\frac{13}{16}$ |
| 8. | $\frac{9}{16}$ | by 3. | $\frac{3}{16}$ | 18. | $9\frac{3}{7}$ | by 11. | $\frac{6}{7}$ |
| 9. | $\frac{3}{4}$ | by 7. | $\frac{3}{28}$ | 19. | 100 | by $4\frac{7}{8}$. | $20\frac{20}{39}$ |
| 10. | 5 | by $\frac{7}{10}$. | $7\frac{1}{7}$ | 20. | $4\frac{1}{5}$ | by $3\frac{1}{4}$. | $1\frac{19}{65}$ |

# CHAP. XVIII.

## THE RULE OF THREE IN FRACTIONS.

THIS Rule differs in nothing from the Rule of Three Direct, already treated of, but that the operations in that were performed by whole numbers, and in this, by broken numbers or Fractions.

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* The reaſon of performing Diviſion by Multiplication, may be ſhewn thus: Suppoſe it were required to divide $\frac{3}{4}$ by $\frac{2}{5}$ now $\frac{3}{4}\div2$ is manifeſtly $\frac{1}{2}$ of $\frac{3}{4}$ or $\frac{3}{4\times2}$ but $\frac{2}{5}=\frac{1}{5}$ of 2; therefore, $\frac{1}{5}$ of 2, or $\frac{2}{5}$ muſt be contained 5 times as often in $\frac{3}{4}$ as 2 is, that is, $\frac{3\times5}{4\times2}$ the *Anſ.* which is according to the Rule, and will anſwer in all caſes.

*Note.* A Fraction is divided by an integer, by dividing the numerator by it, if poſſible; but if it will not exactly divide, then multiply the denominator by it.

*Rule.* Reduce the numbers as before directed in Reduction, ſtate the queſtions as in the Rule of Three, in whole numbers, and invert the firſt number or term in the proportion, then multiply the three terms continually together, and the product will be the anſwer.

### EXAMPLES.

1. If $\frac{3}{4}$ of a yard coſt $\frac{5}{8}$ £. what will $\frac{9}{10}$ of a yard come to at that rate?

If $\frac{4}{3} : \frac{5}{8} :: \frac{9}{10}$ then per Multiplication $\frac{4\times5\times9}{3\times8\times10}=\frac{180}{240}$

or $\frac{3}{4}$l. = 15s.

| | *Anſwers.* | | |
|---|---|---|---|
| | £. | *s.* | *d.* |
| 2. If $\frac{5}{6}$ of a yard coſt $\frac{2}{3}$ of a £. what will $\frac{11}{12}$ of a yard coſt? | | 14 | 8 |
| 3. If $\frac{3}{4}$ of a yard of lawn coſt 11s. 3d. what will $10\frac{1}{3}$ yards coſt? | 7 | 15 | 0 |
| 4. What will $\frac{1}{2}$ cwt. come to, if $6\frac{3}{4}$ cwt. coſt $21\frac{3}{4}$l | 1 | 12 | $2\frac{2}{3}$ |
| 5. If 1 lb. of any thing coſt $5\frac{1}{4}$s. what will $\frac{7}{8}$ come to? | | 4 | $7\frac{1}{8}$ |
| 6. How much will $\frac{1}{4}$ cwt. come to, at the rate of $15\frac{3}{4}$ per cwt.? | | 3 | $11\frac{1}{4}$ |
| 7. If $\frac{3}{5}$ ell of Holland coſt $\frac{1}{3}$l. what will $12\frac{2}{3}$ ells coſt at that rate? | 7 | 0 | $8\frac{8}{9}$ |
| 8. If 3 yards of broad cloth coſt $2\frac{4}{5}$l. what will $10\frac{2}{7}$ yards coſt? | 9 | 12 | 0 |
| 9. If 1 lb. of cochineal coſt 1l. 5s. what will $36\frac{7}{10}$ lb. come to? | 45 | 17 | 6 |
| 10. If 1 yard of broad cloth coſt $15\frac{5}{9}$s. what will 4 pieces coſt, each piece containing $27\frac{3}{7}$ yards? | 85 | 6 | 8 |
| 11. Bought 3 pieces and $\frac{1}{2}$ of ſilk, each containing $24\frac{3}{8}$ ells, at 6s. $0\frac{3}{4}$d. per ell, I deſire to know what the whole quantity coſt? | 25 | 17 | $2\frac{31}{64}$ |
| 12. A merchant has $\frac{3}{5}$ ſhare in a ſhip, and ſells $\frac{3}{4}$ of his ſhare for 171l. what is the whole ſhip worth? | 380 | 0 | 0 |
| 13. If $\frac{3}{16}$ of a piece of land be let for 273l. 2s. 6d. how much will the rent be of the $\frac{5}{32}$ part of it? | 227 | 12 | 1 |

| | Answers. |
|---|---|
| | £. s. d. |
| 14. At $\frac{3}{4}$ per cent. per annum, what is the intereſt of 273l. 15s.? | 8 17 $11\frac{1}{4}$ |
| 15. What does the commiſſion of 530l. 2s. 9d. amount to, at 2s. 6d. per cwt.? | 13 $3\frac{33}{506}$ |
| 16. If 1 lb. of any thing coſt $12\frac{1}{4}$d. how many lb. can I have 36l. 17s. 6d.? | $722\frac{22}{49}$ lb. |
| 17. How much will $\frac{11}{12}$ lb. Troy come to, if $\frac{1}{3}$ of a lb. coſt 19s. 6d.? | 2 13 $7\frac{1}{2}$ |

## *Queſtions for Exerciſe.*

18. What is that number, to which if $\frac{3}{10}$ of $\frac{18}{7}$ of $\frac{141}{213}$ be added, the total will be 1? *Anſ.* $\frac{1216}{2485}$.

19. What number is that, from which if you deduct the $\frac{1}{25}$ of $\frac{7}{8}$, and to the remainder add $\frac{1}{16}$ of $\frac{47}{19}$, the ſum will be 3. *Anſ.* $2\frac{6691}{7600}$.

20. What number is that, to which if you add $\frac{1}{11}$ of 12 more $\frac{1}{19}$ of 27, and from the total ſubtract $\frac{1}{3}$ of $7\frac{1}{2}$ leſs $\frac{29}{30}$ of $1\frac{1}{4}$ the remainder will be 8? *Anſ.* $6\frac{3911}{5016}$.

21. What is the number, which, if multiplied by $\frac{3}{4}$ of $\frac{7}{8}$ of $2\frac{3}{4}$, will produce no more than 1? *Anſ.* $\frac{128}{231}$.

22. There is a number, which, if divided by $\frac{12}{5}$ of $\frac{3}{10}$ will quote $9\frac{28}{37}$; pray what is the number? *Anſ.* $\frac{361}{296}$.

23. A perſon making his will, gave to one child $\frac{19}{30}$ of his eſtate; to another $\frac{11}{39}$, and when the legacies came to be paid, one turned out 540l. 10s. more than the other: what did the teſtator die worth? *Anſ.* 1538l. 12s. $11\frac{3}{4}$ $\frac{267}{411}$.

24. In diſtreſs at ſea, a ſhip threw out 17 hhds. of ſugar, worth 34l. per hhd. the worth of which came up to but $\frac{4}{7}$ of the indigo they caſt overboard, beſides which, they threw out 13 iron guns, worth 18l. 10s. a piece; the value of all amounted to $\frac{3}{7}$ of $\frac{9}{13}$ of that and the ſhip and lading; pray what of the value came into the port? *Anſ.* 4337l. 15s. $6\frac{2}{3}$d.

25. A father dying left his ſon a fortune, $\frac{3}{16}$ of which he ran out in 6 months, $\frac{2}{3}$ of the remainder held him a twelvemonth longer, at which time he had bare 348l. left; pray what did his father bequeath him? *Anſ.* 1284l. 18s. $5\frac{1}{2}$ $\frac{2}{13}$.

*Solution.* Firſt, $\frac{16}{16}-\frac{3}{16}=\frac{3}{16}$ left at 6 months' end; then $\frac{2}{3}$ of $\frac{13}{16}=\frac{13}{24}$ and $\frac{13}{16}-\frac{13}{24}=\frac{13}{48}$ which muſt be equal to 348l. left; then, as $\frac{48}{13}$ : $\frac{348}{1}$ :: $\frac{48}{48}$ or 1 to 1284l. 18s. $5\frac{1}{2}\frac{2}{13}$.

26. A younger brother received 2200l. which was juſt $\frac{5}{12}$ of his brother's fortune, and $3\frac{1}{8}$ times the elder's money was $\frac{1}{2}$ as much again as the father was worth: what was that?

*Anſ.* 11000l.

*Solution.* As $\frac{12}{5}$ : 1200 :: $\frac{12}{12}$ or 1 to 5280l. brother's fortune, and $5280\times3\frac{1}{8}=16500$; this divided $1\frac{1}{2}=11000$ the *Ans.* required.

27. How many ſtones, of $1\frac{3}{4}$ foot long, $\frac{2}{3}$ foot broad, and $\frac{3}{4}$ foot thick, are equal to 50 ſtones, of $3\frac{1}{3}$ feet long, $2\frac{1}{2}$ feet broad, $1\frac{1}{5}$ thick? *Anſ.* $571\frac{3}{7}$.

28. A father deviſed $\frac{34}{83}$ of his eſtate to one of his ſons; $\frac{34}{83}$ of the reſidue to another, and the ſurplus to his relict for life; the children's legacies were found to be 257l. 3s. 4d. different, what money did he leave the widow the uſe of?

*Anſ.* 534l. 2s. $7\frac{231}{289}$d.

*Solution.* His whole eſtate $\frac{83}{83}-\frac{34}{83}=\frac{49}{83}$ then $\frac{34}{83}$ of $\frac{49}{83}=\frac{1666}{6889}$ the youngeſt ſon's part, and $\frac{34}{83}+\frac{1666}{6889}=\frac{4488}{6889}=$both ſons' ſhare; then from $\frac{6889}{6889}$ the whole eſtate, take $\frac{4488}{6889}=\frac{2401}{6889}$ widow's part, $\frac{34}{83}-\frac{1666}{6889}=\frac{1156}{6889}$ the difference of the ſons ſhare, then by rejecting the common denominator, we ſhall have the following proportion, As 1156 : 257l. 3s. 4d. :: 2401 to 534l. 2s. $7\frac{231}{289}$ the *Anſ.*

29. If $\frac{3}{7}$ of $\frac{4}{5}$ of $\frac{7}{8}$ of a ſhip be worth $\frac{4}{9}$ of $\frac{6}{7}$ of $\frac{11}{13}$ of the cargo, valued at 12000l. what did both ſhip and cargo ſtand the owner in? *Anſ.* 15223l. 8s. $10\frac{306}{819}$d.

30. If a wedge of gold, weighing $17\frac{3}{7}$lb. be worth $679\frac{5}{7}$l. what is the value of $1\frac{3}{13}$ grains? *Anſ.* 2d.

*Solution.* $17\frac{3}{7}$ lb.$=100388\frac{4}{7}$ grs. then,
As $\frac{7}{702720}$ : $\frac{1141920}{7}$ :: $\frac{16}{13}$ to 2d. the *Anſ.* required.

31. A man dying gave to his eldeſt ſon $\frac{2}{3}$ of $\frac{1}{4}$ of his eſtate; to his ſecond ſon, $\frac{1}{5}$ of $\frac{1}{2}$, and when they counted their proportions, the one had 40l. more than the other; the remainder was given to the wife and younger children: how much had each?

*Anſ.* The eldeſt, 100l. the ſecond, 60l. and the wife and younger children, 440l.

*Solution.* $\frac{2}{3}$ of $\frac{1}{4} = \frac{1}{6}$ and $\frac{1}{5}$ of $\frac{1}{2} = \frac{1}{10}$

nu. com. den. 30.

$$\left.\begin{array}{ll} \text{Eldeſt ſon,} & \frac{1}{6} = 5 \\ \text{Second,} & \frac{1}{10} = 3 \end{array}\right\} \frac{2}{30} \text{ difference of ſons' ſhare, 40l.}$$

Sum of ſons' $\frac{8}{30}$ ſhare. From $\frac{30}{30}$ take $\frac{8}{30} = \frac{22}{30}$ and by rejecting the com. den. we have this proportion:

As 2 parts : 40l. : : 5 parts to 100l. eldeſt ſon's ſhare.

As 2 parts : 40l. : : 3 parts to 60l. the ſecond ſon's ſhare, and As 2 parts : 40l. : : 22 to 440l. wife and younger children.

32. In the year I wrote this, if to my age you add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$ thereof, with $\frac{4}{5}$ more,
The number 74 will then be had. Ingenious youths my age explore? *Anſ.* 36 years.

*Solution.* Suppoſe his age 1 then from 74

$$\text{To which add} \left\{\begin{array}{l} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{array}\right. \qquad \text{take} \quad 0\tfrac{4}{5}$$

$73\frac{1}{5} = \frac{366}{5}$

$2\frac{1}{30} = \frac{61}{30}$ then $\frac{30}{61} : \frac{365}{5} :: \frac{1}{1}$ to 36. the *Anſ.*

33. A. in a ſcuffle, ſeized on $\frac{2}{3}$ of a parcel of ſugar plumbs; B. catched $\frac{1}{8}$ out of his hands, and C. laid hold on $\frac{3}{10}$ more; D. ran off with all A. had, except $\frac{1}{7}$, which E. afterwards ſecured ſlily for himſelf; then A. and C. jointly ſet upon B. who in the conflict, ſcattered $\frac{1}{2}$ of what he had, which were equally picked up by D. and E. who lay perdue: B. then tacked down C.'s hat, and to work they all went anew for what it contained, of which A got $\frac{1}{4}$; B. $\frac{1}{2}$; D. 2/[illegible], and C. and E. equal ſhares of what was left of that ſtock; D. ſtruck $\frac{3}{4}$ of what B. laſt acquired, out of their hands, they, with difficulty recovered $\frac{5}{8}$ of it in equal ſhares again, but the other three carried off $\frac{1}{3}$ apiece of the ſame; upon this they all called a truce, and agreed, that the $\frac{1}{3}$ of the whole, left by A. ſhould be equally divided among them; how much of the prize, after this diſtribution, remained with each of the competitors?

*Anſ.* A. got 2863; B. 6535; C. 2438; D. 10294; E. 4950.

*Solution.* $\frac{3}{8}$ of $\frac{2}{3}=\frac{1}{4}$ B.'s } Firſt acquiſition.
and $\frac{3}{10}$ of $\frac{2}{3}=\frac{1}{5}$ C.'s } Their ſum $\frac{9}{20}$
then $\frac{2}{3}-\frac{9}{20}=\frac{13}{60}=\frac{91}{420}$ left
$\frac{1}{7}$ of $\frac{13}{60}=\frac{13}{420}$ E.'s firſt acquiſition.
Alſo, $\frac{91}{420}-\frac{13}{420}=\frac{78}{420}$ D.'s; thus ended the firſt heat.

Again, $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$ B.'s
Retained, $\frac{1}{5}$ C.'s
and $\frac{78}{420}+\frac{1}{16}=\frac{139}{560}$ D.'s
alſo, $\frac{13}{420}+\frac{1}{16}=\frac{157}{1680}$ E.'s
} part at the end of the ſecond ſcuffle.

Proceeding $\frac{1}{4}$ of $\frac{1}{5}$ $=\frac{1}{20}$ A.'s
$\frac{1}{3}$ of $\frac{1}{5}=\frac{1}{15}+\frac{1}{8}$ $=\frac{23}{120}$ B.'s
$\frac{2}{7}$ of $\frac{1}{5}=\frac{2}{35}+\frac{139}{560}$ $=\frac{171}{560}$ D.'s
Then $\frac{1}{20}+\frac{1}{15}+\frac{2}{35}=\frac{73}{420}$ to be—from
C.'s. Thus $\frac{1}{5}-\frac{13}{420}=\frac{11}{420}$ and $\frac{1}{2}$ of $\frac{12}{420}=\frac{11}{840}$ } C.'s
And $\frac{157}{1680}+\frac{11}{840}$ $=\frac{179}{1690}$ E.'s
} part of the third ſcuffle

A. B.
Further $\frac{1}{20}+\frac{1}{15}=\frac{7}{60}$, and $\frac{3}{4}$ of $\frac{7}{60}=\frac{7}{80}$ loſt by A. and B.
Then $\frac{5}{16}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{20}$ $=\frac{51}{1280}$ A.'s
Alſo $\frac{5}{16}$ of $\frac{7}{80}+\frac{1}{4}$ of $\frac{1}{15}+\frac{1}{8}$ $=\frac{649}{3840}$ B.'s
And $\frac{1}{8}$ of $\frac{7}{80}+\frac{11}{840}$ $=\frac{323}{13440}$ C.'s
$\frac{1}{8}$ of $\frac{7}{80}+\frac{171}{560}$ $=\frac{1417}{4480}$ D.'s
$\frac{1}{8}$ of $\frac{7}{80}+\frac{179}{1680}$ $=\frac{1579}{13440}$ E.'s
} Part of the laſt ſcuffle

Then $\frac{51}{1280}+\frac{1}{15}$ $=\frac{2863}{26880}$ A.'s
$\frac{649}{3840}+\frac{1}{15}$ $=\frac{6335}{26880}$ B.'s
$\frac{323}{13440}+\frac{1}{15}$ $=\frac{2438}{26880}$ C.'s
$\frac{1417}{4480}+\frac{1}{15}$ $=\frac{10294}{26880}$ D.'s
$\frac{1579}{13440}+\frac{1}{15}$ $=\frac{4950}{26880}$ E.'s
} Part after the ſcuffle

So that if the number of ſugar plumbs were 26880,

*Ans.*
A. got - 2863
B. — - 6335
C. — - 2438
D. — - 10294
E. — - 4950
} =26880 in all.

*Cr thus:*

A. having laid on $\frac{2}{3}$ of the whole parcel, B. caught $\frac{3}{8}$ of it from him, that is, $\frac{1}{4}$ of the whole; C. at the ſame time, ſeized on $\frac{3}{10}$ of the ſame, anſwering alſo to $\frac{1}{5}$ of the whole; D. ran off with what A. had left, ſave $\frac{1}{7}$ of the ſame; A. originally poſſeſſed $\frac{2}{3}$, after B. and C. had ſerved themſelves, of $\frac{5}{20}$ and $\frac{4}{20}$, had only $\frac{13}{60}$ left of which was carried off by E.; this $\frac{1}{7}$ or $\frac{13}{420}$, taken from his $\frac{13}{60}$, leaves $\frac{78}{420}$ for D.'s part of the ſcuffle; and thus ends the firſt heat.

B. having gotten $\frac{1}{4}$ as before, is attacked by A. and C. together, who make him drop $\frac{1}{2}$ of them, or $\frac{1}{8}$; this was equally picked up by E. and D. that is, by each, $\frac{1}{16}$, B. ſtill retaining $\frac{1}{8}$ himſelf; and thus ended the ſecond heat.

B. then kicked over C.'s hat, wherein was $\frac{1}{5}$ of the parcel poured out at firſt; of this, A. got $\frac{1}{4}$, B. $\frac{1}{3}$ and D. $\frac{2}{7}$, together $\frac{73}{420}$, leaving C. and E. equal ſhares of the remainder, $\frac{11}{420}$, that is to each $\frac{11}{840}$; and ſo ended the third heat.

D. then kicked down $\frac{3}{4}$ of what A. and B. had laſt obtained, viz. $\frac{7}{60}$, of which $\frac{3}{4}$ is $\frac{7}{80}$, and of which they are ſaid to recover $\frac{5}{8}$ between them, is $\frac{7}{256}$ per man, and C., D., and E. got each $\frac{1}{8}$ of $\frac{7}{80}$, or $\frac{7}{640}$ a piece more, the remaining $\frac{1}{4}$ ſaved by A. and B. in this part of the conflict, viz. $\frac{1}{4}$ of $\frac{1}{20}$ and B. $\frac{1}{4}$ of $\frac{1}{15}$ was ſeverally retained by each of them; and thus ended the fourth heat.

The original $\frac{1}{3}$ at firſt miſſed of by A. is agreed to be equally divided among them, that is, to each competitor the items which when collected together, belonging to each, will aſſign the part they ſeverally gained of the prize, which will run thus:

A. $\frac{2963}{26880}$, B. $\frac{6335}{26880}$, C. $\frac{2438}{26880}$, D. $\frac{10294}{26880}$, E. $\frac{4956}{26880}$, which added together, make the whole quantity or 1.

The firſt anſwer ſuppoſes the whole number to be 26880 and no ſuch ſuppoſition in the queſtion; the real anſwer is found out laſt, being the leaſt multiple.

34. A gentleman dying left his wife pregnant, and an eſtate, valued at 9875l. 15s. 6d.; ordered by his teſtament, that if his wife ſhould be delivered of a ſon, $\frac{3}{4}$ of the fortune ſhould go to him, and the reſt to the widow; but if a daughter, the widow ſhould enjoy $\frac{3}{4}$, and the daughter the remainder: it happened that ſhe had both a ſon and a daughter at a birth; in this caſe, how ought the fortune to be divided?

*Anſ.* The daughter's ſhare, 759l. 13s. 6d.; the mother's ſhare, 2279l. 0s. 6d.; the ſon's ſhare, 6837l. 1s. 6d.

35. A merry young fellow in a ſhort time, got the better of $\frac{1}{5}$ of his fortune; by advice of his friends he then gave 2200l. for an exempt place in the guards; his profuſion continued till he had no more than 880 guineas left, which he found by computation, was juſt $\frac{3}{20}$ parts of his money, after the commiſſion was bought; pray what was his fortune at firſt? *Ans.* 10450l. allowing the guineas at 21 ſhillings.

*Solution.* 880 guineas=924 Engliſh, then $\frac{20}{3}$ : $\frac{924}{1}$ :: $\frac{1}{1}$ to 6160l. conſequently 2200+6160=836=$\frac{4}{5}$ of his whole eſtate; now As $\frac{4}{5}$ : $\frac{8360}{1}$ :: $\frac{1}{1}$ to 10450l. the *Ans.*

36. In 81034 runlets of brandy, each 18 gallons, how many groſs of bottles, each $\frac{8}{9}$ of a quart?

*Ans.* 45581 groſs, 7 doz. and 6 bottles.

*Solution.* 81034×18×4=5834448 quarts; then As $\frac{9}{8}$ : $\frac{1}{1}$ :: 5834448 to 6563754; this divided by 144= 45581 groſs, 7$\frac{1}{2}$ doz. the *Anſ.*

37. It is a rule in ſome pariſhes to aſſeſs the inhabitants in proportion to $\frac{8}{10}$ of the rents, what is the yearly rent of that houſe which pays 8l. 10s. to the king under this limitation, at 5s. per £.? *Ans.* 42l. 10s.

*Solution.* From $\frac{10}{10}$ take $\frac{8}{10}$=$\frac{1}{5}$ then As $\frac{5}{1}$s. : 1l. :: 17l. 10s. : 42l. 10s. the *Anſ.*

38. A merchant buys 37$\frac{3}{4}$ pieces of cloth, at 23$\frac{1}{2}$l. the piece; pays in ready money, 235$\frac{1}{4}$l. and the reſt he is to pay in wool, at the rate of 7$\frac{2}{3}$s. the ſtone: I demand how many ſtone of wool he muſt deliver? *Anſ.* 1700$\frac{25}{46}$ ſtone.

39. Two pieces of cloth, contain both together, 79$\frac{3}{4}$ yards; I demand how many yards are in each piece, if $\frac{2}{3}$ yard of the one are as long as $\frac{3}{4}$ of the other?

*Anſ.* 42$\frac{15}{16}$ and 37$\frac{9}{17}$ yards.

40. How much will 2 bags of wool come to, No. 1, wt. 94$\frac{3}{16}$ ſtone; No. 2, wt. 205$\frac{7}{9}$ ſtone, at 10s. 6$\frac{2}{3}$d. per ſtone; but 4$\frac{2}{3}$ ſtone of No. 2, are worth but 2$\frac{1}{4}$ ſtone of No. 1?

*Anſ.* 127l. 10s. 4$\frac{59}{84}$d.

41. Four figures of 9 may be ſo placed and diſpoſed of as to denote and read for 100, neither more nor leſs, pray how is that to be done?

42. Five figures of nine may be ſo placed and diſpoſed of as to denote and read for 1000, neither more nor leſs; pray how is that to be done?

# CHAP. XVIII.

## MERCANTILE ARITHMETIC.

### PRACTICE

IS ſo called from the general uſe thereof by all perſons concerned in trade and buſineſs. All queſtions in this rule are performed by aliquot, or even parts, by which means many tedious Reductions are avoided, by the help of compendious methods for ſolving all queſtions in the Rule of Three, where the firſt term is an unit.

*Caſe* 1ſt. When the rate is ſome number of pounds ſterling.

*Rule.* Multiply the number of things propoſed by the rate or price of 1, the product is the anſwer.

EXAMPLES.

| | *Anſwers.* |
|---|---|
| 1. 190 cwt. of butter, at 5l. per cwt. | 190×5=950*l.* |
| 2. What is the amount of 193 cwt. of hops, at 11l. per? | 2123. |
| 3. What is the amount of 120 cwt. at 6l. per? | 720. |
| 4. What is the value of 140 cwt. of indigo, at 12l. per? | 1680. |

*Caſe 2d.* When the rate is an even number of ſhillings.

*Rule.*[*] Multiply the number of yards or things propoſed by half of the number of ſhillings, doubling the firſt figure of the product for ſhillings, and the reſt of the product will be pounds.

EXAMPLES.

1. 140 yards at 2s. per?
1
*l.*14 : 0 *s.* *Anſ.*

2. 140 at 4s. per?
2
*l.* 28 : 0 *s.* *Anſ.*

3. 259

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[*] It is adviſable to write each figure of the product one place farther to the right hand than the correſponding figure of the multiplicand, in order that the units of the pounds may fall directly under the units of the number propoſed; though this caution is of no conſequence in the preſent caſe, it is good to have the cuſtom of obſerving it, as it will be of uſe in many other caſes.

| | *Answ.* | | *Answ.* |
|---|---|---|---|
| 3. 259 at 4s. per. | *l.*51 10*s.* | 7. 460 yards at 12s. | *l.*276 0*s.* |
| 4. 5013 at 8s. per. | 2005 4 | 8. 627 lbs. at 14s. | 438 18 |
| 5. 872 at 10s. per. | 436 0 | 9. 598 cwt. at 16s. | 478 8 |
| 6. 460 at 12s. per. | 276 0 | 10. 214lbs at 18s. | 192 12 |

*Case* 3. When the price is an odd number of ſhillings under twenty.

*Rule.* Multiply the given quantity by the price, and the product will be the anſwer in ſhillings, which divided by 20 gives the pounds.

Or thus; work for the greateſt even number of ſhillings in it by Caſe 2; and for the odd ſhilling, take one twentieth of the given quantity, whoſe price is required; theſe two reſults added together give the anſwer.

EXAMPLES.

1. 732 at 3s. per. or 732
3 1

2,0) 219|6 73 4 for 2s.
$\frac{1}{20}$ 36 12 for 1s.

£.109 : 16s. *Ans.*

*Ans.* £.109 16s.

| | *Anſ.* | | *Anſ.* |
|---|---|---|---|
| 2. 3344lb. at 13s. | £.2173 12*s.* | 6. 240lb. at 13s. | £.156 0*s.* |
| 3. 147 yds. at 7s. | 51 9 | 7. 652 yds. at 15s. | 489 0 |
| 4. 371 lbs. at 9s. | 166 19 | 8. 897 cwt. at 17s. | 762 9 |
| 5. 586 ells at 11s. | 322 6 | 9. 1046 lb. at 19s. | 993 14 |

*Caſe* 4. To find the price of any number of hundreds, yards, feet, &c. when the rate is an aliquot part of a pound.

*Rule.** Take ſuch parts of the quantity propoſed as the given rate is of a pound ſterling; that is, divide the quantity conſidered as ſo many pounds ſterling, by the denominator or lower number of the fraction, and the quotient is the anſwer. But note, that when there is a remainder, proceed as in diviſion of money, &c.

*Table*

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* When the denominator or diviſor has a cypher in the unit place, cut off no figure on the paper, but in your mind omit the cypher of the diviſor, and, dividing by the reſt, write each figure of the quotient one place farther to the right than the reſpective figure of the dividend, that the units of the anſwer may fall under the units of the dividend.

### *Table of the aliquot parts of a Pound Sterling.*

| s. | d. | | | | s. | d. | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | | is the | $\frac{1}{2}$ | of a £. | 1 | 3 | is the | $\frac{1}{16}$ | of a £. | |
| 6 | 8 | —— | $\frac{1}{3}$ | —— | 1 | 0 | —— | $\frac{1}{20}$ | —— | |
| 5 | 0 | —— | $\frac{1}{4}$ | —— | | 8 | —— | $\frac{1}{30}$ | —— | |
| 4 | 0 | —— | $\frac{1}{5}$ | —— | | 6 | —— | $\frac{1}{40}$ | — | $\frac{1}{2}$ of a s. |
| 3 | 4 | —— | $\frac{1}{6}$ | —— | | 4 | —— | $\frac{1}{60}$ | — | $\frac{1}{3}$ —— |
| 2 | 6 | —— | $\frac{1}{8}$ | —— | | 3 | —— | $\frac{1}{80}$ | — | $\frac{1}{4}$ —— |
| 2 | 0. | —— | $\frac{1}{10}$ | —— | | 2 | —— | $\frac{1}{120}$ | — | $\frac{1}{6}$ —— |
| 1 | 8 | —— | $\frac{1}{12}$ | —— | | 1$\frac{1}{2}$ | —— | $\frac{1}{160}$ | — | $\frac{1}{8}$ —— |
| 1 | 4 | —— | $\frac{1}{15}$ | —— | | 1 | —— | $\frac{1}{240}$ | — | $\frac{1}{12}$ —— |

Aliquot parts are ſuch as may be found a certain number of times exactly, without a remainder, in the number of which they are parts: ſuch are, with regard to the pounds ſterling, the ſhillings and pence in this table, which muſt be well-retained in memory.

EXAMPLES.

1. 7898 lbs. at 2s. 2s. is $\frac{1}{10}$ ) 7898

*Ans.* £. 789: 16*s.*

| | *Anſwers.* £. | s. | d. |
|---|---|---|---|
| 2. 2345 yards at 6s. 8d. per yard? | 781 | 13 | 4 |
| 3. 4567 pounds at 4s. per pound? | 913 | 8 | 0 |
| 4. 540 hundred weight, at 5s. per cwt.? | 135 | 0 | 0 |
| 5. 540 pounds, at 2s. 6d. per pound? | 67 | 10 | 0 |
| 6. 5678 pounds, at 3s. 4d. per pound? | 946 | 6 | 8 |
| 7. 7898 yards, at 2s. per yard? | 789 | 16 | 0 |
| 8. 7898 pounds, at 1s. per. pound? | 394 | 18 | 0 |
| 9. 400 pounds, at 1s. 8d. per pound? | 33 | 6 | 8 |

*Note.* When the rate is 8d. then, after dividing by the denominator 30, the quotient is pounds; the remainder doubled is groats, which divided by 3, the quote is ſhillings, and 1 or 2 remaining are ſo many groats.

When the rate is an aliquot part of a ſhilling as well as of a pound, there are two diviſors; firſt divide by the large diviſor, the quotient is pounds; the remainder divided by the other diviſor, the quotient there is ſhillings, and the ſecond remainder gives ſo many times ſaid rate.

### EXAMPLES.

1. 112 at 4d. per. 4d. is $\frac{1}{60}$l. and $\frac{1}{3}$s. ) 112

*Anſ.* £. 1 17 4

| | Anſw. | | | | Anſw. | | |
|---|---|---|---|---|---|---|---|
| | £. | s. | d. | | £. | s. | d. |
| 2. 379 at 6d. per. | 9 | 9 | 6 | 5. 976 at 1d. per. | 4 | 1 | 4 |
| 3. 758 at 2d. per. | 6 | 6 | 4 | 6. 5797 at 3d. per. | 72 | 9 | 3 |
| 4. 112 at 1½d. per. | 0 | 14 | 0 | 7. 9753 at 1½d. per. | 60 | 19 | 1½ |

## *Caſe* V.

### COMPOUND RATES.*

*Rule*. Having made a proper choice of ſimple rates, to make up a given compound rate, ſet the ſimple numbers in a column under the compound rules. 2dly, Find the given numbers of yards, &c. according to ſuch ſimple rules. 3dly, Add, ſubtract, multiply, or divide the amounts,

According as the ſimple rates aſſumed require to be managed, in order to make up the given compound rate, the manner of working the examples of the five caſes following, will be a ſufficient explanation of this rule.

1ſt. When the rate is a total, made of ſimple rates, by Addition.

EXAMPLES.

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* When the rate is not one of thoſe comprehended under the four preceding caſes, it muſt be conceived as compounded of them, either by Addition, Subtraction, Multiplication, or Diviſion, or by ſome combination of thoſe different operations; that is to ſay, every compound rate muſt be either the total of two or more ſimple rules added; or the difference of two ſimple rates ſubtracted one from the other; or the product of a ſimple rate multiplied by ſome number, or the quotient of a ſimple rate divided by ſome number; or in fine, a mixture of theſe.

*Note*. In compound rates, it is not convenient to chooſe odd ſhillings for one part, unleſs it be 5s. or 1s. whoſe anſwer may be had in one line; nor is it convenient to mix Subtraction with Addition.

### EXAMPLES OF POUNDS AND EVEN SHILLINGS.

£. s.
1ſt 140 at 6 6

849 at 6 0
42 at 0 6 by *Caſe* II.

£. 882 the *Ans.*

| | | at | per | *Anſwers.* £. | s. | d. |
|---|---|---|---|---|---|---|
| 2. | 123 | 4l. | 6s. | 528 | 18 | 0 |
| 3. | 234 | 3 | 8 | 795 | 12 | 0 |
| 4. | 345 | 1 | 12 | 552 | 0 | 0 |
| 5. | 8435 | 16l. | 8s. | 138334 | 0 | 0 |
| 6. | 193 | 18l. | 4s. | 3512 | 12 | 0 |
| 7. | [illegible]416 | 19 | 8 | 27470 | 8 | 0 |
| 8. | 112 | 4 | 18 | 548 | 16 | 0 |

### *Even ſhillings and aliquot parts of a £.*

*Rule.* Find the amount of the given quantity at the given number of ſhillings, by *Caſe* II. to which add the amount of the aliquot parts, by *Caſe* IV. and the ſum will be the anſwer.

#### EXAMPLES.

s. d.
1ſt, 790 at 18 6
9

d.
6 is $\frac{1}{40}$ & $\frac{1}{2}$ 711 0 at 18s. by *Caſe* II.
19 15 at 6d. by *Caſe* IV.

£. 730 15 *Anſ.*

| | | at | per | *Anſwers.* £. | s. | d. |
|---|---|---|---|---|---|---|
| 2. | 2710 | 6s. | 8d. | 903 | 6 | 8 |
| 3. | 789 | 14 | 6 | 572 | 0 | 6 |
| 4. | 876 | 16s. | 3d. | 711 | 15 | 0 |

### *Various aliquot parts of a £.*

*Rule.* This is done by dividing the given price into two parts, each of which ſhall be aliquot parts of a £.

EXAMPLES.

*lb.* *s. d.*  
*Or thus:* 1ſt, 765 at 7 4  
*s. d.*  
765 at 7 4 £. 229 10 at 6 0  
51 0 at 1 4  
£. 280 10 *Anſ.*

| | | at per | *Anſwers.* £. | *s.* | *d* |
|---|---|---|---|---|---|
| 2. | 324 | 6s. 4d. | 102 | 12 | 0 |
| 3. | 2752 | 4 8 | 642 | 2 | 8 |

| | gui. | at per | *Anſwer.* £. | *s.* | *d.* |
|---|---|---|---|---|---|
| 4. | 432 | 1l. 2s. 9d. | 491 | 8 | 0 |

*Caſe* 6. When the rate is the difference of two ſimple rates,

*Rule.* Find the amount of the given quantity at the firſt rate, more the difference, and from that amount ſubtract the value of ſaid difference, and the remainder will give the anſwer; this is the caſe for ſubtraction.

£. *s.*  
1ſt, 3142 at 2 10  
9426 at 3 0  
Subtract 1571 at 0 10  
£. 7855 the *Anſ.*

| | | at | per | *Anſwers.* £. | *s.* | *d.* |
|---|---|---|---|---|---|---|
| 2. | 2107 | 2l. 8s. | | 5056 | 16 | 0 |
| 3. | 135 | 9 14 | | 1309 | 10 | 0 |
| 4. | 357 | 3 17 6d. | | 1383 | 7 | 6 |
| 5. | 579 | 19 6 | | 564 | 10 | 6 |
| 6. | 797 | 19 9 | | 787 | 0 | 9 |

*Caſe* 7. When the price of any hundreds, yards, &c. is the product of a ſimple rate and ſome number,

*Rule.* Find the amount of the given quantity at the ſimple rate, by *Caſe* 2.; then multiply that amount by the other number, as is taught in Compound Multiplication, *page* 70, and you have the anſwer

EXAMPLES.

£. *s.*  
1ſt, 987 at 2 8 or 48  
4  
$8 \times 6 = 48$  
394 16  
6  
£. 2368 16 the *Anſ.*

2. 765 at 4l. 16s. or 96s. *Anſ.* 3672l. Find amount at 12s. (by *Caſe* 2.) and multiply by 8.

3. 543 at 1l. 13s. 4d. *Anſ.* 905l. Find amount at 6s. 8d. (by *Caſe* 4.) and multiply by 5

4. 321 at 17s 6d. *Anſ.* 280l. 17s. 6d. Find amount at 2s. 6d. and multiply by 7.

5. 876 at 3l. 12s. *Anſ.* 3153l. 12s. Find amount at 6s. (by *Caſe* 2.) and multiply by 12.

6. 654 at 1l. 15s. *Anſ.* 1144l. 10s. Find amount at 5s. (by *Caſe* 4.) and multiply by 7.

7. 432 at 16s. 8d. *Anſ.* 360l. Find amount at 3s. 4d. (by *Caſe* 4.) and multiply by 5.

8. 123 at 1s. 4½d. *Anſ.* 8l. 9s. 1½d. Find amount at [illegible]d. (by *Caſe* 4.) and multiply by 11.

*Caſe* 8. When the rate is the quotient of ſimple rates,

*Rule.** Conceive the rates as parts of other aliquot parts, or of rates more manageable, then having wrought for the greater, take ſuch parts of the amount as the leſſer is of the greater.

### EXAMPLES.

| | | |
|---|---|---|
| | | d. |
| 1ſt | 123 | at 10 per |
| s. d. | | d. |
| 2 6 is $\frac{1}{8}$ | 15 7 6 | at 30 |
| d. d. | | d. |
| 10 is $\frac{1}{3}$ of 30 | £.5 2 6 | at 10 *Anſ.* |

2. 234 at 2½d. per. *Anſ.* 2l. 8s. 9d. Find amount at 20d. (by *Caſe* 4.) and $\frac{1}{8}$ of that amount=that of 2½d.

3. 354 at 7½d. per. *Anſ.* 11l. 1s. 3d. Find amount at 30d. (by *Caſe* 4) and $\frac{1}{40}$ of that amount=amt. of 7½.

4. 567 at 3¼d. per. *Anſ.* 7l. 13s. 6¾d. Find amount at 3d. and $\frac{1}{4}$ of that amount=amount of 3¼d.

5. 789

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* This caſe occurs often and admits of ſome uſeful remarks: 1. Beſides thoſe aliquot parts of a £. which are mentioned in the Practice Table, there are many more which were deſignedly omitted there on account of the troubleſome diviſor they require, when conſidered as a part of a £. to avoid ſuch diviſor.

5. 789 at 5d. *Anſ.* 16l. 8s. 9d. Find amount at 30d. (by *Caſe* 4.) and $\frac{1}{6}$ thereof=amount of 5d.

6. 324 at $\frac{1}{4}$d. *Anſ.* 6s. 9d. Find amount at 3d. (by *Caſe* 4.) and $\frac{1}{12}$ amount=amount at $\frac{1}{4}$d.

7. 456 at 3$\frac{3}{4}$d. *Anſ.* 7l. 2s. 6d. Find amount at 2s. 6d. and $\frac{1}{8}$ thereof=amount of 3$\frac{3}{4}$d.

8. 897 lb. at $\frac{3}{8}$d. per. *Anſ.* 1l. 8s. 0$\frac{3}{8}$d. Find the amount at 3d. (by *Caſe* 4.) and $\frac{1}{8}$ thereof=amount of $\frac{3}{8}$.

9. 976 at $\frac{1}{2}$d. per. *Anſ.* 2l. 0s. 8d. Find amount at 3d. (by *Caſe* 4.) and $\frac{1}{6}$ thereof=amount of $\frac{1}{2}$d.

2. *When the rate is an aliquot part of an even number of ſhillings,*

10. 789 at 9d. per. *Anſ.* 29l. 11s. 9d. Find amount at 6s. (by *Caſe* 2.) and $\frac{1}{8}$ thereof=amount of 9d.

11. 898 at 1s. 4d. per. *Anſ.* 59l. 17s. 4d. Find amount at 4s. (by do.) and $\frac{1}{3}$ thereof=amount of 1s. 4d.

12. 987 at 3s. 6d. per. *Anſ.* 172l. 14s. 6d. Find amount at 14s.; $\frac{1}{4}$ thereof=amount of 3s. 6d.

3. *When the rate is an aliquot part of ſome number of pounds*

## EXAMPLES.

s. d.
13. 876 at 7 6

s. d.
7 6 $\frac{1}{8}$ of 3l. 2628 at 3l.

£. 328 10 at 7s. 6d. the *Anſ.*

14. 765 at 8s. 4d. per. *Anſ.* 318l. 15s. Find amount at 5l.; $\frac{1}{12}$ thereof=amount at 8s. 4d.

15. 654 at 13s, 4d. per. *Anſ.* 436l. Find amount at 2l.; $\frac{1}{3}$ thereof=amount of 13s. 4d.

*Caſe* 9. When the rate is a total made up of ſimple rates and of the parts of ſimple rates,

*Rule.* Multiply the given quantity by the greateſt rate, and add the amount of the leaſt rate as it is of the greateſt.

### EXAMPLES.

£. s.
1ſt, 123 at 3 3
s. £. 369
3 $\frac{1}{20}$ of 3 18 9
£. 387 9 the *Ans.*

2d. 234 at 9l. 18s. per. *Ans.* 2316l. 12s. Find amount at 9l.; $\frac{1}{20}$ thereof added=amount of 9l. 18s.

3. 345 at 5l. 12s. 6d. per. *Anſ.* 1940l. 12s. 6d. Find amount at 5l.; $\frac{1}{8}$ thereof added=amount of 5l. 12s. 6d.

4. 456 at 6s. 9d. per. *Ans.* 153l. 18s. Find amount at 6s. (by *Caſe* 2.) $\frac{1}{8}$ added=amount of 6s. 9d.

5. 567 at 8s. 8d. per. *Anſ.* 245l. 14s. Find amount at 8s. (by do.); $\frac{1}{12}$ added=amount of 8s. 8d.

6. 678 at 15s. 2d. per. *Anſ.* 514l. 3s. Find amount at 14s. (by ditto); $\frac{1}{12}$ added=amount of 15s. 2d.

7. 789 at 1s. 10$\frac{1}{2}$d. per. *Anſ.* 73l. 19s. 4$\frac{1}{2}$d. Find amount at 1s. 8d. (by *Caſe* 4.); $\frac{1}{8}$ added=amount of 1s. 10$\frac{1}{2}$d.

8. 987 at 2s. 8$\frac{1}{2}$d per. *Anſ.* 133l. 13s. 1$\frac{1}{2}$d. Find amount at 2s. 6d. (by ditto); $\frac{1}{12}$ added=amount of 2s. 8$\frac{1}{2}$d.

9. 176 at 5s. 5d. per. *Anſ.* 237l. 5s. Find amount at 5s. (by ditto); $\frac{1}{12}$ added=amount of 5s. 5d.

10. 654 at 2s. 9$\frac{3}{4}$d. per. *Anſ.* 91l. 19s. 4$\frac{1}{2}$. Find amount at 2s. 6d. (by ditto); $\frac{1}{8}$ added=amount of 2s. 9$\frac{3}{4}$d.

11. 543 at 3s. 1$\frac{1}{2}$d. per. *Anſ.* 84l. 16s. 10$\frac{1}{2}$d. Find amount at 2s. 6d. (by ditto); $\frac{1}{4}$ added=amount of 3s. 1$\frac{1}{2}$d.

12. 432 milreas at 5s. 11$\frac{3}{4}$d per. *Anſ.* 128l. 5s. 1$\frac{1}{2}$d. Find amount at 5d.; $\frac{1}{20}$ and $\frac{1}{8}$ of that $\frac{1}{20}$=amount of 5s. 11$\frac{3}{4}$d.

13. 321 at 3s. 10$\frac{7}{8}$d. *Ans.* 62l. 13s. 10$\frac{7}{8}$d. Find amount at 2s. 6d.; $\frac{1}{8}$ of that $\frac{1}{2}$ added=amount of 3s. 10$\frac{7}{8}$d.

14. 765 guineas at 1l. 2s. 9d. Leave the guineas as pounds and add $\frac{1}{8}$ thereof and $\frac{1}{20}$ of that $\frac{1}{8}$; the two laſt added to the number of guineas will give the pounds required=870l. 3s. 9d.

*Caſe* 10. When the compound rate cannot be made up of a ſmall number of eaſy ſimple rates, but may be eaſily multiplied by the given number of things, *Rule.* Multiply the given pounds, ſhillings and pence by as many tens as there are places of figures in the given quantity, (leſs 1) then multiply the laſt product by the ſignificant figure to the left of the given quantity, the next by 10 is then to be multiplied by the next figure of the given quantity, and ſo on throughout all the figures, collect the different (or ſeveral) ſums thus found, and it will give the anſwer.

EXAMPLE.

| | £. | s. | d. |
|---|---|---|---|
| 15 tons of tobacco at | 23 | 4 | 5½ |
| | | | 3 |
| | 69 | 13 | 4½ |
| | | | 5 |
| | £.348 | 6 | 10½ *Anſ.* |

This being ſo largely inſiſted on in Compound Multiplication, it will be ſufficient here.

To multiply pounds, ſhillings and pence by an even number, (not exceeding 12) with one or two cyphers to the right hand.

*Rule.* Under the pounds ſet the given multiplier; under the ſhillings the $\frac{1}{20}$ of that multiplier; under the pence the $\frac{1}{12}$ of the ſecond multiplier; multiply each denomination of the money by its reſpective multiplier, beginning at the pence, and carrying ſtill each product to the next.[*]

20l.

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[*] As the multiplier for the pence is generally a fraction, you are to multiply by the numerator, or upper number of the fraction, and to divide by the lower; but ſince, in the preſent, the numerator is 1, or 10, or 100, the Multiplication is done by conceiving as many cyphers to the right of the pence as there happen to be cyphers in the reſpective numerator; or if there be odd farthings, for every twenty-five farthings conceive 2½, inſtead of 1 cypher and 25, inſtead of two cyphers. 2. In dividing by the denominator or lower number of the fraction, if any pence remain, for them ſet down ſo many ſuch parts of a lb. as the denominator expreſſes; for example, if 5 pence remain, when the denominator is 12 write down $\frac{5}{12}$ of a £. that is, 8s 4d.; if 3 pence remain when the denominator is 4, write down ¾, that is, 15s.; if 2¾ remain, when the denominator is 6, write down $\frac{2}{6}$ and ¾ of a ſixth, viz. 6s. 8d. and 2s. 6d. that 9s. 2d.

| £. | s. | d. | £. | s. | d. |
|---|---|---|---|---|---|
| 20 | 1 | $\frac{1}{12}$ | 200 | 10 | $\frac{10}{12}$ |
| 40 | 2 | $\frac{1}{6}$ | 400 | 20 | $\frac{10}{6}$ |
| 60 | 3 | $\frac{1}{4}$ | 600 | 30 | $\frac{10}{4}$ |
| 80 | 4 | $\frac{1}{3}$ | 800 | 40 | $\frac{10}{3}$ |
| 100 | 5 | $\frac{10}{24}$ | 1000 | 50 | $\frac{100}{24}$ |
| 120 | 6 | $\frac{1}{2}$ | 1200 | 60 | 5 |

| | £. | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|
| 1ſt, 20 at | 123 | 4 | 5 | 2d, 40 at | 234 | 5 | 6 |
| | 20 | 1 | $\frac{1}{12}$ | | 40 | 2 | $\frac{1}{6}$ |
| | £. 2464 | 8 | 4 *Anſ.* | | £. 9371 | 0 | 0 *Anſ.* |

| | £. | s. | d. | *Anſwers.* £. | s. | d. |
|---|---|---|---|---|---|---|
| 3. 60 at | 345 | 6 | 7. | 20719 | 15 | 0 |
| 4. 100 at | 567 | 8 | 9. | 56743 | 15 | 0 |
| 5. 200 at | 789 | 10 | 11 | 157909 | 3 | 4 |
| 6. 80 at | 456 | 7 | 8. | 36510 | 13 | 4 |
| 7. 120 at | 678 | 9 | 10. | 81419 | 0 | 0 |
| 8. 400 at | 898 | 7 | 6. | 359350 | 0 | 0 |

*Caſe* II. When the quantity whoſe amount is required, conſiſts of ſeveral ſubordinate denominations, whereof only one high denomination is rated,

*Rule.* Work for the rated denomination, ſingly, by ſome of the foregoing rules; and for the lower denominations and ſuch parts of the rates as the lower denominations are of the rated denomination.

| *Aliquot parts of an cwt.* | | | | | | *Parts of ½ cwt.* | | | *Parts of ¼ cwt.* | |
|---|---|---|---|---|---|---|---|---|---|---|
| *qr.* | *lb.* | | | *qr.* | *lb.* | *qr.* | *lb.* | | *lb.* | |
| 2 | 0 | is $\frac{1}{2}$ | Complement. | 2 | 0 | 1 | 0 | is $\frac{1}{2}$ | 14 | is $\frac{1}{2}$ |
| 1 | 0 | is $\frac{1}{4}$ | ——— | 3 | 0 | 0 | 14 | is $\frac{1}{4}$ | 7 | is $\frac{1}{4}$ |
| 0 | 16 | is $\frac{1}{7}$ | ——— | 3 | 12 | 0 | 8 | is $\frac{1}{7}$ | 4 | is $\frac{1}{7}$ |
| 0 | 14 | is $\frac{1}{8}$ | ——— | 3 | 14 | 0 | 7 | is $\frac{1}{8}$ | 3½ | is $\frac{1}{8}$ |
| 0 | 8 | is $\frac{1}{14}$ | ——— | 3 | 20 | 0 | 4 | is $\frac{1}{14}$ | 1¾ | is $\frac{1}{16}$ |
| 0 | 7 | is $\frac{1}{16}$ | ——— | 3 | 21 | 0 | 3½ | is $\frac{1}{16}$ | | |

EXAMPLES.

| | *cwt.* | *qr.* | *lb.* | | | | £. | *s.* | *d.* | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1ſt, | 8 | 3 | 16 | at | | | 3 | 8 | 4 | per cwt. |
| | | | | | | | | | 8 | |
| | | *qr.* | | | | | 27 | 6 | 8 | |
| | | 2 | | is | $\frac{1}{2}$ | | 1 | 14 | 2 | |
| | | 1 | | is | $\frac{1}{2}$ | | 0 | 17 | 1 | |
| | | 16 lb. | | is | $\frac{1}{7}$ | | 0 | 9 | $9\frac{1}{7}$ | |
| | | | | | | £. | 30 | 7 | $8\frac{1}{7}$ | the *Anſ.* |

| | | *Anſwers.* £. | *s.* | *d.* |
|---|---|---|---|---|
| 2. | 10 cwt. 2 qrs. 14 lb. at 2l. 16s. 8d. per. | 30 | 2 | 1 |
| 3. | 17 cwt. 2qrs. at 1l. 12s. 6d. per. | 28 | 8 | 9 |
| 4. | 1 cwt. 1qr. 8lb. at 1l. 17s. | 2 | 8 | $10\frac{5}{7}$ |
| 5. | 2 cwt. 2qrs. at 3l. 14s. 3d. | 9 | 5 | $7\frac{1}{2}$ |
| 6. | 15lb. 2 ounces, 5dwts. at 4l. 7s. per lb. | 66 | 1 | $3\frac{3}{4}$ |
| 7. | 7 ounces, 15 dwts. 12 grains, at 6l. 3s. per. | 2 | 8 | $8\frac{3}{8}$ |
| 8. | 13 yards, 1 quarter, at 17s. 6d. per yard. | 11 | 11 | $10\frac{6}{7}$ |
| 9. | 64 acres, 2 roods, 28 perches, at 1l. 16s. per. | 116 | 8 | $3\frac{3}{5}$ |

*Caſe* 12. When the number, the price of which is required, is of divers denominations, and the price of one of the loweſt denomination is given to find the amount of the compound Number.

*Rule.* Reduce the compound number to that name or denomination of which the price of one is given, and then find the amount by the foregoing rules.

EXAMPLES.

1. How much will 14cwt. 1qr. 16lb. of cheeſe come to, at $3\frac{1}{2}$d. per pound?

| | cwt. | qr. | lb. |
|---|---|---|---|
| | 14 | 1 | 16 |
| | 212 | | |
| | 1612 | | |
| 3*d.* is $\frac{1}{80}$*l.* & $\frac{1}{4}$*s.* | *l.* 20 | 3 | 0 |
| $\frac{1}{2}$*d.* is $\frac{1}{6}$ of 3*d.* | 3 | 7 | 2 |
| £. | 23 | 10 | 2 *Anſwer.* |

by compendiums in Reduction for reducing cwt. qrs. lbs. into lbs. in one line, ſee page 108, 109, of Reduction

2. 3 hogſ-

| | *Anſwers.* £. | s. | d. |
|---|---|---|---|
| 2. 3 hogſheads, 36 gallons, 2 quarts of rum, at 1s. 8d. per quart? | 75 | 3 | 4 |
| 3. 136 yards, 3 quarters, 2 nails, at 1s. 3d. per nail? | 136 | 7 | 6 |
| 4. 1 pound, 4 ounces, 15 penny-weights, 18 grains, at 1$\frac{1}{2}$d. per grain? | 50 | 7 | 3 |
| 5. 3 tons, 16 cwt. 1 quarter, 16 pounds, at 4$\frac{1}{2}$d. per pound? | 160 | 8 | 6 |
| 6. 2 hogſheads of tobacco, weighing 24 cwt. 3 quarters, 22 lb. at 9$\frac{3}{4}$d per lb. | 113 | 10 | 1$\frac{1}{2}$ |
| 7. 112 pounds, 4 ounces, at 3s. per ounce? | 269 | 8 | 0 |
| 8. 6 hogſheads, 18 gallons of Brandy, at 8s. per gallon? | 158 | 8 | 0 |

## *Troy Weight.*

*Caſe* 13. The ounces and penny-weights are in the proportion of pounds and ſhillings, but the grains are as halfpence; therefore, taking half the grains, the whole is reduced to the form of pounds, ſhillings, and pence.

### EXAMPLES.

| oz. | dwt. | gr. |
|---|---|---|
| 67 | 8 | 9 of gold, at 4l. 2s. per ounce? |

| | | | |
|---|---|---|---|
| or, *l.* | 67 | 8 | 4$\frac{1}{2}$*d.* |
| | | | 4 |
| | 269 | 13 | 6 |
| 2s. is $\frac{1}{10}$l. | 6 | 14 | 10 |
| *Anſw.* £. | 276 | 8 | 4*d.* |

*or thus, by the common way:*

| | oz. | dwt. | gr. |
|---|---|---|---|
| | 67 | 8 | 9 at 4l. 2s. per oz. |
| | 4 | 2 | |
| | £.268 | 0 | 0 |
| 2s. are $\frac{1}{10}$l. | 6 | 14 | 0 |
| 5dwts. are $\frac{1}{4}$oz. | 1 | 0 | 6 |
| 2 are $\frac{1}{10}$ | 0 | 8 | 2$\frac{2}{5}$ |
| 1 is $\frac{1}{2}$ | 0 | 4 | 1$\frac{1}{5}$ |
| 8 grains are $\frac{1}{3}$ | 0 | 1 | 4$\frac{2}{5}$ |
| 1 is $\frac{1}{8}$ | 0 | 0 | 2$\frac{5}{16}$ |
| | £. 276 | 8 | 4$\frac{1}{4}$*d.* *Anſ.* nearly. |

2. 45 ounces,

| | *Answers.* £. | *s.* | *d.* |
|---|---|---|---|
| 2. 45 ounces, 6 dwts. 7 grains, at 5s. 10d. per ounce? | 13 | 4 | 4$\frac{1}{48}$ |
| 3. If 1lb. of ſilver coſt 3l. 4s. what will 100lb. 11 ounces come to? | 322 | 18 | 8 |
| 4. If 1 ounce of ſilver coſt 5s.6d. how much will 10lb. 6 dwts. 14 grains come to? | 33 | 1 | 9$\frac{29}{40}$ |

## *Avoirdupois Weight.*

*Caſe* 14. Tons, hundreds, and quarters are reduced to the form of pounds, ſhillings, and pence by multiplying the quarters by 3.†

### EXAMPLE.

Tons cwt.qrs. l. s. d.
33 3 3 at 33 3 4 per Ton?

| | l. | s. | d. | | l. | s. | d. | |
|---|---|---|---|---|---|---|---|---|
| or | 33 | 3 | 9 | at | 33 | 3 | 4 | per. |
| | | | 11 | | | | | |
| | 365 | 1 | 3 | | | | | |
| | | | 3 | | | | | |
| | 1095 | 3 | 9 | | | | | |
| 3s.4d. is $\frac{1}{6}$l. | 5 | 10 | 7$\frac{1}{2}$ | | | | | |
| *Anſw.* £. | 1100 | 14 | 4$\frac{1}{2}$*d.* | | | | | |

*or thus, by the common way:*

Tons cwt.qrs. l. s. d.
33 3 3 at 33 3 4 per

| | | | |
|---|---|---|---|
| | 33 | | |
| | 99 | | |
| | 99 | | |
| | 1089 | | |
| 3s. 4d. is $\frac{1}{6}$l. | 5 | 10 | |
| 2 cwt. is $\frac{1}{10}$ | 3 | 6 | 4 |
| 1 is $\frac{1}{2}$ | 1 | 13 | 2 |
| 2qrs. is $\frac{1}{2}$ | 0 | 16 | 7 |
| 1 is $\frac{1}{2}$ | 0 | 8 | 3$\frac{1}{2}$ |
| *Anſwer as before,* £. | 1100 | 14 | 4$\frac{1}{2}$ |

Hundreds, quarters, and pounds Avoirdupois weight are reduced to the form of pounds, ſhillings, and pence, by multiplying the quarters by 5, and the pounds by 2$\frac{1}{7}$: begin thus, take

---

† Tons and hundreds are in the ratio of pounds and ſhillings, but the ratio of hundreds to quarters being that of 4 to 1, muſt be reduced to that of 12 to 1, that is 4×3=12 to 1.

take $\frac{1}{7}$ of the pounds, which add to twice the pounds, divide the total by 12, the remainder write down as pence, and carry the quotient to 5 times the quarters.*

EXAMPLES.

| cwt. | qrs. | lbs. | | l. | s. | d. | |
|---|---|---|---|---|---|---|---|
| 54 | 3 | 21 | at | 1 | 13 | 4 | per cwt. |
| | 5 | $2\frac{1}{7}$ | | | | | |
| 54 | 18 | 9 | at | 1 | 0 | 0 | |
| 18 | 6 | 3 | at | 0 | 6 | 8 | |
| 18 | 6 | 3 | at | 0 | 6 | 8 | |

£.91 11 3*d* *Anſ.*

*or thus,*
*by the common way:*

| | cwt. | qrs. | lbs. | | l. | s. | d. | |
|---|---|---|---|---|---|---|---|---|
| | 54 | 3 | 21 | at | 1 | 13 | 4 | per cwt. |
| | 1 | 13 | 4 | | | | | |
| | 54 | 0 | 0 | | | | | |
| 10s. is $\frac{1}{2}$ | 27 | 0 | 0 | | | | | |
| 2 is $\frac{1}{5}$ | 5 | 8 | 0 | | | | | |
| 1 is $\frac{1}{2}$ | 2 | 14 | 0 | | | | | |
| 4d. is $\frac{1}{3}$ | 0 | 18 | 0 | | | | | |
| 2qrs. is $\frac{1}{2}$ | 0 | 16 | 8 | | | | | |
| 1 is $\frac{1}{2}$ | 0 | 8 | 4 | | | | | |
| 14lb. is $\frac{1}{2}$ | 0 | 4 | 2 | | | | | |
| 7 is $\frac{1}{2}$ | 0 | 2 | 1 | | | | | |

Here you may plainly ſee the vaſt difference between both methods.

*Anſw.* as before, £.91 11 3*d.*

2. Reduce 19 ton, 14 cwt. 2 qrs. to the form of pounds, ſhillings, and pence. *Anſw.* 19l. 14s. 6d.

If a ton coſt 1l. the coſt of 19 tons, 14 cwt. 2 qrs. would be 19l. 14s. 6d. therefore we have an eaſy method for finding the amount of tons, hundreds, and quarters, when the rate per ton is more or leſs than 1l.

3. Reduce 24 tons, 15 cwt. 3 qrs. 14lb. into the form of pounds, ſhillings, and pence, and find the amount at 3l. 12s. 6d per ton. *Anſw.* 24l. 15s. $10\frac{1}{2}$d. *Amt.* 89l. 17s. $6\frac{9}{16}$d.

*Promiſcuous*

---

* Pounds are to ſhillings as 20 to 1, but hundreds are to quarters as 4 to 1; therefore $4 \times 5 =$ the form of quarters. But as they muſt retain the ſame value, it is $\frac{4 \times 5}{4} = \frac{20}{4}$ but quarters are to pounds as 28 to 1, which muſt be reduced to the ratio of 12 to 1; therefore $\frac{20}{4} \div 28 = \frac{20}{4 \times 28}$ which multiplied by 12 is $\frac{20 \times 12}{4 \times 28} = \frac{60}{28} = 2\frac{1}{7}$, the multiplier, as per rule.

## *Promiſcuous Queſtions.*

1. At 19l. 19s. 11¾d. per ton, what will 19 tons. 19 cwt. 3 qrs. 27½lb. come to? *Anſw.* 399l. 19s. 5$\frac{16641}{17920}$d.

Tons.
20 leſs by ½ a pound.
at 20l. leſs by ¼d.

| | | | |
|---|---|---|---|
| | 400 | 0 | 0 |
| 20 farthings | | | 5 ſubtract |
| | l.399 | 19 | 7 |
| ½lb coſt | | | 1$\frac{1279}{17920}$ |
| *Anſw.* | £.399 | 19 | 5$\frac{16641}{17920}$*d.* |

*or thus,*
*by the common way:*

| | Tons | cwt. | qrs. | lb. |
|---|---|---|---|---|
| | 19 | 19 | 3 | 27½ |
| | 19 | 19 | 11¾ | |
| | 171 | | | |
| | 19 | | | |
| | 361 | | | |
| 10s. is ½ | 9 | 10 | | |
| 5 is ½ | 4 | 15 | | |
| 4 is ⅕ | 3 | 16 | | |
| 6d. is ⅛ | 0 | 9 | 6 | |
| 3 is ½ | 0 | 4 | 9 | |
| 2 is ⅓ of 6. | 0 | 3 | 2 | |
| ½ is ¼ | 0 | 0 | 9½ | |
| ¼ is ½ | 0 | 0 | 4¾ | |
| 10 cwt. is ½ | 9 | 19 | 11⅞ | |
| 5 is ½ | 4 | 19 | 11$\frac{15}{16}$ | |
| 4 is ⅕ | 3 | 19 | 11$\frac{19}{20}$ | |
| 2 qrs. is ⅛ | 0 | 9 | 11$\frac{159}{160}$ | |
| 1 is ½ | 0 | 4 | 11$\frac{319}{320}$ | |
| 14lb. is ½ | 0 | 2 | 5$\frac{639}{640}$ | |
| 7 is ½ | 0 | 1 | 2$\frac{1279}{1280}$ | |
| 4 is ⅐ | 0 | 0 | 8$\frac{1279}{2240}$ | |
| 2 is ½ | 0 | 0 | 4$\frac{1279}{4480}$ | |
| ½ is ¼ | 0 | 0 | 1$\frac{1279}{17920}$ | |
| *Anſw.* | £.399 | 19 | 5$\frac{16641}{17920}$ | |

The difference between both methods is evident to the meaneſt capacity, the one having but 26 figures, excluſive of the anſwer and ſtating; the other, when the fractions are added, will be upwards of 400.

| | *Anſwers.* £. | s. | d. |
|---|---|---|---|
| 2. What will 193 cwt. 3qrs. coſt at the rate of 17s. 9½d per cwt.? | 172 | 7 | 1⅝ |
| 3. At 9s. 4d. per cwt. what will 106 cwt. 3 qrs. 14 lb. come to? | 49 | 17 | 6 |

| | *Answers.* | | |
|---|---|---|---|
| | £. | s. | d. |
| 4. At 21s. 6d. what will 306cwt. 3 qrs. 21lb. come to? | 329 | 19 | $1\frac{7}{8}$ |
| 5. At 25s. 11d. per cwt. what will 499 cwt. 3qrs. 25lb. come to? | 647 | 17 | $7\frac{75}{112}$ |
| 6. At 19l. 19s. 6d. per ton, what will 20 ton 19 cwt. 3qrs. 14lb. come to? | 419 | 7 | 9 |
| 7. At 25l. 10s. 9d. per ton, what will 10 ton 17 cwt. 3 qrs. come to? | 278 | 0 | $9\frac{39}{80}$ |
| 8. At 6s. 9d. the yard, what will 7 yards, $3\frac{1}{2}$ quarters come to? | 2 | 13 | $1\frac{7}{8}$ |
| 9. If 1cwt. coſt 19s. $11\frac{3}{4}$d. what will 11cwt. 3 qrs. $27\frac{1}{2}$lb. come to? | 11 | 19 | $7\frac{110}{128}$ |
| 10. At 19s. 9d. per cwt. what will 109 cwt. 0 qrs. 15lbs. come to? | 107 | 15 | $4\frac{83}{112}$ |
| 11. At 22s. 4d. the cwt. what coſt 18 cwt. 3 qrs. 14lb? | 21 | 1 | $6\frac{1}{2}$ |
| 12. If 1 ounce of ſilver coſt 5s. 6d. what will 20lb. 13 dwt. 14 grs. coſt? | 66 | 3 | $8\frac{33}{40}$ |
| 13. At 18s. $6\frac{1}{2}$d. per cwt. what will 50 cwt. 2 qrs. 14lb. come to? | 46 | 18 | $8\frac{1}{16}$ |
| 14. At 50s. per cwt. what will 2qrs. 14lb. coſt? | 1 | 11 | 3 |
| 15. At 17l. 10s. 8d. per ton, what will 3 cwt. 3 quarters come to? | 3 | 5 | 9 |
| 16. If 1lb of ſilk coſt 3l. 6s. 5d. what will 80lb. 10 ounces coſt? | 267 | 14 | $10\frac{1}{8}$ |
| 17. If 1 ounce of ſilver coſt 5s. 6d. what will 10lb. 6 dwts. 14 grains come to? | 33 | 1 | $9\frac{29}{40}$ |
| 18. If 42 yards of cloth are bought for 38l. 17s. $6\frac{1}{2}$d. how much will 15 yards $1\frac{1}{2}$ quarters come to? | 14 | 4 | $7\frac{141}{224}$ |
| 19. If 1lb. of ſpice coſt 12s. 8d. what will $7\frac{7}{8}$ ozs. come to? | | 6 | $2\frac{3}{16}$ |
| 20. At 27l. 10s. 6d. the cwt. what will be the price of $14\frac{3}{4}$lb.? | 3 | 12 | $5\frac{221}{224}$ |
| 21. At 17s. 6d. the yard, what will 3 quarters, 2 nails coſt? | | 15 | $3\frac{3}{4}$ |
| 22. How much will be the price of 3 qrs. 21lb. of tallow, at 27s. 8d. per cwt? | 1 | 5 | $11\frac{1}{4}$ |
| 23. At 16s. $5\frac{1}{4}$d. per cwt. what will be the coſt of 203 cwt. 3 quarters? | 167 | 9 | $11\frac{11}{14}$ |
| 24. If 1 dozen of candles coſt 4s. 4d. what will a box containing $6\frac{1}{2}$ dozen coſt? | 1 | 8 | 2 |

| | | Answers. | | |
|---|---|---|---|---|
| | | £. | s. | d. |
| 25. | What will 499 cwt. 3 qrs. 25lb. coſt at 25s. 11d. per cwt.? | 647 | 17 | $7\frac{75}{112}$ |
| 26. | How much will 9 yards, 1 qr. 2 nails, come to, at 1l. 2s. 4d. per yard? | 10 | 9 | $4\frac{1}{2}$ |
| 27. | How much will be the yearly rent of a farm containing 44 acres, 1 rood, 15 perches, at 2l. 12s. 10d. the acre? | 117 | 2 | $9\frac{15}{16}$ |
| 28. | What will 35 gallons, 5 pints of rum coſt, at 7s. 6d. per gallon? | 13 | 7 | $2\frac{1}{4}$ |
| 29. | What will 48765 ſkains of worſted come to, at 1l. 4s. 9d. per 100 ſkains? | 603 | 9 | $4\frac{1}{20}$ |
| 30. | At 9s. 4d. per cwt. what will 53 cwt. 1 qr. 21lb. come to? | 24 | 18 | 9 |
| 31. | At 7l. 9s. 3d. per ton, what will 3 ton, 5 cwt. 2 qrs. come to? | 24 | 8 | $9\frac{21}{40}$ |
| 32. | At 2l. 16s. 10d. per cwt. what will 3 qrs. $12\frac{1}{2}$lb. come to? | 2 | 8 | $11\frac{69}{112}$ |
| 33. | How much will 17lb. 5 ounces, 14 dwts. come to at 3l. 6s. 9d. per lb.? | 58 | 6 | $5\frac{19}{40}$ |
| 34. | How much will 32 acres, 1 rood, 14 perches come to, at 1l. 16s. per acre? | 58 | 4 | $1\frac{4}{5}$ |
| 35. | How much will 4 cwt. 1 qr. 7lb. come to, at 6s. 6d. per lb.? | 156 | 19 | 6 |
| 36. | How much will 14 cwt. 2 qrs. 8lb. come to, at 8l. 9s. 2d. per cwt.? | 123 | 5 | 0 |
| 37. | How much will 72 cwt. 3 qrs. 11lb. come to, at 2l. 4s. 4d. per cwt.? | 161 | 9 | $7\frac{1}{4}$ |
| 38. | At 16l. 4s. 6d. per ton, what will 1 ton 4 cwt. 2 qrs. 8lb. come to? | 19 | 18 | $8\frac{2}{35}$ |

## *Practice applied to the calculation of Bills of Parcels, Book-debts, &c.*

It is uſual, when goods are ſold, for the ſeller to deliver to the buyer, with the goods, a bill of parcels which is a note of the contents and prices, with a total caſt up, &c.

Theſe bills ought to be handſomely written, and in a methodical order, according to the beſt and cuſtomary way of each particular trade or buſineſs.

We ſhall therefore for the learners' ſake, ſhew the form of bills of parcels in moſt trades and profeſſions, and alſo the total value of each bill at the bottom, and leave the remaining articles for the learners' exerciſe.

1. *A Linen*

### 1. *A Linen-draper's Bill.*

*Dublin, Feb.* 14, 1804.

The Hon. Lady Goodhouſekeeper bought of Coleman and Co. Linen-drapers,

| | | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|
| 16 yards of dowlaſs, | at | 1 | 4 | per yard, | | | |
| 22½ Do. Holland, | — | 3 | 4 | — | | | |
| A piece of muſlin, | | | | | 2 | 17 | 3 |
| 85½ yards of diaper, | — | 2 | 4 | — | | | |
| 19½ yards of damaſk, | — | 3 | 6 | — | | | |
| 1 piece of cambrick, | | | | | 3 | 4 | 6 |
| | | | | | £. 24 | 5 | 10 |

### 2. *A Milliner's Bill.*

*Dublin,* 1804.

Madam Strawberry bought of Manywords, Milliner.

| | | | | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| Feb. | 15. Silver ribbon, | 21 yards, | at | 2 | 2 | per | | | |
| | 17. Fine lace, | 11½ do. | — | 10 | 6 | — | | | |
| | Sarſnet hoods, | 8 | — | 4 | 3 | — | | | |
| March | 10. Indian Fans | 17 | — | 3 | 10 | — | | | |
| | 12. Kid gloves, | 9 pair | — | 2 | 2 | — | | | |
| | 12. Lamb do. | 9 do. | — | 1 | 2 | — | | | |
| | 17. Robbin, | 12 pies. | — | | 6 | — | | | |
| | | | | | | | £. 15 | 1 | 5 |

### 3. *A Hoſier's Bill.*

*Dublin,* 1804.

Mrs. Mary Slipſtocking bought of John Hoſier,

| | | *pair* | | | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|
| Jan. | 4. | 15 | of women's worſted, | at | 5 | 7 | per | | | |
| | 10. | 23 | — men's ſilk, | — | 17 | 4 | — | | | |
| | | 22 | — men's yarn, | — | 3 | 2 | — | | | |
| | | 18 | — Conamara, | — | 4 | 9 | — | | | |
| | | 38 | — thread, | — | 3 | 4 | — | | | |
| | | 13 | — Limerick gloves, | — | 4 | 4 | — | | | |
| | | | | | | | | £. 41 | 0 | 7 |

When the money is paid down then there is a receipt given as follows:

Received, the 10th of February, 1804, Forty-one pounds, and ſeven pence ſterling, in full for this bill, for my maſter, *John Hoſier.* *James Faithful.*

### 4. *A Brazier's Bill.*

*Dublin,* 1804.

Mr. Martin Forreſter bought of George Reed,

| | | | £. | s. | d. |
|---|---|---|---|---|---|
| May | 3. | 50 pewter plates, wt. 50 lb. at 1s. 4d. per lb. | | | |
| | 16. | 8 do. diſhes, wt. 40 lb. at 1s. 6d. per. | | | |
| | | A copper with a diſcharging cock, wt. 66½ lb. at 1s. 4½d. per. | | | |
| | 23. | Iron work to do. and a crane, wt. 97 lb. at 2¾d. per. | | | |
| | — | Stove-grate, with ſhovel, tongs, poker, and fender, | 1 | 18 | 6 |
| June | 1. | A large braſs pot and ſauce-pan, wt. 38 lb. at 1s. 7d. per, | | | |
| | | | £.16 | 19 | 0 |

If a ſervant receive the amount let the receipt run thus:

Received, the 1ſt of July, 1804, from Mr. Martin Forreſter, by the hands of Mr. Thomas Truſty, the ſum of ſixteen pounds, nineteen ſhillings ſterling, for my maſter, *George Reed.* *William Faithful.*

### 5. *A Cabinet-maker's Bill.*

*Dublin,* 1804.

His Grace the Duke of Dalky, to Thos. Cabinet-maker, Dr.

| | | | £. | s. | d. |
|---|---|---|---|---|---|
| Mar. | 10. | To a chimney-glaſs and a pair of ſcreens, | 5 | 18 | 0 |
| April | 1. | To a pair of pier-glaſſes, 72 inches, in gilt frames, | 60 | 0 | 0 |
| | | To a pair of Indian cabinets, at 43l. 10s. each, | | | |
| | | A fine Indian four-leaved ſcreen and a fire-ſcreen, | 17 | 10 | 0 |
| May | 4. | A book-caſe with glaſs doors, board and corner-board ditto, | 21 | 0 | 0 |
| | | A walnut table and a ſet of dreſſing boxes, japanned, | 3 | 4 | 10 |
| | | Carried over, | | | |

Brought

| | | £. | s. | d. |
|---|---|---|---|---|
| | Brought over, | | | |
| May 4. | A dozen and half of fine matted chairs, at 18s. 6d. each, | | | |
| | 12 Elbow chairs, ſtuffed with hair, at 1l. 15s. each, | | | |
| June 10. | A dreſſing table, with implements for writing, | 6 | 16 | 6 |
| | | £. 239 | 2 | 4 |

A receipt where there is a company.

Received, 1ſt July, 1804, two hundred and thirty-nine pounds, two ſhillings and four pence, in full for this bill for my maſter, *Thomas Cabinet-maker & Co.*

per me, *Thomas Never-to-doubt.*

### 6. *A Wine-Cooper's Bill.*

Bernard Jones, Eſq. Bought of William Wine-Cooper,

| Feb. 1, 1804. | | *gall.* | | *s.* | *d.* | | £. | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|
| | Palm ſack, | 18 | at | 8 | 6 | per, | | | |
| March 10. | Port, red, | 35 | — | 5 | 4 | — | | | |
| 19. | Sherry, | 17 | — | 6 | 6 | — | | | |
| | Rheniſh, | 19 | — | 6 | 8 | — | | | |
| | Liſbon white, | 32 | — | 4 | 10 | — | | | |
| Dec. 30. | A double cheſt of Florence, agreed for, | | | | | | 4 | 0 | 0 |
| | 10½ dozen of beſt Burgundy, at 50s. | | | | | | | | |
| April 3. | 7 do. of Colerotie, 2l. 2s. | | | | | | | | |
| | | | | | | | £. 81 | 10 | 6 |

### 7. *A Silver Smith's Bill.*

Sir Thos. Bibant, bought of Leonard Silverſmith,

| May 7, 1804. | *oz.* | *dwt.* | *gr.* | | *s.* | *d.* | | £. | *s.* | *d.* |
|---|---|---|---|---|---|---|---|---|---|---|
| A punch-bowl, wt. | 23 | 4 | 0 | at | 5 | 10 | per oz. | | | |
| A tankard, | 10 | 3 | 6 | — | 6 | 2 | — | | | |
| A tea-pot and lamp | 30 | 5 | 12 | — | 7 | 3 | — | | | |
| 6 plates, | 73 | 11 | 5 | — | 6 | 1 | — | | | |
| 18 ſpoons, | 41 | 0 | 10 | — | 6 | 3 | — | | | |
| | | | | | | | | £. 56 | 1 | 4 |

8. *A Grocer's*

## 8. *A Grocer's Bill.*

*Dublin*, 1804.

William Williams, Eſq. to Charles Groder, Dr.

| | | lbs. | | | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|
| Auguſt | 1. | 15½ | of currants, | at | 0 | 4 | per lb. | | | |
| | | 17¼ | — Malaga raiſins, | — | 0 | 5½ | — | | | |
| | | 19¾ | — raiſins, | — | 0 | 6 | — | | | |
| | 10. | 17 | — rice, | — | 0 | 3⅐ | — | | | |
| Sept. | 1. | 3 | — ſugar, | — | 0 | 8½ | — | | | |
| | 3. | 8¼ | — pepper, | — | 1 | 6 | — | | | |
| | 16. | 13 | oz. of cloves, | — | 0 | 9 | per oz. | | | |
| | | | | | | | | £. 2 | 12 | 1⅞ |

## 9. *A Woollen-draper's Bill.*

*Dublin*, *Sept.* 4, 1804.

Mr. Thomas Dilatory bought of William Broadcloth,

| | | | yds. | | s. | d. | | £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 10 pieces of cloth, each | 42 | at | 12 | 6 | per | | | |
| Oct. | 10. | 62 yards of Spaniſh black | | — | 15 | 1½ | — | | | |
| Dec. | 19. | 12 pieces of drab, each | 36 | — | 11 | 4½ | — | | | |
| | | 74 yards of broadcloth | | | 16 | 3 | — | | | |
| | | | | | | | | £. 615 | 4 | 3 |

If part of the bill is paid, write thus:

Received from Mr. *Thomas Dilatory*, two hundred pounds, nineteen ſhillings and nine pence, in part payment for my maſter, *William Broadcloth*. *Jeffry Truſty.*

## 10. *A Tobacconiſt's Bill.*

*Dublin*, 1804.

Mr. Henry Virginia, to Joſeph Tobacconiſt, Dr.

| | £. | s. | d. |
|---|---|---|---|
| 1 hhd. beſt bright tobacco, qt. wt. 5½ cwt. at 10⅛d. per lb. | | | |
| 1 box of Oronooko, qt. wt. 75½ lbs. at 11¾d. per, | | | |
| 5 bags of old Spaniſh, qt. wt. 684 lb. at 4⅛d. per, | | | |

Carried over,

| | £. s. d. |
|---|---|
| Brought over, | |
| $\frac{1}{2}$ hhd. qt. groſs, 335 lb. tare, 42 lb. qt. wt. 293 lb. at $5\frac{3}{8}$ per. | |
| 2 rolls, qt. wt. 94 lb. at 1s. $5\frac{1}{8}$d. per, | |
| A parcel of Patomeck River, qt. wt. 113 lb. at 1s. 8d. | |
| | £.64 2 6 |

### 11. *A Bricklayer's Bill.*[*]

The Rt. Hon. Ed. Thompson to Jonath. Bricklayer Dr.
For work and materials at his houſe in Mountjoy-ſquare.

| | | | s. | d. | | £. s. d. |
|---|---|---|---|---|---|---|
| March 20. | Bricks, 25 thouſand, | at | 15 | 7 | per | |
| 22. | Tiles, 11 do. | — | 19 | 5 | — | |
| | Lime, 28 hundred | — | 15 | 11 | — | |
| April 4. | Sand, 19 load, | — | 3 | 10 | — | |
| | Work for myſelf, 90 days, | | 3 | 0 | — | |
| | Do. a labourer, 90 days, | | 1 | 8 | — | |
| | Do. for my man, 90 days, | | 2 | 6 | — | |
| | | | | | | £.88 6 8 |

### 12 *A Builder's Bill.*[†]

*June* 1, 1804.

The Right Hon. Bernard Earl of Dalkey, Dr.
To Henry Builder, for work and materials in his Houſe, Mt. Pellier,

| | | | £. s. d. | | £. s. d. |
|---|---|---|---|---|---|
| Oak timber, | 12 load | at | 2 5 0 | per load, | |
| Fir timber, | 35 ton | — | 1 12 10 | — load, | |
| | | | Carried over, | | |

Oak

---

[*] A brick ought to be 9 inches long, $4\frac{1}{2}$ broad, and $2\frac{1}{2}$ thick; 500 bricks are a load; 1000 tiles, do.; 25 buſhels are a hundred of lime; about 4500 bricks will make a rod of Brick-work, viz. $272\frac{1}{4}$ ſquare feet, a brick and a half thick.

[†] *Note.* Deals and nails are 120, or 6 ſcore to the hundred, 50 feet are a load, and 40 feet a ton of timber; 100 feet are the ſquare of carpenter's tiling, &c.

| | | | £. | s. | d. | | Brought forward, £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| Oak plank, | 96 feet | at | 0 | 0 | 3 | per foot, | | | |
| Norway deals, | 590 do. | — | 6 | 15 | 0 | — hund. | | | |
| Sixpenny nails, | 25 thou. | — | 0 | 3 | 10 | — thouſ. | | | |
| Ten groat nails, | 3 hun. | — | 0 | 14 | 10 | — — | | | |
| Work for myſelf, | 90 days | — | 0 | 3 | 4 | — day, | | | |
| Ditto for 3 men, | 90 — | — | 0 | 2 | 6 | — — | | | |
| Wainſcot, | 73 yds. 7 ft. | | 0 | 3 | 2 | — yd. agrd. | | | |
| Double quarter, | 58 feet, | — | 0 | 0 | 4 | — foot, | | | |
| Chriſtiana deals, | 135 — | — | 7 | 10 | 0 | — hun. | | | |
| Riga Timber, | 180 — | — | 0 | 1 | 1 | — foot, | | | |
| | | | | | | | £.207 | 13 | 3½ |

# CHAP. XIX.

## TARE AND TRETT.

GROSS weight is the weight of the goods in hundreds and pounds, with the weight of the hogſheads, caſks, cheſts, bag, bale, &c. that contain the goods. Tare is allowed to the buyer for the weight of the hogſhead, caſk, cheſt, bag, bale, box, &c. wherein the goods are contained, and is ſometimes reckoned ſo much per bag, bale, barrel, caſk, &c. as in ſilks, cottons and raiſins. At other times, at ſo much per cwt. as 14, 16, 20, or 24 lb. per cwt. There is alſo a diſtinction of cuſtom-houſe and invoice tare, as in tobacco, indigo, &c. which is ſometimes uncertain, happening according to the ſize of the caſk; but in general, the weight of the caſk, bag, bale, &c. is marked on the outſide thereof.

Trett is an allowance made for waſte, duſt, &c. in ſundry ſorts of goods, as tobacco, ſugar, ſpices, &c. and is in moſt places 4 lb. per 104 lb. ſuttle, and is found by dividing the ſuttle pounds by 26, (becauſe 4 times 26 is 104); when the groſs weight is brought into pounds, and before the tare is ſubtracted, they are called lbs. groſs. When the tare is ſubtracted,

tracted, and not the Trett, the remainder is called the ſuttle weight, and when the Trett is taken away, the remainder is the neat wt. (or nett).

But according to the cuſtom of the city of Cork, the Trett is 1 per cwt.; that is 1 lb. out of every 112 lb. to be ſubtracted from the groſs wt. of all ſorts of merchandizes, except for beef, wool, butter and hides. For wool, there is allowed 8 lb. for every 20 ſtone to be deducted after the Tare is ſubtracted, but in Dublin, 8 lb. for every 21 ſtone; for butter, 2 lb. at the beam for every large caſk; for raw hides, 4 lb. per piece; and for beef, 8 lb. per carcaſe.

*Note.* There is an allowance of 2 lb. for every draft above 3 cwt. to the citizens of London, and this they call Cloff.

The foregoing cuſtoms and remarks being premiſed, this moſt uſeful rule in the occurrences of Mercantile Buſineſs, will be divided into five different caſes, as follow, viz.

### *Caſe* I.

When the Tare is ſo much per box, barrel, bag, &c. to find the neat wt. and amount, at any given rate.

*Rule.* Multiply the number of boxes, barrels, &c. by the Tare of one, and ſubtract the product from the groſs, the remainder will be the neat wt. required; then calculate it at the given rate by the rules in Practice.

### EXAMPLES.

1. What will 241 barrels of figs, each 3 qrs. 19 lb. come to at 4½d. per lb. neat, tare 10 lb. per barrel?

*qrs. lbs.* *bar.*
3 19 241
84 10

the lbs. in 1 bar. 103 2410 the lbs. tare in all.
241
*d.* *l.* *s.* *d.*
103 6 is $\frac{1}{40}$ and $\frac{1}{2}$)22413 at 4½ per lb.
412
206 560 6 6
of 6d. 1½ is ¼ 140 1 7½ ſubtr.
From 24823 groſs. Practice
Take 2410 tare. *Anſ.* £. 420 4 10½ *Caſ* 6.

22413 nt. wt.

2. What will 14 caſks of tobacco come to, each weighing 5 cwt. 2 qrs. 17 lb. groſs; tare, 100 lb. per caſk, at 9l. 6s. per cwt.? *Anſ.* 621l. 16s. 8d.

3. What will 400 raw hides come to, wt. groſs, 306 cwt. 3 qrs. 15 lb.; tare, 4 lb. per hide, at 19s. 10d. per cwt.?

*Anſ.* 290l. 3s. $2\frac{2}{3}$d.

## *Caſe.* II.

To find the amount of any quantity of goods, at a given rate, when the Tare is ſo much per cwt.

*Rule.* 1. When the Tare of one hundred is an aliquot part of it, divide the groſs weight by the denominator, and ſubtract the quotient from the groſs wt. the remainder will be the neat weight.

2. But if the tare be not an aliquot part of an cwt. take ſuch parts as make it up, and ſubtract their ſum from the groſs weight, the remainder will be the neat weight, then find the amount by the rules in Practice.

### EXAMPLES.

4. What will 85 caſks of butter come to, wt. groſs, 173 cwt. 3 qrs. 14 lb. at 3l. 4s. 6d. per cwt. tare being 16 lb per cwt.?

*Anſ.* 480l. 12s. $9\frac{9}{14}$d.

| | *cwt.* | *qrs.* | *lbs.* | |
|---|---|---|---|---|
| | 173 | 3 | 14 | |
| 16 is $\frac{1}{7}$ | 24 | 3 | 10 | tare. |
| | 149 | 0 | 4 | nt. wt. |
| | | 5 | $2\frac{1}{7}$ | (by *Caſe* 14, Practice.) |
| | 149 | 0 | $8\frac{4}{7}$ | |
| | | | 3 | |
| *s.* | 447 | 2 | $1\frac{5}{7}$ | |
| *lb.* 4 is $\frac{1}{5}$ | 29 | 16 | $1\frac{5}{7}$ | |
| 6 is $\frac{1}{8}$ of 4 | 3 | 14 | $6\frac{3}{14}$ | |
| £. | 480 | 12 | $9\frac{9}{14}$ | the *Anſ.* required. |

5. What is the amount of 13 hhds. of tobacco, wt. groſs, each containing 4 cwt. 3 qrs. 17 lb.; tare, 13 lb. per cwt. at 5l. 10s. per cwt.? *Anſ.* 304l. 19s. $1\frac{525}{1568}$d.

6. If 1 cwt. of butter coſt 21s. 6d. what will 250 firkins coſt, wt. groſs, 127 cwt. 2 qrs. 21 lb.; tare, 11 lb. per firkin? *Anſ.* 110l. 17s. $4\frac{3}{5}\frac{1}{8}$d.

### *Caſe* III.

When there is an allowance of both Tare and Trett; to find the amount of the neat weight of any quantity of goods, at any rate propoſed.

*Rule.* Find the tare, by ſome of the foregoing rules, ſubtract it from the groſs; then find the trett, and ſubtract it from the remainder (or ſuttle) and you have the nt. wt. the amount of which may be calculated by the rules in Practice.

EXAMPLES.

7. A merchant bought 4 hhds. of indigo, wt. as follows, viz. No. 1, 10 cwt. 2 qrs. 4 lb.; tare, 3 qrs. 4 lb.; No. 2, 11 cwt. 0 qrs. 10 lb.; tare, 3 qrs. 10 lb.; No. 3, 12 cwt. 1 qr. 0 lb.; tare, 3 qrs. 14 lb.; No. 4, 11 cwt. 2 qrs. 14 lb.; tare 3 qrs. 18 lb.; trett 1 lb. per, and at 1s. per ounce, or 89l. 12s. per cwt. how much do they amount to?

| *No.* | *cwt.* | *qrs.* | *lb.* | | | *qrs.* | *lb.* |
|---|---|---|---|---|---|---|---|
| 1.— | 10 | 2 | 4 | tare, | | 3 | 4 |
| 2.— | 11 | 0 | 10 | — | | 3 | 10 |
| 3.— | 12 | 1 | 0 | — | | 3 | 14 |
| 4.— | 11 | 2 | 14 | — | | 3 | 18 |
| groſs, | 45 | 2 | 0 | | 3 | 1 | 18 |
| trett, | | 1 | $17\frac{1}{2}$ | | | | |
| ſut. wt. | 45 | 0 | $10\frac{1}{2}$ | | | | |
| tare, | 3 | 1 | 18 | | | | |
| nt. wt. | 41 | 2 | $20\frac{1}{2}$ | | | | |
| | 89 | 12 | | | | | |
| | 369 | | | | | | |
| | 328 | | | | | | |
| | 3649 Carried over. | | | | | | |

*or thus: by taking the tare firſt out of the gross wt. and then the trett.*

| *No.* | *cwt.* | *qrs.* | *lb.* | | | *qrs.* | *lb.* |
|---|---|---|---|---|---|---|---|
| 1.— | 10 | 2 | 4 | tare, | | 3 | 4 |
| 2.— | 11 | 0 | 10 | — | | 3 | 10 |
| 3.— | 12 | 1 | 0 | — | | 3 | 14 |
| 4.— | 11 | 2 | 14 | — | | 3 | 18 |
| groſs wt. | 45 | 2 | 0 | | 3 | 1 | 18 |
| tare, | 3 | 1 | 18 | | | | |
| ſuttle wt. | 42 | 0 | 10 | | | | |
| trett, | | 1 | $14\frac{5}{58}$ | | | | |

nt. wt. 41 2 $23\frac{5}{5}\frac{1}{8}$ at 89l. 12s. per cwt. come to 3737l. 10s. $6\frac{6}{7}$d. the *Anſ.*;

Remark, the difference is 2l. 14s. $6\frac{6}{7}$d.

| | | | | | |
|---|---|---|---|---|---|
| s. | | | 3649 | | Brought over. |
| 10 | is | $\frac{1}{2}$ | 20 | 10 | |
| 2 | is | $\frac{1}{5}$ | 4 | 2 | |
| 2qr. | is | $\frac{1}{2}$ | 44 | 16 | |
| 16lb. | is | $\frac{1}{7}$ | 12 | 16 | |
| 4 | is | $\frac{1}{4}$ | 3 | 4 | |
| $\frac{1}{2}$ | is | $\frac{1}{8}$ | 0 | 8 | |

* *Anſ.* £.3734 16

When the trett is according to the cuſtom of the city of Cork, viz. 1 lb. per cwt. it is no more than to take 1 lb. for every cwt. *(i. e.)* 1 lb. for every 112 lb.; $\frac{1}{4}$ lb. for 1 qr.; $\frac{1}{2}$ lb. for 2 qrs.; $\frac{3}{4}$ lb. for 3 qrs. &c. which reduce to cwts. qrs. and lbs. if need be, and ſubtract it from the ſuttle wt. the remainder will be the nt. wt.

8. What will 2 hhds. of tallow come to, weighing groſs 21 cwt. 2 qrs.; tare 28 lb. per hhd.; trett 1 lb. per cwt. and at 1l. 10s. per cwt.? *Anſ.* 23l. 19s. $5\frac{19}{112}$d.

9. What will 2 bags of merchandizes come to, wt. groſs 6 cwt. 2 qrs. trett, 1 lb. per cwt. tare 1 qr. 4 lb. and at 1s. 8d. per lb.? *Anſ.* 57l. 9s. $7\frac{5}{7}$d.

10. What will 150 carcaſes of beef come to, wt. groſs 675 cwt. 0 qrs. 7 lb. trett, 8 lb. per carcaſe; at 8s. 4d. per cwt. and what is the nt. and amount?

*Anſ.* nt. wt. 664 cwt. 1 qr. 11 lb. amount 276l. 16s. $2\frac{23}{28}$d.

11. Bought 500 firkins of butter, weight groſs, 243 cwt. 3 qrs. tare, 13lb. per firkin; more 364 do. weight groſs, 198 cwt. 3 qrs. 12 lb. tare, $12\frac{1}{4}$ lb. per firkin; more 1728 ditto, each wt. groſs, 2 qrs. 26 lb. tare $12\frac{1}{2}$lb. per firkin; trett, 1 lb. per cwt. and at 32s. 8d. per cwt. I demand the nt. wt. and amount?

*Anſ.* 1250 cwt. 2 qrs. $19\frac{9}{56}$ lb. neat; its amount 2042l. 15s. $3\frac{1}{13}$d.

12. Bought 264 firkins of butter, wt. groſs 314 cwt. 2 qrs. 19 lb. tare, 13 lb. per firkin+514 ditto, wt. 238 cwt. 1 qr. 18 lb. tare, $12\frac{3}{4}$ lb. per firkin+196 ditto, wt. 89 cwt. 3 qrs. 19

---

* We have exhibited the common method, as taught in moſt of our ſchool-books, viz. by deducting the trett from the groſs weight before the tare is firſt taken away, and contraſted it with that method which reaſon and demonſtration manifeſtly point out as juſt and truly ſcientific, viz. by taking the tare firſt from the groſs, and then, from the remainder, which is the abſolute weight of the goods, uſually called the *ſuttle weight*, deducting the trett and finding the amt.; by which it appears that there is a material difference in the anſwers, viz. 2l. 14s. $6\frac{6}{7}$d. ſo that in high-priced articles, ſuch as indigo, ſilk, &c. a very conſiderable error is introduced, when the common method is uſed, as may be ſeen by the foregoing example.

19 lb. tare, $12\frac{3}{4}$ lb. per firkin; trett, 1 lb. per cwt. and at 23s. 4d. per cwt. neat. What will they amount to?

*Anſ.* 615 lb. 11s. $10\frac{4}{8}\frac{7}{4}$d.

13. Received 120 large caſks of butter, each weight, 1 cwt. 2 qrs. 7 lb. tare, 32 lb. per + 117 ditto, each weight 1 cwt. 2 qrs. 6 lb. tare, $34\frac{1}{2}$ lb. per + 283 ditto, each weight, 1 cwt. 1 qr. 18 lb. tare 27 lb. per: I demand the nt. wt. and amount, at 25s. 8d. per cwt.?

*Anſ.* 623 cwt. 0 qr. 10 lb. neat; amount 799l. 12s. $7\frac{1}{2}$d.

## *Caſe* IV.—*Of Wool.*[*]

14. Bought 3 bags of wool, wt. viz. No. 1, 3 cwt. 1 qr. 12 lb.; No. 2, 3 cwt. 3 qrs. 7 lb.; No. 3, 3 cwt. 2 qrs. 15 lb. tare 30 lb. per bag, and trett, 8 lb. per 20 ſtone, at 10s. 6d. per ſtone: what do they amount to? *Ans.* 35l. 16s. $7\frac{1}{2}$d.

### EXAMPLES.

```
                                cwt. qrs. lb.
30                 No. 1.—3      1    17
 3                 —   2:—3      3     7
—   qr. lbs.       —   3.—3      2    15
90=3  6                      ─────────────
                              10      3     6
                                      3     6
                              ─────────────
                              10 .    0     0
                               7      0     0
                              ─────────────
                              70      0     0
8 lb. is 1/40 of 20 ſtone,     1     12     0
                              ─────────────
                   cwt. 68    4     0 at 10s. 6d. Find amt.
                                     per rules in Practice.
```

15. What will 5 packs of wool come to, No. 1, 4 cwt. 2 qrs. 15 lb.; No. 2, 4 cwt. 2 qrs. 0 lb.; No. 3, 3 cwt. 3 qrs. 21 lb. No. 4, 3 cwt. 3 qrs. 14 lb.; No. 5, 4 cwt. 0 qrs. 14 lb. tare, 28 lb. per pack; trett, 8 lb. for every 20 ſtone, at 11s. 6d. per ſtone? *Anſ.* 77l. 15s. $8\frac{1}{7}\frac{3}{8}$d.

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[*] After you have deducted the tare as before, multiply the cwts. by 7 (becauſe there are 7 ſtone of wool in 112 lbs.) to which add the ſtones in the qrs. and lbs. (if any) then becauſe there are 8 lb. trett allowed for every 20 ſtone; ſay 8 lb. is $\frac{1}{40}$, and the ſuttle wt. divided thereby, and the quotient taken from the ſuttle wt. will give the nt. wt. in ſtones, &c. then find the amount by the rules in Practice.

## *Promiſcuous Queſtions.*

16. Sold 2 hhds. of tallow, No. 1, wt. 11cwt. 3qrs. 17lb. groſs tare 3 qrs. 20lb. No. 2, wt. 11cwt. 1 qr. 17lb. groſs tare 3qrs. 14lb. trett 1lb. per cwt. How much is the neat wt. and its amount at 1l. 10s. per cwt.? *Anſ.* Neat weight 21 cwt. 1qr. 6½lb. and amount, 31l. 19s. $2\frac{25}{28}$d.

17. Bought 4 packs of wool, each wt. groſs 7cwt. 3qrs. 24lb. tare 32lb. per; more 9 packs each wt. 50 cwt. 3 qrs. 21lb. tare 30lb. per pack; more 11 packs each wt. 5 cwt. 1qr. 20lb. tare 26lb. per pack; and at 13s. 4d. per ſtone: what will it come to? *Anſ.* 636l. 10s. 10d.

18. Invoice of 15 bags of wool bought for my own account, viz.

| | ct. | qrs. | lb. | | ct. | qrs. | lb. | | ct. | qrs. | lb |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. 1, | 4 | 3 | 16 | No. 6, | 5 | 3 | 20 | No. 11, | 5 | 2 | 19 |
| 2, | 5 | 1 | 12 | 7, | 6 | 2 | 12 | 12, | 6 | 1 | 8 |
| 3, | 3 | 3 | 25 | 8, | 6 | 1 | 14 | 13, | 6 | 2 | 5 |
| 4, | 6 | 1 | 24 | 9, | 7 | 1 | 11 | 14, | 7 | 1 | 18 |
| 5, | 7 | 2 | 4 | 10, | 4 | 3 | 26 | 15, | 7 | 2 | 12 |

Tare 30lb. per bag, trett 8lb. for every 3 cwt. at 10s. 3d. per ſtone: how much do they come to? *Ans.* 311l. 13s. 8⅔d.

19. Bought 12 bags of wool, viz.

| | ct. | qrs. | lb. | | ct. | qrs. | lb. | | ct. | qrs. | lb. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| No. 1, | 5 | 2 | 24 | No. 5, | 6 | 2 | 13 | No. 9, | 6 | 1 | 17 |
| 2, | 2 | 1 | 0 | 6, | 5 | 1 | 19 | 10, | 4 | 2 | 26 |
| 3, | 7 | 1 | 18 | 7, | 7 | 2 | 14 | 11, | 5 | 3 | 13 |
| 4, | 4 | 3 | 22 | 8, | 5 | 3 | 24 | 12, | 6 | 1 | 19 |

Tare 29lb. per bag, trett 8lb. for every 20 ſtone, at 10s. 9d. per ſtone: what do they come to? *Anſ.* 243l. 1s. 4d.

20. What will 3 bales of coffee come to, No. 1, weight 2cwt. 3 qrs. 12lb. tare 16lb. No. 2, 3 cwt. 1 qr. 20lb. tare 19lb. No. 3, 3 cwt. 1 qr. 24 lb. tare 18lb. trett 1lb. per cwt. and at 18l. 10s. per cwt.? *Anſ.* 170l. 0s. $2\frac{23}{36}$d.

## CHAP. XX.

## ESTIMATES, IMPORTS, AND EXPORTS.

GOODS Imported and Exported have ſeveral charges attending them, which are to be added to the firſt coſt, in order to aſcertain what they ſtand the owner in:

The following examples will explain the method of making the neceſſary calculation.

Cuſtomers make trade, and the opulent are the chief conſumers; it is by their expatriation that trade is to be benefited. Foreign trade conſiſts in exports and imports; exports are of two kinds, raw materials and manufacture.

The exports muſt always bear a proportion to the imports, if the former decreaſe ſo muſt the latter. Markets make trade, and cuſtomers make markets.

1. Samuel Champion ſhipped in the Cardiff of Dublin, bound for Cadiz, 400 barrels of beef, each weighing 2 cwt. 3 qrs. 14 lb. which being bought at the rate of $2\frac{1}{4}$d. per lb. ſalting and cooperage ſtood him in 3s. 6d. per barrel, carriage to the quay, boat hire, &c amounted to 2l. 14s. 9d. he alſo bought 250 tubs of butter on the quay, each weighing 1 cwt. 2 qrs. 18 lb. which when on board ſtood him in 3l. 4s. 6d. the cwt. Required the whole amount of the butter and beef?

*Anſw.* 2619l. 2s. $9\frac{3}{14}$d.

2. A merchant in Cork imports from London 5 bales of paper, each containing 10 reams, which with charges there, amounted to 29l. he pays in Cork 6d. per ream duty, for freight 19s. porterage 1s. What does it ſtand him in per ream? *Ans.* 12s. 6d.

3. Bought 200 carcaſſes of beef, weight neat 850cwt. 2qrs. at 18s. 8d. per cwt. and ſalted them up in 400 barrels, which ſtands me in 3s. 4d. a piece, paid for 50 barrels of ſalt at 1l. 3s. 8d. per barrel, paid for ſalting, labourers, &c. 4l. 3s. 4d. I want to know what one barrel ſtands me in.

*Anſ.* 2l. 6s. $2\frac{7}{25}$d.

4. Shipped for Briſtol 20 packs of yarn, which coſt me 408l. 10s. paid for 59 yards of coarſe linen at 7d. per yard, for packing 5d. per pack, porterage to the quay 4d. per pack, for licenſe, duty, and other charges, 5l. 13s. 4d. lighterage

11s. 2d. I deſire to know what they ſtand me in per pack on board. *Ans*. 20l. 17s. 2$\frac{7}{25}$d.

5. Imported from Spain 10 tun of wine at 10l. per hhd. paid duty 1s. 1d. per gallon, freight 15s. per tun, the charges for lighterage, porterage, &c. came to 11l. 9s. by accident one pipe containing 126 gallons entirely leaked, What doth the remainder ſtand me in per gallon? How muſt I ſell it per hhd. to gain 10 per cent.? And what then will be the gain upon the whole? *Ans*. the remainder ſtands me in 4s. 7d. per gallon; I muſt ſell it at 15l 17s. 7½d per hhd. and gain by the whole, 54l. 17s. 3d.

6. Imported 6 bags of hops each weighing 50lb. which ſtand me in 15l. 4s. 9d. beſides 2d. per lb. duty, freight 25s. and porterage home 1s. 6d. I demand what 1lb ſtands me in? *Ans*. 15¼d.

# CHAP. XXI.

## COMMISSION AND BROKERAGE.

COMMISSION is a premium allowed by an employer to his factor for tranſacting his buſineſs. Brokerage is an allowance of ſo much per cent to perſons called Brokers, whoſe employment is to find out cuſtomers, to buy goods, and procure merchandize for ſuch as want to purchaſe. The allowance made to perſons who tranſact the buſineſs of merchants that deal in foreign trade is called factorage and is little different from commiſſion.

Inſurance is an allowance of ſo much per cent, given to certain perſons and offices, who engage to make good the loſs of ſhips, houſes, and merchandizes, which may happen from ſtorms, fire, &c.

By having the amount of ſales of goods diſpoſed of by a factor, broker, or perſon commiſſioned, the amount of goods inſured, and the premium or commiſſion, &c. allowed on every 100l. &c. to find the commiſſion on the whole.

*Rule* 1. Multiply the ſum (on which the commiſſion, brokerage, inſurance, &c. is to be charged or calculated, by the integer or whole number of the rate, and for the fractional part of the rate, take the like part of the ſum.

2. Divide the product by 100, thus, from the pounds of the dividend point off two figures to the right hand. Theſe figures ſo cut off, multiply by 20, adding the ſhillings, (if any) and from this product again cut off two figures to the right. Theſe two now cut off, multiply by 12, adding the pence of the dividend, (if any) and here again cut off 2 figures to the right and multiply by 4, adding the farthings of the dividend (if any) and ſtill cut off two figures as before, theſe ſeveral ſums to the left of the ſeparating points, are the pounds, ſhillings, pence and farthings of the anſwer required. Or thus:

*A ſhort rule of dividing £. s. d. by* 100.

Omit the units and tens of the pounds, ſetting the remaining figures of the pounds underneath each figure two places further to the right, and you have the pounds of the quotient.

2. The 5th of the units and tens ſo omitted, are the ſhillings.

3. The remainder of the units (if any) take as tens, with half the ſhillings of the dividend, as units, then caſt away 1 for every 25, the remainder here is farthings, which reduce to pence.

EXAMPLES.

William Jones of Dublin, ſold 400 barrels of oats, belonging to Mr. Nicholas Walſh of Waterford, which amounted to 345l. 6s. 7¾d. and for which he allowed 2½ per cent commiſſion on the whole: how much muſt he remit his employer after deducting his commiſſion?

*Anſ.* 336l. 13s. 11¾d.

| £ | s. | d. | |
|---|---|---|---|
| 345 | 6 | 7¾ | |
| | | 2½ | |
| 690 | 13 | 3½ | at 2. |
| 172 | 13 | 3⅞ | at ½ |
| 863 | 6 | 7⅜ | at 2½ |
| 8 | 12 | 8 | the *Anſ.* |

*or thus; by the common way:*

| | | | |
|---|---|---|---|
| 345 | 6 | 7¾ | 863 6 7⅜ |
| | | 2½ | 20 |
| 690 | 13 | 3½ | 12,66 |
| 172 | 13 | 3⅞ | 12 |
| 863 | 6 | 7⅜ | 7,99 |
| | | | 4 |
| | | | 3,99 |

From ſales 345l. 6s. 7¾d. take the commiſſion, 8l. 12s. 8d. ; nearly, remains 336l. 13s, 11¾d. the ſum to be remitted.

*Note*. You ſee the difference between both methods, in dividing by 100, the one is done at firſt ſight, the other having 15 figures.

| | | £. | s. | d. | at | Anſwers. £. | s. | d. |
|---|---|---|---|---|---|---|---|---|
| 2. | What is the commiſ. on | 456 | 7 | 8 | at $\frac{1}{2}$? | 2 | 5 | $7\frac{1}{2}$ + |
| 3. | ——— | 567 | 8 | 9 | — $\frac{1}{4}$? | 1 | 8 | $4\frac{1}{2}$. + |
| 4. | ——— | 678 | 9 | 10 | — $\frac{1}{8}$? | 0 | 16 | $11\frac{1}{2}$. + |
| 5. | ——— | 789 | 10 | 11 | — $\frac{1}{3}$? | 2 | 12 | $7\frac{1}{2}$. |
| 6. | ——— | 380 | 10 | 5 | — $3\frac{3}{4}$? | 14 | 5 | $4\frac{11}{16}$. |
| 7. | ——— | 199 | 3 | 6 | — $2\frac{1}{2}$? | 4 | 19 | $7\frac{1}{20}$. |
| 8. | ——— | 1009 | 18 | 0 | — $2\frac{1}{4}$? | 22 | 14 | $5\frac{23}{50}$. |
| 9. | ——— | 336 | 10 | 0 | — $2\frac{1}{2}$? | 8 | 8 | 3. |

10. What muſt I pay a broker who ſells goods for me to the amount of 500l. 10s. 7d. being allowed 7s. per ct. for his trouble? *Anſ.* 1l. 15s. $0\frac{389}{1000}$d.

11. Inſured on goods in the William, of Dublin, bound for Alicant, in Spain, amounting to 10000l. at 10 per cent. *Anſ.* 1000l.

*To calculate Commiſſion, Brokerage, &c. by Aliquot parts of* 100.

*Rule*. Take ſuch part or parts of the ſum propoſed, as the rate is of 100.

EXAMPLES.

£. s. d.
(12.) 123 4 5 at 5

5 is $\frac{1}{20}$ 6 3 $2\frac{1}{2}$ *Anſ.*

£. s. d.
(13.) 234 5 6 at $2\frac{1}{2}$

$2\frac{1}{2}$ is $\frac{1}{40}$ 5 17 $1\frac{1}{2}$ *Anſ.*

£. s. d.
(14.) 345 6 7 at $7\frac{1}{2}$

5 is $\frac{1}{20}$ 17 5 4 at 5
$2\frac{1}{2}$ is $\frac{1}{2}$ 8 12 8 at $2\frac{1}{2}$

£. 26 18 0 *Anſ.*

## *Of Inſurance.*

Inſurance is a ſecurity or writ, called a policy, given by perſons, called Inſurers, or Under-writers, to indemnify the inſured, or perſons inſuring, from ſuch loſſes, as are expreſſed in the policy for which a premium is given to the former, and with the reſtrictions particularized in the writ.

In caſe of loſs, the inſured recovers, generally, two per cent. ſhort of the ſum under-written, that is, inſtead of 100l. he recovers but 98l.; but as an equivalent, the inſured may have this two per cent alſo; and this is called covering his outſet. This draw-back of 2 per cent. is called the diſcount per cent.

In caſes where partial loſs is incurred for the preſervation of the whole, all concerned in the ſhip, cargo, &c. are to bear a proportional part of what was ſacrificed for the common good, and which muſt be made good by the under-writers, in ſuch proportion as they have under-written, and this is called average.

*Prob.* 1. The ſum to be inſured given together with the rate per cent. to find the premium upon that ſum.

*Rule* As 100l. are to the given rate per cent. ſo is the given ſum to the required premium.

*Note.* This may be done as in page 196.

### EXAMPLE.

What is the premium of 347l. 12s. 6d. at 15l. 15s. per cent? *Anſ.* 54l. 15s. 0$\frac{9}{40}$d.

*Prob.* 2. The ſum inſured, and the diſcount per cent given, to find the ſum to be recovered, and the diſcount upon the whole.

*Rule.* As 100l. are to 100 leſs, the diſcount per cent, ſo is the ſum inſured to the ſum recovered, ſometimes called ſhort recovery.

2. The difference between the ſum inſured and the recovered ſum, is the diſcount.

### EXAMPLE.

At 2½ per cent. diſcount, what is the recovery upon 1597l. 13s. 4d.? *Anſ.* 1557l. 14s. 6d.

*Prob* 3. The rate per cent, premium, and diſcount, and the real value at riſque given, to find what ſum muſt be inſured to recover the outſet in a ſingle voyage.

*Rule.* Add the rate per cent of the premium and diſcount together; call this the ſum.

2. Find the difference between this ſum and 100l.; then ſay, As this difference : is to 100 :: ſo is the real value riſque : to the ſum neceſſary to be inſured to recover it.

I want to inſure in Amſterdam, 574l. 18s. premium, 7l. 9s. per cent. In Edinburgh, 345l. 10s. premium, 9l. 8s. In London, 459l. 11s. 9d. premium, 7l. 17s. 6d. Diſcount. at Amſterdam is $2\frac{1}{2}$ per cent, and in each of the other two, but 2 per cent; required the covered outſet at each place?

*Anſ.* At Amſterdam, 638l. 8s. $5\frac{79}{1801}$d. Edinburgh, 389l. 19s. $1\frac{73}{4453}$d. London, 509l. 18s. $10\frac{494}{721}$d.

*Prob.* 4. The real value at riſque, the rates per cent, premium and diſcount given, to find the premium out and home, and the ſum to be inſured to recover the inſet.

*Rule.* 1. Find what ſum will be inſured to recover the outſet, by *Prob.* 3.

2. Find the diſcount upon this ſum by *Prob.* 2.

3. The difference of theſe ſums gives the premiums and outſet.

4. The difference between this laſt ſum and the value at riſque, gives the premium out.

Then ſay, As 100 leſs the premium per cent : 100+2 the premium per cent :: ſo is the premium and outſet formed as above : the required premium home.

Laſtly, As 100—premium per cent+2 : 100 :: the given premium, found as above : the ſum to be inſured to recover the inſet.

### EXAMPLE.

What ſum muſt I inſure to recover 5780l. on a voyage out and home, premium 8 per cent, diſcount 2 per cent, in caſe of loſs? *Anſ.* 6993l. 1s. 8¾d nearly.

## CHAP. XXII.

## SIMPLE INTEREST.

SIMPLE INTEREST is an allowance made by the Borrower, of any ſum of money to the Lender, according to a certain rate per cent per annum, which by law, muſt not exceed 6

6 per cent. in Ireland, or 5 per cent in England, that is in Ireland, 6l. are allowed for the uſe of 100l. for 1 year; 12l. for the uſe of 100l. for 2 years, &c.

The principal is the money lent.

The rate is the ſum per cent per annum, agreed on.

The amount is the principal and intereſt added together.

## *Caſe* I.

To find the intereſt of any ſum of money, for years, and aliquot parts of a year, at any given rate per cent per annum.

*Rule.* 1. Multiply the rate by the years, and for the parts of a year take the like part of the rate, which add together.

2. With the intereſt ſo found, work as for commiſſion, by the method in commiſſion, page 195. Or thus: divide the principal by 20, the quotient is one year's intereſt, at 5 per cent, which multiply by the number of years for the anſwer; and for any other rate take proportional parts.

### EXAMPLES.

1. What is the intereſt of 555l. 5s. 5d. for 2$\frac{1}{2}$ years, at 4$\frac{1}{2}$ per cent per annum? *Anſ.* 62l. 9s. 4$\frac{1}{2}$d.

| *years.* | | | £. | s. | d. | |
|---|---|---|---|---|---|---|
| 2 | 9 | Multiply | 555 | 5 | 5 | at 11$\frac{1}{4}$ |
| $\frac{1}{2}$ | 2$\frac{1}{4}$ | | | | | |
| | 11$\frac{1}{4}$ | | 6107 | 19 | 7 | at 11 |
| | | | 138 | 16 | 4$\frac{1}{4}$ | at $\frac{1}{4}$ |
| | | | 100)6246 | 15 | 11$\frac{1}{4}$ | |
| | | | £. 62 | 9 | 4$\frac{1}{2}$ | the *Anſ.* required. |

*or thus:*

| | | £. | s. | d. |
|---|---|---|---|---|
| | | 555 | 5 | 5 |
| 10 is | $\frac{1}{10}$ | 55 | 10 | 6$\frac{1}{2}$ |
| 1$\frac{1}{4}$ is | $\frac{1}{8}$ | 6 | 18 | 9$\frac{3}{4}$ |
| | | £. 62 | 9 | 4$\frac{1}{2}$ |

the ſame as before.

*or thus:*

£. s. d. £. s. d.

555 5 5 ÷ 20 = 27 15 3$\frac{1}{4}$ × 2$\frac{1}{2}$ = 69l. 8s. 2$\frac{1}{8}$ — $\frac{1}{10}$ thereof = 62l. 9s. 4$\frac{1}{2}$d. As before.

*Or thus, by the common way:*

£. £. s. £. s. d.
If 100 : 4 10 :: 555 5 5
20 20 20

2000 90 11105
12 12

24000 133265
90

24000)11993850(499 s.
96000

239385
216000

233850
216000

17850
12

24000)214200(8
192000

22200 = $\frac{111}{120}$

2,0)49,9 $8\frac{111}{120}$

24 19 $8\frac{111}{120}$
$2\frac{1}{2}$

49 19 $5\frac{17}{20}$
12 9 $10\frac{37}{41}$

£. 62 9 $4\frac{1}{2}$
the ſame as before.

*In the firſt* 13; *in the latter,* 117.

2d. What is the intereſt and amount of 277l. 12s. 8½d. for 4½ years, at 5 per cent.

*Anſ.* Intereſt, 62l. 9s. $4\frac{5}{16}$d.; amount, 340l. 2s. $0\frac{13}{16}$d.

3. What is the intereſt and amount of 642l. 10s. for 3⅓ years, at 5¾ per ct.?

*Ans.* Intereſt 123l. 2s. 11d. amount 765l. 12s. 11d.

4. What is the intereſt and amt. of 379l. 2s. 6d. for 4⅓ yrs. at $5\frac{5}{9}$ per cent? *Ans.* Inter. 91l. 5s. 5d. amt. 470l. 7s. 11d.

5. What is the intereſt and amt. of 258l. 11s. 3d. for 7½ years, at $5\frac{5}{9}$ per cent?

*Anſ.* Intereſt 107l. 14s. 8¼d. amount 366l. 5s. 11¼d.

6. What is the intereſt and amt. of 212l. 8s. 1½d. for 6⅔ years, at 5¾ per ct.?

*Anſ.* Intereſt 81l. 8s. 5¾d. amt. 293l. 16s. 6⅞d.

### *Caſe.* II.

To calculate intereſt for years and months, at 5 per cent per annum.

* *Rule.* Suppoſe the years ſhillings, and the months pence; then take ſuch part or parts of the principal as thoſe ſhillings and pence are of a £.

1. Or multiply the months of the time given by the rate of intereſt, divide this laſt product by 1200, this fraction, reduced to its loweſt terms, will give the intereſt of 1l. for the given term.

2. Multiply the given principal by this laſt fraction, and you have the anſwer.

EXAMPLE.

1. What is the intereſt of 161l. 15s. 4d. at 5 per cent per annum for 5 years?

| | £. | s. | d. |
|---|---|---|---|
| | 161 | 15 | 4 |
| 5 years taken as ſhill.=5s. is $\frac{1}{4}$ | £.40 | 8 | 10 *Anſ.* |

2. What is the intereſt of 161l. 15s. 4d. for 6 years, 3 months, at 5 per cent per an.?

| yrs. | | £. | s. | d. | |
|---|---|---|---|---|---|
| | | 161 | 15 | 4 | |
| 5 | is $\frac{1}{4}$ | 40 | 8 | 10 | for 5 yrs. |
| 1 | is $\frac{1}{5}$ | 8 | 1 | $9\frac{1}{5}$ | do. 1. |
| 3m. | is $\frac{1}{4}$ | 2 | 0 | $5\frac{3}{4}$ | 3 months |
| | | £.50 | 11 | $0\frac{17}{20}$ | the *Anſ.* |

*Or thus, by the ſecond rule:*

yrs. m. m.
6. : 3=75
5
375 ÷ by 1200=$\frac{5}{16}$

Or

---

* The firſt rule demonſtrated in the firſt example, thus: find the intereſt of 1l. for 1 year, thus:

| *prin.* | | *time* | | *gain.* | |
|---|---|---|---|---|---|
| 100 | : | 1 | :: | 5 | |
| 1 | : | 5 | : | * *Blank.* | |

Then multiply 1ſt and 2d for a diviſor, and the other 3 for a dividend, per rule page 138, that $100\times1=100$ and $1\times5\times5=25$; therefore $\frac{25}{100}=\frac{1}{4}$l. or thus: $5\times5\div100=\frac{1}{4}$l. hence the rule is manifeſt.

*Note.* The rule for 5 per cent may be applied to any rate. 1. Find the intereſt at 5 per cent. 2. For the difference between 5 and the propoſed rate, take ſuch part of the intereſt at 5, as the difference is of 5. 3. If the propoſed rate be more than 5, add this part; if leſs than 5, ſubtract the part found, from the intereſt, at 5 per cent.

£. s. d.
161 15 4
5

4)808 16 8

202 4 2

£.50 11 0½
the ſame as before

159
20

288)3180(11
288

300
288

12
12

144(0
4

576(½
576

*Or thus, by the common way;*

| £. | yr. m. | £. s. d. |
|---|---|---|
| 100 | 6 3 | 161 15 4 |
| 20 | 12 | 20 |
| 2000 | 75 | 3235 |
| 12 | | 12 |

If 24000 > 5 < 38820
12 75

288000 194120
271768

2911800
5

288,000) 14559,000 (50
1440

159

Anſwer, £.50 11*s.* 0½*d.* the ſame as before. Remark the difference: *The one having* 132 *fig. the other only* 20.

Find the intereſt and amount of the following principals:

3. 242l. 10s. for 5 years, 7 months, at 5 per cent.
*Ans.* Intereſt 67l. 13s. 11½d. amt. 310l. 3s. 11½d.
4. 242l. 10s. for 3 years, 1 month, at 5 per cent
*Anſ.* Intereſt 37l. 7s. 8½d.; amount 279l. 17s. 8½d.
5. 242l. 10s. for 6 years, 5 months, at 5 per cent.
*Anſ.* Intereſt 77l. 16s. 0½d.; amount 320l. 0s. 0½d.
6. 175l. 10s 6d. for 1 year, 7 months, at 6 per cent
*Anſ.* Intereſt 16l. 13s. 5$\frac{97}{100}$d.; amount 192l. 3s. 11$\frac{97}{100}$d.
7. 340l. for 1 year 6 months, at 8 per cent.
*Anſ.* Intereſt 40l. 16s.; amount 380l 16s.
8. 215l. 4s. 2d. for 7 years, 3 months, at 6 per cent.
*Anſ.* Intereſt 93l. 12s. 3¾d.; amount 308l. 16s. 5¾d.

9. 215l 4s. 2d. for 4 years, 7 months, at 6 per cent.
*Anſ.* Intereſt 59l. 3s. 7¾d.; amt. 274l. 7s. 9¾d.

10. 125l. 1s. 9d. for 4 years, 2 months, at 6 per cent?
*Anſ.* Intereſt 31l. 5s. 5¼d.; amt. 156l. 7s. 2¼d.

### *Intereſt for months, at 6 per cent per annum.*

* *Rule.* 1. Multiply the principal by the months.

2. Divide the product by 200, the quotient will be the anſwer.

*Note.* If the given number of months be even, you may multiply the principal by half the months, and the 100dth of the product is the anſwer, (which may be done by the ſhort method in Commiſſion, page 196); or thus: omit the tens and units of the pounds, half the reſt of the pounds gives the pounds of the intereſt; the remainder of the hundreds together with the tens, gives the ſhillings; to the remainder of the dividend add $\frac{1}{5}$ of the ſame, (which may be done in the mind) the pounds of the total give the pence, and 5 ſhillings give a farthing.

### EXAMPLES.

1. What is the intereſt of 117l. 5s. 2½d. for 5 years, at 6 per cent per annum?

| £. | s. | d. | | m. |
|---|---|---|---|---|
| 117 | 5 | 2½ | for 5 yrs. or | 60 |
| | | 5 | | |
| 586 | 6 | 0½ | | |
| | | 12 | | |
| 2,00)7035 | 12 | 6 | | |
| £. 35 | 3 | 6¾ *Anſ.* | | |

*or thus, by the 2d rule:*

$\frac{5\times6}{100}=\frac{3}{10}$ the intereſt of a £. for one year.

| £. | s. | d. |
|---|---|---|
| 117 | 5 | 2½ |
| | | 3 |
| 10)351 | 15 | 7½ |
| £. 35 | 3 | 6¾ ſame as before. |

*or*

---

* The rule at 6 per cent for months demonſtrated thus: Let $n$ = any number of months

| | *prin.* | *time* | *gain* |
|---|---|---|---|
| | 100 : | 12 : | 6 |
| | 1 : | 1n : | 0 *Blank.* |

Then $\frac{6n}{1200}$ or $\frac{n}{200}$ the intereſt of 1l. for $n$ time: hence the rule is manifeſt

*or thus, by the common way:*

| | |
|---|---|
| 100 | 117 5 2½ |
| 20 | 20 |
| 2000 | 2345 |
| 12 | 12 |
| 24000 | 28142 |
| 2 | 2 |
| 48000 | 56285 |

then, If 48000 *halſpence* : 1 £. 6 : 56285 *halſpence*

48000   56285 × 5 = 281425

281425
6

4800,0)168855,0(35
14400
24855
24000
855
20
)17100(3
14400
2700
12
)32400(6
28800
3600
4
)14400(¾
14400
.. .. ..

The *Anſ.* 35l. 3s. 6¾d. as before: here indeed is a great difference; *the one having* 168 *figures; the other only* 30.

| | *Anſwers.* £. | *s.* | *d.* |
|---|---|---|---|
| 2. Find the intereſt of 161l. 15s. 4d. for 3 months, at 6 per cent per annum. | 2 | 8 | $6\frac{9}{25}$ |
| 3. How much will the intereſt be of 1161l. 10s. for 6 months, at 6 per cent? | 34 | 16 | $10\frac{4}{5}$ |
| 4. How much is the intereſt of 1090l. for 8½ months, at 6 per cent? | 46 | 6 | 6 |
| 5. Find the intereſt of 215l. 4s. 2d. for 3 years and 5 months, at 6 per cent. | 44 | 2 | $4\frac{3}{4}$ |

6. What

| | *Anſwers.* £. | *s.* | *d.* |
|---|---|---|---|
| 6. What is the intereſt of 876l. 5s. 4d. for 10 months, at 6 per cent? | 43 | 16 | 3$\frac{1}{5}$ |
| 7. What is the intereſt of 571l. 15s. for 8 months, at 6 per cent per annum? | 22 | 17 | 4$\frac{4}{5}$ |
| 8. Tell me the intereſt of 344l. 17s. 6d. for 12 months, at 6 per cent per annum? | 20 | 13 | 10$\frac{1}{5}$ |
| 9. Tell me the intereſt of 1090l. for 17 months, at 6 per cent. | 92 | 13 | 0 |
| 10. What is the intereſt of 197l. 11s. for 3 months, at 6 per cent? | 2 | 19 | 3$\frac{9}{50}$ |

*Note.* This rule of 6 per cent may be applied univerſally to other rates.

*Rule.* Find the intereſt at 6 per cent. Secondly for the difference between 6 and the rate propoſed, take ſuch parts, of the intereſt at 6 as the ſaid difference is of 6. Thirdly, if the rate propoſed be more than 6, add, if leſs, ſubtract the part from the intereſt at 6.

11. Caſt up the intereſt of 1000l. for 15 months, at 8 per cent. *Anſ.* 100l.

12. At 8 per cent. what will 468l. 16s. 8d. come to, for 1 month? *Anſ.* 3l. 2s. 6$\frac{2}{15}$d.

13. At 7 per cent. tell me what will 3800l. 14s. 4d. come to for 1 year? *Anſ.* 266l. 1s. 0$\frac{1}{25}$d

14. What will 3020l come to, for 5$\frac{1}{2}$ months, at 8 per cent? *Anſ.* 110l. 14s. 8d.

15. Caſt up the intereſt of 886l. 16s. for 1 year 11$\frac{1}{2}$months at 8 per cent. *Anſ.* 138l. 18s. 7$\frac{17}{25}$d.

### *Intereſt for Days, at 5 per cent.*

*Rule.* Multiply the principal by the number of days, the product divided by 7300,[*] the quotient is the intereſt of the principal for that time at 5 per cent per annum.

EXAMPLE.

---

[*] 7300 becomes a conſtant diviſor becauſe it is the intereſt of 100£. for 73 days, or of 73£. for 100 days at 5 per cent.

Some Bankers proceed by the following rule, Multiply the ſum by the days and divide the product by 30, the quotient gives pence, which reduce to ſhillings and for every 6*s.* abate 1*d*

*N. B.* As the intereſt of a pound for 12 months is one ſhilling, for a twelfth of a year, (or one month) it is one penny. Therefore if a month's intereſt on any ſum be required, we may reckon a penny in every pound: thus a month's intereſt on 3£ is 3 *d.* or 2*s.* 6*d.* 2 months 5*s.* 3 months 7*s.* 6*d.* &c. This method would be exact if each month were alike, however when February is not included in the time the error in moſt caſes will not be regarded in buſineſs.

### EXAMPLE.

Given at intereſt 410l. 12s. 6d. for 147 days at 5 per cent per annum; required the intereſt?

*Solution*, 410l. 12s 6d. × 147 ÷ 7300 = 8l. 5s. $4\frac{1}{4}$d. *Ans.*

But in this, or like caſes, as it may be troubleſome to multiply the odd ſhillings and pence by the days, it will be beſt to proceed thus:

Multiply the pounds of the principal by the days, and add in as you proceed, ſuch parts of the days as thoſe ſhillings and pence are of a pound.

### EXAMPLE.

410
147

2872
1649
410

73|00) 603|62 ( 8l.
584

1962
20

392|40 ( 5s.
365

2740
12

328|80 ( 4
292

$\frac{36}{73}$ ( $\frac{1}{2}$

147

10s. is $\frac{1}{2}$ $73\frac{1}{2}$
2s. 6d. is $\frac{1}{4}$ $18\frac{3}{8}$

$91\frac{7}{8}$ or 92 nearly.

*or thus,*
*by the common way:*

| £. | s. | d. |
|---|---|---|
| 410 | 12 | 6 |

20

8212
12

98550

If 24000
365 > 5 < 98550
147

120000
144000
72000

8760000

689850
394200
98550

14486850
5

Carried over, 72434250

100
20

2000
12

24000

Brought over, £. *s.* *d.*

876000)72434250(8 5 4½

70080000

---

2354250

20

---

47085000(5s.

43800000

---

3285000

12

---

39420000(4d.

35040000

---

$4380000)\frac{4380000}{8760000}\left(\frac{1}{2}\right.$

*Anſ* 8l. 5s. 4½d. the ſame as before, the difference is upwards of 120 figures.

Or thus: Multiply the principal by the number of days; divide the product by 100; to the quotient add $\frac{1}{12}$ of itſelf, the ſum is the intereſt required in three pences. This rule is conciſe and uſeful, but when the number of days exceed 100, a ſmall correction may be neceſſary.

| | *Anſwers.* £. | *s.* | *d.* |
|---|---|---|---|
| 2. What is the intereſt of 50l. for 41 days at 5 per cent per annum? | | 5 | 7$\frac{29}{73}$ |
| 3. What is the intereſt of 60l 17s. 6d. for 3 days, at 5 per cent? | | | 6 |
| 4. What is the intereſt of 72l. 18s. 6d. at 5 per cent, for 41 days? | | 8 | 2+ |
| 5. Required the intereſt of 87l. 9s. 1d. at 5 per cent, for 50 days? | | 11 | 11$\frac{111}{146}$ |
| 6. What is the intereſt of 1254l. 5s. 6d. for 95 days, at 5 per cent? | 16 | 6 | 5+ |
| 7. What is the intereſt of 45l. 16s. 8d. for 21 days, at 5 per cent? | | 2 | 7½ |

*8. What is the intereſt of 6l. for 91 days, at 5 per cent per annum? £. s. d. 1 $5\frac{1}{4}$+

9. How much is the intereſt of 150l. 15s. 6d. for 53 days, at 5 per cent per annum? 1 1 $10\frac{1291}{1460}$ nearly.

## *Intereſt for Days at Six per Cent per Annum.*

†*Rule.* Multiply the principal by the days of its continuance at intereſt, divide the product by 300, and from the quotient ſubtract its $\frac{1}{73}$, the remainder will be the anſwer in ſhillings.

EXAMPLES.

1. What is the intereſt of 850l. for 210 days, at 6 per cent per annum?

---

* The two foregoing rules for intereſt for days may be thus demonſtrated,

£ *principal. time. gain.*
100 : 365 : 5
6 : 91 : 0 *blank.*

that is, $\frac{6\times5\times91}{365\times100}$ the required intereſt, but $\frac{5}{365\times100}=\frac{1}{7300}=\frac{1}{73}\times\frac{1}{100}$ and $\frac{1}{73}$£.$=\frac{246}{73}d.=3\frac{1}{4}\times5{,}46d.$

Hence the following operations, $5{,}46\times3\frac{1}{4}=17{,}715$ the anſwer 1s. $5\frac{3}{4}$d. but $\frac{1}{4}$ being the $\frac{1}{12}$ of 3*d*, if the $\frac{1}{12}$ of 5,46 were added to it, the ſum would be $5{,}46+{,}455=5{,}915$ which being multiplied by 3, the product will be $5{,}915\times3=17{,}745$ the anſwer as before; hence the rule is manifeſt.

*Note.* 1. As $\frac{21}{73}$ is not exactly $\frac{1}{4}$ this rule will not give the anſwer ſtrictly true, yet when the days are few, and the principal ſmall, it will give the anſwer ſufficiently near. 2. As in the 6th example, where the ſum is large and the time long, proceed by the firſt rule. 3. The firſt rule is univerſal for any other rate of intereſt propoſed either above 5 or under 5, by dividing the given rate into aliquot parts of 5, if more, add, if leſs, ſubtract. 4. When the ſhillings and pence are leſs than 10*s*. they may be omitted in the proceſs, as not being productive of any intereſt in ſhort dates.

£ *princ. time. gain.*
† 100 : 365 :: 6
850 : 214 :: 0 *blank.*

then $\frac{6\times850\times214}{100\times365}$ (ſee page 138)$=856\times214\times\frac{6}{100\times365}=$the intereſt required; and $\frac{6}{36500}$£.$=\frac{120}{36500}$s.$=\frac{24}{7300}=\frac{24}{73}\times\frac{1}{100}$ but to divide any number by 73 is the ſame as to divide by 72 and diminiſh the quotient by its one 73d part nearly; therefore the expreſſion $\frac{24}{73}\times\frac{1}{100}$ when 72 is made the diviſor will be $\frac{24}{72}\times\frac{1}{100}=\frac{1}{300}$; then the expreſſion for the intereſt above will

$$\begin{array}{r} 850 \\ 210 \\ \hline 3|00)\;1785|00 \\ \hline 595 \\ \frac{1}{73}\qquad 8\frac{11}{73} \\ \hline \end{array}$$

The common method would take upwards of 100 figures more, which has been ſhewn in page 205.

*Anſ.* $586\frac{62}{73}$ = 29£. 6*s*. $10\frac{14}{73}$*d*.

| | Answers. | | |
|---|---|---|---|
| | £. | s. | d. |
| 2. What is the intereſt of 205l. 6s. 3d. for 237 days, at 6 per cent? | 7 | 19 | 11¾ nearly. |
| 3. How much is the intereſt of 50l. 12s. 6d. for 40 days, at 6 per cent? | | 6 | $7\frac{65}{73}$ |
| 4. How much is the intereſt of 345l. 6s. 8d. for 73 days, at 6 per cent? | 4 | 2 | 10½ nearly |
| 5. How much is the intereſt of 234l. 5s. 6d. for 123 days, at 6 per cent? | 4 | 14 | 8¾+ |
| *6. Caſt up 240l. for 1 year and 135 days, at 7 per cent per annum. | 23 | 0 | $3\frac{31}{73}$ |
| 7. Tell me the intereſt of 564l. for 700 days, at 8 per cent. | 86 | 10 | $7\frac{41}{73}$ |
| 8. Tell me the intereſt of 100l. from the 10th of May to the 30th of Auguſt, at 8 per cent. | 2 | 9 | $1\frac{14}{73}$ |
| 9. How much is the intereſt of 175l. for 438 days, at 8 per cent? | 16 | 16 | 0 |
| 10. Caſt up the intereſt of 100l. at 8 per cent, from the 6th of June to the 28th of December. | 4 | 9 | $10\frac{26}{73}$ |

A TABLE

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will become $850\times214\times\frac{1}{300}$ which is the required intereſt in ſhillings, greater than the true intereſt by $\frac{1}{73}$ (nearly) part of itſelf. Hence the rule is manifeſt. Q.E.D.

*Note.* For every 25 cut off by dividing by 300, count a penny; for every 6, a farthing.

* The ſame rule for intereſt for days at 6 per cent is univerſal for other rates; take ſuch parts of the intereſt at 6, as the ſaid difference is of 6, then if the rate be more add, if leſs ſubtract.

## A TABLE

Shewing the number of days from one day in a Month to the ſame day in any following Month, in a common year.

| | JANUARY, | FEBRUARY, | MARCH, | APRIL, | MAY, | JUNE, | JULY, | AUGUST, | SEPTEMBER, | OCTOBER, | NOVEMBER, | DECEMBER, |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| JANUARY, | 365 | 334 | 306 | 275 | 245 | 214 | 184 | 153 | 122 | 92 | 61 | 31 |
| FEBRUARY, | 31 | 365 | 337 | 301 | 276 | 245 | 215 | 184 | 153 | 123 | 92 | 62 |
| MARCH, | 59 | 28 | 365 | 334 | 304 | 273 | 243 | 212 | 181 | 151 | 120 | 90 |
| APRIL, | 90 | 59 | 31 | 365 | 335 | 304 | 273 | 243 | 212 | 182 | 151 | 121 |
| MAY, | 120 | 89 | 61 | 30 | 365 | 334 | 304 | 273 | 242 | 212 | 181 | 151 |
| JUNE, | 151 | 120 | 92 | 61 | 31 | 365 | 335 | 304 | 273 | 243 | 212 | 182 |
| JULY, | 181 | 150 | 122 | 91 | 61 | 30 | 365 | 334 | 304 | 273 | 242 | 212 |
| AUGUST, | 212 | 181 | 153 | 122 | 92 | 61 | 31 | 365 | 334 | 304 | 273 | 243 |
| SEPTEMBER, | 243 | 212 | 184 | 153 | 123 | 91 | 62 | 32 | 365 | 335 | 304 | 274 |
| OCTOBER, | 273 | 242 | 214 | 183 | 153 | 122 | 92 | 61 | 30 | 365 | 334 | 304 |
| NOVEMBER, | 304 | 273 | 245 | 214 | 184 | 153 | 123 | 92 | 61 | 31 | 365 | 335 |
| DECEMBER, | 334 | 303 | 275 | 244 | 214 | 183 | 153 | 122 | 91 | 61 | 30 | 365 |

*Example.* To find the number of days from the 1ſt, 10th, or 20th of June to the 1ſt, 10th, or 20th of March; *Solution,* Find June at the head, and down that column in the line even with March is 273: or if it was from the 15th of June to the 26th of March, then the number of days would be 273+11=284; or if it was from the 15th of June to the 4th of March, 273—11=262 days.

To compute intereſt on a debt when partial payments are made, multiply the ſum, and the ſeveral balances in their order, by the number of days from each payment, and divide the ſum of the products by 7300, the quotient is the legal intereſt at 5 per cent.

If a bill of 360l. be due April 10th, of which 180l. was paid May 10th, 80l. July the 4th, and the balance on the 13th of Auguſt, required how much is due at 6 per cent?

| | £. | days. |
|---|---|---|
| April 10th due | 360×30=10800 | |
| May 10th paid | 180 | |
| Balance, | 180×55=9900 | |
| July 4th paid | 80 | |
| Balance, | 100×40=4000 | |
| Auguſt 13th paid | 100 | |

73|00) 247|00

3 7 8 at 5 per cent.
$\frac{1}{5}$ 0 13 6 Fractions omitted.

£. 4 1 2 at 6 per cent.

In calculating Intereſt on current and caſh accounts, add and ſubtract their ſums paid and received in the order of their dates, and if the balance be ſometimes due to one party and ſometimes to the other, extend the products in different columns.

Required the intereſt at 6 per cent due on the following account, to be ſettled 31 December.

*Dr. Mr.* W. Inglis, *his account current with* C. Todd, *Cr.*

| | | £. | s. | | | £. | s. |
|---|---|---|---|---|---|---|---|
| May 15, | To caſh | 182 | 0 | July 5, | By caſh | 198 | 10 |
| June 29, | To do. | 136 | 10 | Oct. 10, | By do. | 145 | 15 |
| Sept. 16, | To do. | 275 | 15 | Nov. 18, | By do. | 126 | 0 |

£. s. d.

Debtor ſide,
Creditor ſide,

laſt balance, £.

May

| | | £. | s. | days. | |
|---|---|---|---|---|---|
| May 15, | To | 182 | 0 | ×45 | =8190 |
| June 29, | To | 136 | 10 | | |
| | | 318 | 10 | × 6 | =1911 |
| July 5, | By | 198 | 10 | | |
| | | 120 | 0 | ×73 | =8760 |
| Sept. 16, | To | 275 | 15 | | |
| | | 395 | 15 | ×24 | =9498 |
| Octr. 10, | By | 145 | 15 | | |
| | | 250 | 0 | ×39 | =9750 |
| Novr. 18, | By | 126 | 0 | | |
| laſt balance, | | 124 | 0 | ×43 | =5332 |
| | | | | 230 | 43441 |

| | days. |
|---|---|
| May | 16 |
| June | 30 |
| July | 31 |
| Auguſt | 31 |
| Sept. | 30 |
| Octr. | 31 |
| Novr. | 30 |
| Decr. | 31 |
| | 230 |

3|00) 434|41

144 9½

$\frac{1}{73}$ = 1 11¾

2|0) 14|2 9¾

£.7 2 9¾ *Anſ.*

Before we find the days, we try if the laſt balance (124l.) be the ſame as the balance of the account, which proves the additions and ſubtractions; and before multiplying by the days, we try if (230) the ſum of the column of days be equal to the number of days that intervene from (15 May) the date the account commences to bear intereſt to (31 December) the day of ſettlement.

Required the intereſt and balance on the following caſh account, allowing 5 per cent when the ballance is due to the Bank, and 4 per cent when due to A. B.

| *Dr.* *Mr.* A. B. | *his Caſh acct.* | *with the* | Bank of Ireland, | | *Cr.* |
|---|---|---|---|---|---|
| Jan. 25, | To caſh dr. | £.230 | Feb. 14, | By caſh rem. | £.185 |
| Mar. 16, | To do. | 175 | Apr. 10, | By do. | 140 |
| June 1, | To do. | 25 | May 20, | By do. | 145 |
| Aug. 2, | To do. | 340 | July 4, | By do. | 300 |
| Sept. 12, | To do. | 176 | Oct. 22, | By do. | 120 |
| Nov. 7, | To do. | 100 | | | |

| Date. | | £. | *days.* | Due the Bank. £. | Due A. B. £. |
|---|---|---|---|---|---|
| Jan. 25 | Drawn, | 230 | 20 | 4600 | |
| Feb. 14 | Remitted, | 185 | | | |
| | By | 45 | 30 | 1350 | |
| March16 | Drawn, | 175 | | | |
| | By | 220 | 25 | 5500 | |
| April, 10 | Remitted, | 140 | | | |
| | By | 80 | 40 | 3200 | |
| May, 20 | Remitted, | 145 | | | |
| | To | 65 | 12 | | 780 |
| June, 1 | Drawn, | 25 | | | |
| | To | 40 | 33 | | 1320 |
| July, 4 | Remitted, | 300 | | | |
| | To | 340 | 29 | | 9860 |
| Auguſt,2 | Drawn, | 340 | | | |
| | | | 41 | | |
| Sept. 12 | Drawn, | 176 | | | |
| | By | 176 | 40 | 7040 | |
| Oct. 22 | Remitted, | 120 | | | |
| | By | 56 | 16 | 896 | |
| Nov. 7 | Drawn, | 100 | | | |
| | By | 156 | 286 | 22586 | 11960 |

22586
9568

3|00) 130|18

£. 1 15 8 Intereſt

From 11960= 5 per cent.
Subtract 2392= 1 per cent.

9568= 4 per cent.

| | £. | s. | d. |
|---|---|---|---|
| Balance due the Bank, | 156 | 0 | 0 |
| Intereſt due do. | 1 | 15 | 8 |
| Due do. | £.157 | 15 | 8 |

11. How much intereſt is due on the following accounts current?

| Richard Cluff of Dublin, Dr. | | | Contra. | | Cr. |
|---|---|---|---|---|---|
| 1804. | | £. | 1804. | | £. |
| Jan. 1. | To Caſh, | 219 | Feb. 10. | By Caſh, | 75 |
| 25. | To ditto, | 146 | March 27. | By ditto, | 120 |
| Feb. 19. | To ditto | 25 | April 17. | By ditto, | 240 |
| Mar. 11. | To ditto, | 120 | May 14. | By do. for bal. | 75 |
| | Sum, | 510 | | Sum, | 510 |

## *Manner of Operation.*

| 1804. | | | £. | *intereſt due on* | | *days* | | |
|---|---|---|---|---|---|---|---|---|
| January | 1. | Paid | 219 | | | | | |
| | 25. | Paid | 146 | £.219 | × | 24 | = | 5256 |
| February | 10. | Received | 75 | 365 | × | 16 | = | 5840 |
| | 19. | Paid | 25 | 290 | × | 9 | = | 2610 |
| March | 11. | Paid | 120 | 315 | × | 20 | = | 6300 |
| | 27. | Received | 120 | 435 | × | 16 | = | 6960 |
| April | 17. | Received | 240 | 315 | × | 21 | = | 6615 |
| | 14. | Received | 75 | 75 | × | 28 | = | 2100 |

30|0)356|81

*s.*118 11¼*d.*

For the reaſon given in page 209, 1 7¼+

11|7 4

Intereſt due, £.5 17 4
nearly.

## *Obſervations on Intereſt Accounts Current.*

1. The legal and cuſtomary manner of ſettling an Intereſt accompt is thus: When money is paid in part, find what intereſt is then due, and if feaſible, take it out of ſuch payment, and apply only the remainder to leſſen the principal; or which is all one, and moſt commodious, add the intereſt to its principal, and from the total ſubtract the ſum paid.

2. Now if ſuch remainder be leſs than the laſt principal, then it is the preſent principal; otherwiſe not.

3. The intercalary day in February, every Leap-year, muſt be accounted for.

4. September, 1752, wanted 11 days by act of Parliament, viz. between the 2d and 14th days.

12. Q. Borrowed of R. the principal ſum of 4685l. January 11, 1799; intereſt 5½ per cent per annum: in diſcharge of his debt, Q. makes the following payments, viz.—

| | | | | |
|---|---|---|---|---|
| 1799, November 19, | Paid R. | £.905 | 5 | 2 |
| 1801, November 19, | Ditto, | 1000 | 12 | 0½ |
| 1803, January 11, | Ditto, | 2984 | 13 | 5½ |

How ſtands the account, January 11, 1803, betwixt Q. and R.?

*Solution.*

Q.'s account of intereſt with R.

| | | £. | | |
|---|---|---|---|---|
| 1799. | | | | |
| January 11. | Borrowed the principal A = | 4685 | | |
| Novem. 19. | The intereſt of A. to this date, 312 days, | 220 | 5 | 2 |
| | | 4905 | 5 | 2 |
| | Firſt payment, | 905 | 5 | 2 |
| 1801. | From principal, B. = | 4000 | 0 | 0 |
| Nov. 19. | The intereſt of B. 731 days, | 440 | 12 | 0½ |
| | | 4440 | 12 | 0½ |
| | Second payment, | 1000 | 12 | 0½ |
| 1803. | Principal, C. = | 3449 | 0 | 0 |
| January 11. | The intereſt of C. 418 days, | 216 | 13 | 5½ |
| | | 3656 | 13 | 5½ |
| | Third payment, | 2984 | 13 | 5½ |
| | Balance due to R. | £.672 | 0 | 0 |

13. A. gives to B. at intereſt, anno, 1717 primo Novembris, 6000l. at 4½ per cent; B., to repay him with intereſt, at the expiration of 2 years, having liberty to pay before that time, as much of the principal as he pleaſes: now B. pays,

The

| | | | £. |
|---|---|---|---|
| The 16th of December, 1717, | | — | 900 |
| The 11th of March, | — | — | 1260 |
| The 30th ditto, | — | — | 600 |
| The 17th of Auguſt | — | — | 800 |
| The 12th of February, 1719, | | — | 1048 |

I would know how much principal and intereſt B., primo Novem. anno 1719, is to pay? *Ans.* 1642l. 9s. $2\frac{1546}{1825}$d.

*Manner of Calculation.*

| | £. | | *days* | *product* |
|---|---|---|---|---|
| November 1ſt, the principal | 6000 | was in uſe | 45 | =270000 |
| 16th Decemr. then received | 900 | | | |
| | 5100 | was in uſe | 85 | =433500 |
| 11th of March then received | 1260 | | | |
| | 3840 | was in uſe | 19 | = 72960 |
| 30th of ditto then received | 600 | | | |
| | 3240 | was in uſe | 140 | =453600 |
| 17th of Auguſt then received | 800 | | | |
| | 2440 | was in uſe | 179 | =436760 |
| 12th of February, 1719, | 1048 | | | |
| | 1392 | was in uſe | 262 | =364704 |

3)20315,24

6771 8

$\frac{7}{73}$ = 92 9

6678 11

$1\frac{1}{2}$ is $\frac{1}{4}$ of 6=1669 $8\frac{1}{4}$

2|0)500|9 $2\frac{1}{4}$

Inter. 250 19 $2\frac{1}{2}$

| | £. | s. | d. |
|---|---|---|---|
| Inter. | 250 | 19 | $2\frac{1}{4}$ |
| Laſt Prin. | 1392 | 0 | 0 |
| | £. 1642 | 19 | $2\frac{1}{2}$ |

the *Anſ.*

*or thus, by the common way:*

| | days |
|---|---|
| November | 29 |
| December | 16 |
| 1ſt number of days | 45 |
| December | 15 |
| January | 31 |
| February | 28 |
| March | 11 |
| 2d number | 85 |
| March | 19 |
| 3d number | |
| March | 1 |
| April | 30 |
| May | 31 |
| June | 30 |
| July | 31 |
| Auguſt | 17 |
| 4th number | 140 |
| Auguſt | 14 |
| September | 30 |
| October | 31 |
| November | 30 |
| December | 31 |

| | days |
|---|---|
| January | 31 |
| February | 12 |
| 5th number | 179 |
| 1ſt number of days | 45 |
| 2d ditto, | 85 |
| 3d ditto, | 19 |
| 4th ditto, | 140 |
| 5th ditto, | 179 |
| | 468 |

From 730
Take 468
6th n. 262

| | | |
|---|---|---|
| Due | 6000 | 1ſt. |
| Paid | 900 | |
| Due | 5100 | 2d. |
| Paid | 1260 | |
| Due | 3840 | 3d. |
| Paid | 600 | |
| Due | 3240 | 4th. |
| Paid | 800 | |
| Due | 2440 | 5th. |
| Paid | 1048 | |
| Due | 1392 | 6th. |

| £. | | s. | | £. |
|---|---|---|---|---|
| 100 | > | 90 | < | 6000 |
| 365 | | | | 45 |
| 36500 | | | | 30000 |
| | | | | 24000 |
| | | | | 270000 Carried over. |

Brought

270000 Brought forward,
90

365|00)243000|00(
2190
2400
2190
2100
1825
275

2|0)66|5
£. 33 5

365)3300(9
3285
$5)\frac{15}{365}(\frac{3}{73}$

£. s. £.
(ii) 100 > 90 < 5100
365 85
36500
25500
40800
433500
90
2,0
365|00) 390150|00 (106|8
365
2515
2190
3250
2920
330
12
365)3960(10
365
$5)\frac{310}{365}(\frac{62}{73}$

53 8

£. s. £.
(iii) 100 > 90 < 3840
365 19
36500
34560
3840
72960
90
2,0
365|00) 65664|00( 17|9
365
2916
2555
3614
3285
329
12
365)3948(10
365
$\frac{298}{365}$

8 19

(iv)

```
      £.        s.    £.
(iv) 100 >      90 <  3240
     365               140
     -----           ------
     36500           129600
                     3240
                     ------
                     453600
                         90
                     ------        2,0
365|00)408240|00(111|8
       365            ----
       ---            55 18
        432
        365
        ---
         674
         365
         ---
         3090
         2920
         ----
          170
           12
          ---  d.
   365)2040(5
       1825
       ----
     5)215/365(43/73
```

```
      £.        s.    £.
(v)  100 >      90 <  2440
     365               179
     -----           ------
     36500            21960
                     17080
                      2440
                     ------
                     436760
                         90
                     ------         2,0
365|00)393084|00(107|6
       365             ----
       ---             53 16
       2808
       2555
       ----
        2534
        2190
        ----
         344
          12
         ---  d.
   365)4128(11
       365
       ---
        478
        365
        ---
        113
        ---
        365
```

```
      £.        s.    £.
(vi) 100 >      90 <  1392
     365               262
     -----            -----
     36500             2784
                      8352
                     2784
                     ------
Carried over         364704
```

```
  364704 Brought forward.
      90
  ------ 2,0
3650|0)3282336|0(89|9
       29200      ----
       -----    44 19
        36233
        32850
        -----
         33836
         32850
         -----
           986
            12
         ----- d.
  3650)11832(3
       10950
       -----
```

$$2)\ \frac{882}{3650}\left(\frac{441}{1825}\right.$$

| | £. | s. | d. |
|---|---|---|---|
| The 1ſt *Anſ.* | 33 | 5 | $9\frac{3}{73}$ |
| 2d — | 53 | 1 | $10\frac{62}{73}$ |
| 3d — | 8 | 19 | $10\frac{298}{365}$ |
| 4th — | 55 | 18 | $5\frac{43}{73}$ |
| 5th — | 53 | 16 | $11\frac{113}{365}$ |
| 6th — | 44 | 19 | $3\frac{441}{1825}$ |
| Laſt principal, | 1392 | 0 | 0 |
| The *Anſ.* £. | 1642 | 9 | $2\frac{1546}{1825}$ |

*The difference between the 1ſt and 2d is upwards of 4000, when the fractions are added.*

☞ Intereſt for days, at 6 per cent per annum.

*Rule.* Multiply the principal by $\frac{1}{3}$ part of the number of days, and cut off two figures to the right, which are half farthings, and the reſt of the figures are ſhillings; deduct 3½d. for every pound of this ſum, and the remainder is the intereſt at 6 per cent per annum.

EXAMPLES.

1. 75l. for 45 days, at 6 per cent; then 45 × 15 = 11s. 3d. deduct 1¾d. = 11s. 1¼d. the *Anſ.*

2. 433l. 10s. for 91 days = 6l. 9s. 8¼d. the *Anſ.* required.

**N. B. The above Rule and Examples ☞ ſhould have been placed page 209.**

14. Suppoſe I borrow at intereſt 300l. at 6 per cent, for 18 months, with condition to pay as much of the principal before the time as I pleaſe, now after 3 months' time I pay 60l.; and 4 months after that 100l.; and 5 months after that I pay 75l. I would know how how much principal and intereſt I have yet to pay at the expiration of ſaid 18 months?

*Anſ.* 79l. 15s.

15. A merchant borrowed at intereſt 250l. at 8 per cent, for 2 years, with condition to pay before that time as much of the principal as he pleaſes; now at the expiration of 9 months he pays 80l. and 6 months after that 70l. leaving the reſt the full time of the aforeſaid 2 years, I would know how much principal and intereſt the ſaid merchant is to pay?

*Anſ.* 127l. 16s.

16. Lent at intereſt 600l. the 13th of May, 1717, for 1 year, at the rate of 5 per cent, with condition that the receiver may diſcharge as much of the principal before the time as he pleaſes; now he pays the 9th of July, 200l. and the 17th of September, 150l.; I would know how much principal and intereſt he is to pay at the expiration of the year, counting it 365 days? *Anſ.* 266l. 13s. 5$\frac{7}{73}$d.

17. 1799, January 14, R. S. lent to T. U. at intereſt, the principal 3580l. ſterling, at 5¼ per cent per annum; now T. U. paid on account, the ſums following, viz.

| | | £. | s. | d. |
|---|---|---|---|---|
| 1799. | April 25, | 1512 | 10 | 2 |
| | November 8, | 19 | 11 | 1¾ |
| 1800. | April 27, | 11 | 16 | 6¼ |
| | Oct. 14, | 11 | 6 | 6 |
| 1801. | Oct. 14, | 2000 | 0 | 0 |
| 1802. | May 14, | 4 | 5 | 11 |
| | Dec. 19, | 69 | 6 | 9½ |
| | | £. | | |

What balance was due on this account after laſt payment?

*Anſ.* 300l.

1803. The Rt. Hon. David Latouche, Dr.

| | | | £. | s. | d. |
|---|---|---|---|---|---|
| Jan. | 15. | To caſh paid on your account, | 123 | 0 | 0 |
| Feb. | 10. | To do. paid I. Keen, on your account, | 90 | 0 | 0 |
| | 22. | To do. | 47 | 0 | 0 |
| Mar. | 23. | To do. paid R. Jackſon, on your acct. | 55 | 0 | 0 |

1803.

1803. Contra Cr.

| | | | £. | s. | d. |
|---|---|---|---|---|---|
| Jan. | 31. | By caſh, per your remittance by G. Harding, | 75 | 0 | 0 |
| Mar. | 3. | By do. per your draft on T. Tronſon, | 100 | 0 | 0 |
| April | 4. | By do. per your order on ſtock, | 100 | 0 | 0 |
| May | 4. | By do. per I. Dunn, being the ballance of my advance, | 40 | 0 | 0 |

How much intereſt was due to me on the 4th of May, 1803 ?

*Anſ.* 1l. 13s. 11¼d at 6 per cent per annum.

1803. Paid for account of William Macklin, viz.

| | | | | £. | s. | d. |
|---|---|---|---|---|---|---|
| May | 15. | Paid | —— | 135 | 6 | 8 |
| | 26. | Do. | —— | 115 | 10 | 0 |
| June | 26. | Do. | —— | 85 | 3 | 4 |
| July | 18. | Do. | —— | 74 | 0 | 0 |
| | | | | £. 410 | 0 | 0 |

1803. Received in ſundry payments :

| | | | | £. | s. | d. |
|---|---|---|---|---|---|---|
| May | 15. | Received, | —— | 100 | 0 | 0 |
| June | 25. | Do. | —— | 150 | 0 | 0 |
| July | 18. | Do. | —— | 100 | 0 | 0 |
| Aug. | 13. | Received the balance, | | 60 | 0 | 0 |
| | | | | £. 410 | 0 | 0 |

How much intereſt was due to me on the 13th of Aug. 1803 ?

*Anſ.* 1l. 7s. 6¼d at 6 per cent per annum.

A TABLE

## A TABLE

For expediting the ſolution of all queſtions in Simple Intereſt; having any principal number of days and rate per cent per annum given, in a conciſe and ſyſtematic manner.

| *Number.* | £. | *s.* | *d.* | *q.* | *Num.* | *s.* | *d.* | *q.* |
|---|---|---|---|---|---|---|---|---|
| 1000000 | 2739 | 14 | 6 | 0.99 | 90 | 4 | 11 | 0.71 |
| 900000 | 2465 | 15 | 0 | 3.29 | 80 | 4 | 4 | 2.41 |
| 800000 | 2191 | 15 | 7 | 1.59 | 70 | 3 | 10 | 0.11 |
| 700000 | 1717 | 16 | 1 | 3.89 | 60 | 3 | 3 | 1.81 |
| 600000 | 1643 | 16 | 8 | 2.19 | 50 | 2 | 8 | 3.51 |
| 500000 | 1369 | 17 | 3 | 0.49 | 40 | 2 | 2 | 1.21 |
| 400000 | 1093 | 17 | 9 | 2.79 | 30 | 1 | 7 | 2.90 |
| 300000 | 821 | 18 | 4 | 1.10 | 20 | 1 | 1 | 0.60 |
| 200000 | 547 | 18 | 10 | 3.40 | 10 |  | 6 | 2.30 |
| 100000 | 273 | 19 | 5 | 1.70 | 9 |  | 5 | 3.67 |
| 90000 | 246 | 11 | 6 | 0.33 | 8 |  | 5 | 1.4 |
| 80000 | 219 | 3 | 6 | 2.96 | 7 |  | 4 | 2.41 |
| 70000 | 191 | 15 | 7 | 1.59 | 6 |  | 3 | 3.78 |
| 60000 | 164 | 7 | 8 | 0.22 | 5 |  | 3 | 1.1 |
| 50000 | 136 | 19 | 8 | 2.85 | 4 |  | 2 | 2.55 |
| 40000 | 109 | 11 | 9 | 1.48 | 3 |  | 1 | 3.89 |
| 30000 | 82 | 3 | 10 | 0.11 | 2 |  | 1 | 1.26 |
| 20000 | 54 | 15 | 10 | 2.74 | 1 |  |  | 2.63 |
| 10000 | 27 | 7 | 11 | 1.37 | 0.9 |  |  | 2.37 |
| 9000 | 24 | 13 | 1 | 3.23 | 8 |  |  | 2.10 |
| 8000 | 21 | 18 | 4 | 1.10 | 7 |  |  | 1.84 |
| 7000 | 19 | 3 | 6 | 2.96 | 6 |  |  | 1.58 |
| 6000 | 16 | 8 | 9 | 0.82 | 5 |  |  | 1.32 |
| 5000 | 13 | 13 | 11 | 2.68 | 4 |  |  | 1.05 |
| 4000 | 10 | 10 | 2 | 0.55 | 3 |  |  | ·79 |
| 3000 | 8 | 4 | 4 | 2.41 | 2 |  |  | ·53 |
| 2000 | 5 | 9 | 7 | 0.27 | 1 |  |  | .26 |
| 1000 | 2 | 14 | 9 | 2.14 | 0·9 |  |  | .24 |
| 900 | 2 | 9 | 3 | 3·12 | 8 |  |  | .21 |
| 800 | 2 | 3 | 10 | 0.11 | 7 |  |  | .18 |
| 700 | 1 | 18 | 4 | 1.10 | 6 |  |  | .16 |
| 600 | 1 | 12 | 10 | 2.8 | 5 |  |  | .13 |
| 500 | 1 | 7 | 4 | 3·7 | 4 |  |  | .11 |
| 400 | 1 | 1 | 11 | 0.5 | 3 |  |  | .8 |
| 300 | 0 | 16 | 5 | 1.4 | 2 |  |  | ·5 |
| 200 | 0 | 10 | 11 | 2·3 | 1. |  |  | ·3 |
| 100 | 0 | 5 | 5 | 3·1 |  |  |  |  |

*Rule.* Multiply the principal by the rate, multiply that product by the number of days, and divide this laſt product by 100, then take from the table the ſeveral ſums which ſtand oppoſite the ſeveral parts of the quotient, and add them together for the intereſt required.

### EXAMPLES.

1. What is the intereſt of 225l. 10s. for 23 days, at 4½ per cent per annum? *Anſ.* 12s. 9d. 1q. 85.

*Solution.*

Principal 22.5
Rate 4.5

1014.75
23

100)23339.25

233.3925

| In the table againſt | | £. | s. | d. | q. |
|---|---|---|---|---|---|
| 200 | is - | 0 | 10 | 11 | 2.03 |
| 30 | — - | 0 | 1 | 7 | 2.90 |
| 3 | — - | 0 | 0 | 1 | 3.89 |
| 0.3 | — - | 0 | 0 | 0 | 0.79 |
| 0.09 | — - | 0 | 0 | 0 | 0.24 |
| | *Anſ.* £. | 0 | 12 | 9.1.85 | |

2. What is the intereſt of 15l. 5s. for 117 days, at 4¾ per cent per annum? *Anſ.* 5s. 3d.+

3. What is the intereſt of 112l. 12s. 6d. from the 8th of May to the 3d of November, at 4 per cent per annum?
*Anſ.* 2l. 4s. 2d.+

## A TABLE

For finding the Intereſt of any ſum of money, for any number of months, weeks, or days, at any rate per cent.

| Years. | Months. | | | Weeks. | | | Days. | | |
|---|---|---|---|---|---|---|---|---|---|
| £. | £. | s. | d. | £. | s. | d. | £. | s. | d. |
| 1 | 0 | 1 | 8 | 0 | 0 | 4½ | 0 | 0 | 0½ |
| 2 | 0 | 3 | 4 | 0 | 0 | 9 | 0 | 0 | 1¼ |
| 3 | 0 | 5 | 0 | 0 | 1 | 1¾ | 0 | 0 | 2 |
| 4 | 0 | 6 | 8 | 0 | 1 | 6½ | 0 | 0 | 2½ |
| 5 | 0 | 8 | 4 | 0 | 1 | 11 | 0 | 0 | 3¼ |
| 6 | 0 | 10 | 0 | 0 | 2 | 3¾ | 0 | 0 | 4 |
| 7 | 0 | 11 | 8 | 0 | 2 | 8¼ | 0 | 0 | 4½ |
| 8 | 0 | 13 | 4 | 0 | 3 | 1 | 0 | 0 | 5¼ |
| 9 | 0 | 15 | 0 | 0 | 3 | 5½ | 0 | 0 | 6 |
| 10 | 0 | 16 | 8 | 0 | 3 | 10¼ | 0 | 0 | 6½ |
| 20 | 1 | 13 | 4 | 0 | 7 | 8¼ | 0 | 1 | 1¼ |
| 30 | 2 | 10 | 0 | 0 | 11 | 6½ | 0 | 1 | 7¾ |
| 40 | 3 | 6 | 8 | 0 | 15 | 4½ | 0 | 2 | 2¼ |
| 50 | 4 | 3 | 4 | 0 | 19 | 2¾ | 0 | 2 | 9 |
| 60 | 5 | 0 | 0 | 1 | 3 | 0¼ | 0 | 3 | 3½ |
| 70 | 5 | 16 | 8 | 1 | 6 | 11 | 0 | 3 | 10 |
| 80 | 6 | 13 | 4 | 1 | 10 | 9¼ | 0 | 4 | 4½ |
| 90 | 7 | 10 | 0 | 1 | 14 | 7¼ | 0 | 4 | 11¼ |
| 100 | 8 | 6 | 8 | 1 | 18 | 5½ | 0 | 5 | 5¾ |
| 200 | 16 | 3 | 4 | 3 | 16 | 11 | 0 | 10 | 11½ |
| 300 | 25 | 0 | 0 | 5 | 15 | 4½ | 0 | 16 | 5¼ |
| 400 | 33 | 6 | 8 | 7 | 13 | 10 | 1 | 1 | 11 |
| 500 | 41 | 13 | 4 | 9 | 12 | 3½ | 1 | 7 | 4¾ |
| 600 | 50 | 0 | 0 | 11 | 10 | 9 | 1 | 12 | 10½ |
| 700 | 58 | 6 | 8 | 13 | 9 | 2¾ | 1 | 18 | 4¼ |
| 800 | 66 | 13 | 4 | 15 | 7 | 8¼ | 2 | 3 | 10 |
| 900 | 75 | 0 | 0 | 17 | 6 | 1¾ | 2 | 9 | 3¾ |
| 1000 | 83 | 6 | 8 | 19 | 4 | 7¼ | 2 | 14 | 9½ |
| 2000 | 166 | 13 | 4 | 38 | 9 | 2¾ | 5 | 9 | 7 |
| 3000 | 250 | 0 | 0 | 57 | 13 | 10 | 8 | 4 | 4½ |
| 4000 | 333 | 6 | 8 | 76 | 18 | 5½ | 10 | 19 | 2 |
| 5000 | 416 | 13 | 4 | 96 | 3 | 0¾ | 13 | 13 | 11½ |
| 6000 | 500 | 0 | 0 | 115 | 7 | 8¼ | 16 | 8 | 9 |
| 7000 | 583 | 6 | 8 | 134 | 12 | 3½ | 19 | 3 | 6½ |
| 8000 | 666 | 13 | 4 | 153 | 16 | 11 | 21 | 18 | 4¼ |
| 9000 | 750 | 0 | 0 | 173 | 1 | 6¼ | 24 | 13 | 1¾ |
| 10,000 | 833 | 6 | 8 | 192 | 6 | 1¾ | 27 | 7 | 11¼ |
| 20,000 | 1666 | 13 | 4 | 384 | 12 | 3½ | 54 | 15 | 10½ |
| 30,000 | 2500 | 0 | 0 | 576 | 18 | 5½ | 82 | 3 | 10 |

*Rule*. Multiply the principal by the rate per cent, and the number of months, weeks, or days which are required, cut off two figures on the right hand of the product and collect from the table the ſeveral ſums againſt the different numbers as when added, will make the number remaining; and the ſeveral ſums together will give the intereſt required.

N. B. For every 10 that is cut off in months, add 2d.; for every 10 cut off in weeks, add an halfpenny; and for every 40 in the days, 1 farthing.

EXAMPLES.

1. What is the intereſt of 2467l. 10s. for 10 months, at 4 per cent per annum?

£. s.
2467 10
4
9870 0
10
987|40

| | £. | s. | d. |
|---|---|---|---|
| 900= | 75 | 0 | 0 |
| 80= | 6 | 13 | 4 |
| 7= | 6 | 11 | 8 |
| 987= | 82 | 5 | 0 *Anſ.* |

2. What is the intereſt of 2467l. 10s. for 12 weeks, at 5 per cent?

£. s.
2467 10.
5
12337 10
12
1480|50

| | £. | s. | d. |
|---|---|---|---|
| 1000= | 19 | 4 | 7¼ |
| 400= | 7 | 13 | 10 |
| 80= | 1 | 10 | 9¼ |
| 50= | 0 | 0 | 2½ |
| 1480\|50= | 28 | 9 | 5 *Anſ.* |

3. What is the intereſt of 2467l. 10s. for 50 days, at 6 per cent?

£. s.
2467 10
6
14805 0
50
7402|50

| | £. | s. | d. |
|---|---|---|---|
| 7000= | 19 | 3 | 4½ |
| 400= | 7 | 13 | 10 |
| 2= | 0 | 0 | 1¼ |
| 50= | 0 | 0 | 0¼ |
| 7402\|50= | 20 | 5 | 7 *Anſ.* |

To find what an eſtate from 1 to 60,000l. per annum will come to for 1 day.

*Rule.* Collect the annual rent or income from the Table for 1 year, againſt which take the ſeveral ſums for 1 day, add them together, it will give the anſwer.

An eſtate of 376l. per annum, what is that per day?

| | *s.* | *d.* | |
|---|---|---|---|
| 300= | 16 | 5¼ | |
| 70= | 3 | 10 | |
| 6= | 0 | 4 | |
| 376=1 | 0 | 7¼ | *Anſ.* |

To find the amount of any income, ſalary or ſervants' wages, for any number of months, weeks or days.

*Rule.* Multiply the yearly income or ſalary, by the number of months, weeks, or days, and collect the product from the table.

What will 270l. per annum come to for 11 months, 3 weeks and 6 days?

*For 11 months.*

| £. | £. | *s.* | *d.* |
|---|---|---|---|
| 270 | 2000=166 | 13 | 4 |
| 11 | 900= 75 | 0 | 0 |
| | 70= 5 | 16 | 8 |
| 2970 | | | |
| | 2970=247 | 10 | 0 |

*For 3 weeks.*

| £. | £. | *s.* | *d.* |
|---|---|---|---|
| 270 | 800=15 | 7 | 8¼ |
| 3 | 10= | 3 | 10¼ |
| 810 | 810=15 | 11 | 6½ |

*For 6 days.*

| £. | £. | *s.* | *d.* |
|---|---|---|---|
| 270 | 1000=2 | 14 | 9½ |
| 6 | 600=1 | 12 | 10½ |
| | 20=0 | 1 | 1 |
| 1620 | | | |
| | 1620=4 | 8 | 9 |

*For the whole time.*

| £. | *s.* | *d.* | |
|---|---|---|---|
| 247 | 10 | 0 | |
| 15 | 11 | 6½ | |
| 4 | 8 | 9 | |
| £. 257 | 10 | 3½ | *Anſ.* |

CHAP.

# CHAP. XXIII.

## OF STOCKS.

BY the word Stock, was originally meant a particular ſum of money contributed to the eſtabliſhing a fund to enable a company to carry on a certain trade, by means of which the contributor became a partner in that trade, and received a ſhare in the profit thereof, in proportion to the money employed.*

EXAMPLES.

1. Required the price of 700l. bank ſtock, at $225\frac{1}{2}$ per cent?

*Solution.* As 100 : 225 10 : : 700

7

£. 1578 10 the *Anſ.*

| | *Anſwers.* £. | s. | d. |
|---|---|---|---|
| 2. What is the purchaſe of 912l. 14s. Bank ſtock, at $127\frac{3}{4}$ per cent? | 1165 | 19 | $5\frac{41}{50}$ |
| 3. What is the purchaſe of 2380l. Indian ſtock, at $147\frac{1}{4}$ per cent? | 3504 | 11 | 0 |
| 4. How much is the purchaſe of 1633l. 4s. Bank annuities at $89\frac{3}{8}$ per cent? | 1504 | 7 | $2\frac{2}{5}$ |
| 5. What is the purchaſe of 995l. 10s. Conſolidated annuities, at $84\frac{7}{8}$ per cent? | 844 | 18 | $7\frac{7}{20}$ |
| 6. What does 2400l. capital ſtock in the 3 per cent Conſolidated bank annuities come to, at $84\frac{1}{8}$ per cent? | 2019 | 0 | 0 |

CHAP.

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* But this term has been extended farther; (though improperly) to ſignify any ſum of money lent to government, on condition of receiving certain intereſt, until it is paid, which makes a part of the national debt. Now as the ſecurity both of government and the public companies, is eſteemed preferable to that of any private perſon, and as the ſtocks are negociable and may be ſold at any time; and ſince the intereſt is generally punctually paid when due; ſo they are thereby enabled to borrow money on a lower intereſt than private perſons, where there muſt be always ſome danger of loſing both principal and intereſt.

But as any capital ſtocks of a company are raiſed for a particular purpoſe, and limited by Parliament to a certain ſum, it follows, that when that ſum is completed, no ſtock can be purchaſed of the company; though ſhares already purchaſed, may be transferred from one perſon to another; this being

# CHAP. XXIV.

## REBATE OR DISCOUNT.

REBATE OR DISCOUNT is when a certain ſum of money, due at any time to come, is ſatisfied by paying ſo much preſent money, as being put out at intereſt would amount to the given ſum in the ſame time.

To find, therefore, the diſcount of any ſum of money, according to the time it would have became due: this is the

*Rule.* 1. Find the intereſt of 100l. at the rate per cent, and for the given time.

2. Add the intereſt ſo found, to the 100, and ſay, As that ſum is to the intereſt, ſo is the given ſum to the rebate (or its intereſt).

3. Subtract the rebate from the whole ſum, and the remainder is the preſent worth. Or, as the ſaid principal and intereſt are to 100l. ſo is the given ſum to its principal, (which is the preſent worth) then ſubtract the preſent worth from the given ſum, and the remainder is the rebate. To prove this rule, find the amount of the preſent payment at the time and rate per cent given, which will be equal to the firſt given ſum.

The following is a conciſe rule for ſolving all queſtions in this rule. To find the rebate,

*Rule.*

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being the caſe, there is frequently a great diſproportion between the original value of the ſhares, and what is given for them when tranſferred: for if there be more buyers than ſellers, a perſon who is indifferent about ſelling will not part with his ſhare without a conſiderable profit to himſelf; and, on the contrary, if many are diſpoſed to ſell and few inclined to buy, the value of ſuch ſhares will naturally fall in proportion to the impatience of thoſe who want to turn their ſtock into ſpecie. Hence we may have ſome idea of the nature of ſtock-jobbing, the myſtery of which conſiſts in nothing more than this; the perſons concerned in that practice, who are denominated ſtock-jobbers, make contracts to buy and ſell, at a certain time, ſome quantity of ſome particular ſtock; againſt which time they endeavour, according as the contract is, either to riſe or lower ſuch ſtock, by raiſing rumours and ſpreading fictitious ſtories, in order to induce people either to ſell out in a hurry, and conſequently cheap, if they are to deliver ſtock, or to become unwilling to ſell, and therefore to make it dearer, if they are to receive ſtock.

As

*Rule.** As 100l. with its intereſt for the given rate and time is to the intereſt of 100l. for the ſame, ſo is the debt to the diſcount; or, as the amount of 100l. is to 100l. ſo is the debt to the preſent value.

### EXAMPLES.

1. How much is the rebate or diſcount of 975l. 11s. for 11 months, at 6 per cent per annum? *Anſ.* 50l. 17s. $2\frac{197}{271}$d.

2. What preſent money will ſatisfy a debt of 85l. 10s. due at the expiration of 10 months, diſcount being allowed at the rate of 6 per cent per annum? *Anſ.* 81l. 8s. $6\frac{6}{7}$d.

3. Suppoſe 810l. were to be paid in 3 months hence, allowing 5 per cent diſcount; what muſt be paid in hand? *Anſ.* 800l.

4. A gentleman has a legacy of 400l. left him by his uncle, due to him the 4th of November, but wanting money, is willing to allow 4½ per cent diſcount for preſent payment, what muſt he receive, this being the 9th October? *Ans.* 398l. 14s. 5+d.

5. Sold

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As the price of the different ſtocks is conſtantly fluctuating above or below its original value; ſo when a perſon unacquainted with things of this nature, ſees in the papers, the prices of ſtocks, where bond-ſtock is marked, perhaps, 127l.; India ditto, 134, 134¼; South Sea ditto, 97½, &c. he is to underſtand that 100.l of thoſe reſpective ſtocks ſell at ſuch a time for thoſe ſeveral ſums. Theſe ſtocks or funds come under many deſignations, as Bank-Stock, India-Stock, India Annuities, India Bonds, South Sea Annuities, Three-per-cent Reduced Annuities, Three-per-cent Conſolidated ditto, Three-and a-half-per-cent ditto, Four-per-cent ditto, Long ditto, Exchequer Bills, Navy Bills, Debentures, &c. &c.

* The method among bankers, merchants, &c. in diſcounting bills, is to find the intereſt of the ſum drawn for, from the time the bill is preſented, to the time it becomes due, (including the days of grace) which intereſt they reckon as the diſcount, and by that means make it more than it ought to be. But when goods are bought or ſold, the diſcount is to be made for preſent payment at any rate per cent, without any regard to time, the intereſt of the ſum, as calculated for a year, is the diſcount.

EXAMPLE.

£.
77 payable in 31 days, at 6 per cent per annum.

|  |  | £. | s. | d. |  |
|---|---|---|---|---|---|
| × 31 | From | 77 | 0 | 0 | Principal ſum, |
| 3) 23,87 | Take | 0 | 7 | 10½ | Intereſt for 31 days nearly. |
| 7 10½ | £. | 76 | 12 | 1½ | the ready money to be paid. |

This

5. Sold goods upon truſt for 8 months, to the amount of 95l. 10s. what is the preſent worth, at 7 per cent? *Ans.* 91l. 4s. 10d.+

6. How much ready money for a note of 18l. due 16 months hence, at 5 per cent per annum? *Ans.* 16l. 18s. 10d. nearly.

7. What is the preſent worth of 52l. due 6 months hence, at 5 per cent per annum? *Ans.* 50l. 14s. $7\frac{25}{41}$d.

8. What is the diſcount of 648l. which is payable in 12 months, at 8 per cent per annum? *Ans.* 48l.

9. How much ready money muſt be paid in lieu of 1500l. whereof 500l. is payable in 6 months, 700l. in 14 months after, and the reſt in 10 months after this, diſcounting at the rate of $5\frac{1}{2}$ per cent per annum? *Ans.* 1431l. 5s. 3d.+

10. Bought goods at a ſale, by inch of candle, for 3000l. whereof paid down the $\frac{1}{4}$, and paſſed a bond for the reſt, payable in 6 months; but am at liberty to clear off the debt immediately, rebate at 4 per cent annum, beſides $\frac{1}{4}$ per cent abatement on preſent worth for prompt payment, how much ready money will do it? *Ans.* 2189l. 12s. 8d.+

# CHAP. XXV.

## EXCHANGE.

EXCHANGE is the receiving of money* in one country for the ſame value paid in another. The par of Exchange is

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This method is more ready than the foregoing, and with very good reaſon, is preferred in practice, when a ſmall ſum is diſcounted for a ſhort time; for on ſuch occaſions, which are thoſe that generally happen ofteneſt, the overcharge in the diſcount is a mere trifle; for example, leſs than three farthings in the diſcount of 100l. for a month, nevertheleſs the advocates for this are highly miſtaken, and it muſt be owned that the method is erroneous in theory, and that its errors might become conſiderable in practice alſo; if it were applied to caſes where large ſums are to be diſcounted for long intervals of time: As for example, What is the difference between the intereſt of 500l. at 5 per cent per annum, for 12 years, and the diſcount of the ſame ſum, at the ſame rate for the ſame time? *Anſ.* 112l. 10s. in favour of intereſt.

* Of Monies ſome are real, others imaginary. The real Monies of every kingdom or ſtate, ſignifies pieces of metal, coined by the authority of the

is always fixed and certain, being the intrinſic value of foreign money compared with ſterling; but the courſe of Exchange riſes and falls upon various occaſions.

Exchange of money is performed by this general

*Rule.* Make the money that is to be exchanged, the third term or number in the Rule of Three, and that term of the Rate of Exchange which is of the ſame kind with it, *i. e.* the country currency, the firſt number, and the other term of the ſaid rate the ſecond number, and then proceed as in the Rule of Three for the anſwer. *N. B.* If the firſt number be an unit the queſtion may be performed by Practice.

As the trading people of Ireland, for the moſt part, have no direct Exchange with other countries, but generally draw their foreign demands through the medium of England, it is neceſſary, firſt to ſhew how Engliſh money is reduced into Iriſh, and Iriſh into Engliſh.

*Caſe* 1. To reduce Engliſh money into Iriſh at par.

*Rule.* To the Engliſh money add $\frac{1}{12}$ of itſelf the total is the Iriſh.

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the prince or ſtate, and rendered current at a certain value, by virtue of that authority.

The imaginary Monies probably had, (but at preſent have not) exiſtence are uſed in keeping accounts, and include all other denominations, which are made uſe of to expreſs a ſum of money of which no ſuch pieces are current, in any reſpective ſtate. Thus a pound ſterling, is an imaginary ſum, no ſuch piece being now current.

Currency is the different coins in circulation, and is what paſſes in the payments or receipts of money between perſons, (excluſive of another ſpecies of imaginary Monies called Bills) and this currency is generally conſidered as caſh.—A Bill of Exchange is a written order for the payment of money at one place for money (or goods) iſſued or received at another.

Bank Notes in the buſineſs of Exchange, are ſuch as are obtained from Bankers for money lodged in their Banks, and this is called Bank Money.—Domeſtic Bankers' notes paſs as caſh in almoſt all tranſactions of trade.—Currency is what paſſes in the paying or receiving ſums of money due from one perſon to another, without the interpoſition of the Bank, and is frequently called Caſh.—The money uſed in the Banks of foreign countries, is in general much finer than that of common circulation. That uſed in the latter is called Bullion, (or Billion) and is a coinage of Gold or Silver, in which Copper or ſome other baſe metal predominates. Bullion of Gold is any Gold beneath the ſtandard of 21 carats, and Bullion of Silver, that is below ten penny wts. of fine Silver to the oz.—In Holland and moſt places in Germany, their current Silver is very baſe Bullion.—Agio in Commerce is a term uſed in Holland and other places for the difference between the value of bank money and current.

Agio is alſo uſed for the profit ariſing from the diſcounting a Note, a Bill, &c.

Agio

### EXAMPLES.

1. 242l. 12s. 6d. Engliſh, how much Iriſh money is it at par?

| | £. | s. | d. |
|---|---|---|---|
| | 242 | 12 | 6 |
| $\frac{1}{12}$ | 20 | 4 | 4½ |
| £. | 262 | 16 | 10½ Iriſh money, *Anſ.* required. |

| | Anſwers. £. | s. | d. |
|---|---|---|---|
| 2. In 219l. 10s. Engliſh, how much Iriſh at par? | 237 | 15 | 10 |
| 3. Sold by my factor at Briſtol fair 50 dozen of ſilk handkerchiefs, at 4s. 6d. per pair. Now ſuppoſing his commiſſion 2½ per cent. how much Iriſh money am I to receive? | 142 | 11 | 10½ |

*Caſe* 2. To reduce Iriſh into Engliſh money at par.

*Rule.* Subtract from the Iriſh money $\frac{1}{13}$ of itſelf, the remainder is the Engliſh money.

1. 262l. 16s. 10½d. Iriſh money, how much Engliſh at par?

| | £. | s. | d. |
|---|---|---|---|
| | 262 | 16 | 10½ |
| $\frac{1}{13}$ | 20 | 4 | 4½ |
| £. | 242 | 12 | 6 *Anſwer*, which is a proof of the firſt Example. |

2. In

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Agio is ſometimes uſed for the Rate of Exchange of a ſums negociated whether profit or loſs. The Exchanges which Britain have directly to negociate with other places may be reduced to three diſtinct heads, namely, when England receives a variable number of pounds from other countries for 100l. ſterling, ſuch as Ireland, the Weſt India Iſlands, the Continent of America, the Iſle of Man, &c.

2d. When Britain receives a variable number of ſhillings or of ſome other Denominations, for the Britiſh pounds ſterling, ſuch as the United Provinces, Hamburg, and the Low Countries.

3d. When Britain pays a variable number of pence for the price of Exchange of ſome other countries, ſuch as France, Spain, Portugal, Italy, Denmark, Sweden, Ruſſia, &c. &c.

In England and Ireland accounts are kept in pounds, ſhillings, pence and farthings. The manner is to give an uncertain number per 100l. more in Dublin than in London, the difference is called Exchange, according to the preſent currency of Britiſh coins in both countries (or nations) 12d or a ſhilling Engliſh makes 13d or 1s. 1d. Iriſh, or 100l. Engliſh=$108\frac{1}{3}$ Iriſh; ſo that the par of Exchange is $8\frac{1}{3}$ per 100l. The courſe is generally from $8\frac{1}{3}$ to $17\frac{7}{8}$ per cent. more or leſs.

| | | *Anſwers.* £. | s. | d. |
|---|---|---|---|---|
| 2. In 163l. 16s. Iriſh, how much Engliſh money at par ? | | 151 | 4 | 0 |
| 3. In 15l. 17s. 6d. Iriſh, how much Engliſh Exchange at par ? | | 14 | 13 | $0\frac{12}{13}$ |

*Caſe* 3. To reduce Engliſh money into Iriſh, at any given Rate.

*Rule.* Multiply the given ſum Britiſh, by the rate per cent. divide the product by 100, the quotient added to the given ſum Britiſh, gives the Iriſh required.

### EXAMPLES.

1. How much Iriſh money in 512l. 16s. 8d. Britiſh, Exchange at 6½ ?

512 16 8
6½
3077 0 0
½ = 256 8 4
100) 3333 8 4
£.33 6 8¼

£. s. d.
512 16 8
33 6 8¼ Exchange.
£.546 3 4¼ *Anſ.* in Iriſh money.

See the Rule given in Commiſſion, &c. page 196, for dividing pounds ſhillings and pence by 100, which will be very uſeful in Exchanges.

*Or thus by the common way.*

33,33 8 4
20
6,68
12
$8,\frac{20}{100} = \frac{1}{5}$

*Anſ.* 33l. 6s. 8¼d. nearly as before the one done mentally the other has 15 figures.

| | £. | s. | d. | | | | *Iriſh.* £. | s. | *Anſwer.* d. |
|---|---|---|---|---|---|---|---|---|---|
| 2. | 199 | 17 | 10 | Engliſh, Exch. at | 5 | per cent. | 200 | 8 | $8\frac{7}{10}$ |
| 3. | 750 | 18 | 8 | —— —— | 7 | —— | 803 | 9 | $11\frac{17}{25}$ |
| 4. | 234 | 5 | 6 | —— —— | $7\frac{7}{8}$ | —— | 252 | 14 | $5\frac{3}{4}$+ |
| 5. | 690 | 13 | 3½ | —— —— | $7\frac{3}{8}$ | —— | 741 | 12 | $0\frac{1}{4}$ |
| 6. | 1974 | 12 | 10 | —— —— | $8\frac{3}{4}$ | —— | 2147 | 8 | $5\frac{1}{2}$+ |

| | £. | s. | d. | | Iriſh. £. | Anſwers. s. | d. |
|---|---|---|---|---|---|---|---|
| 7. | 203 | 5 | $7\frac{1}{2}$ | Engliſh, Exch. at 10 per cent. | 223 | 12 | $2\frac{1}{4}$ |
| 8. | 199 | 19 | 11 | —— —— 11 —— | 221 | 19 | $10\frac{69}{100}$ |
| 9. | 103 | 0 | 11 | —— —— $5\frac{1}{3}$ —— | 108 | 10 | $9\frac{74}{75}$ |
| 10. | 905 | 17 | 7 | —— —— $5\frac{1}{4}$ —— | 953 | 8 | $9\frac{31}{400}$ |
| 11. | 300 | 15 | 0 | —— —— $7\frac{1}{2}$ —— | 323 | 6 | $1\frac{1}{2}$ |
| 12. | 138 | 12 | 10 | —— —— $12\frac{1}{2}$ —— | 155 | 19 | $5\frac{1}{4}$ |
| 13. | 212 | 8 | $1\frac{1}{2}$ | —— —— $6\frac{2}{3}$ —— | 226 | 11 | 4 |
| 14. | 617 | 1 | 0 | —— —— $8\frac{1}{3}$ —— | 668 | 9 | 5 |
| 15. | 207 | 2 | 6 | —— —— $8\frac{1}{3}$ —— | 224 | 7 | $8\frac{1}{2}$ |
| 16. | 125 | 15 | 5 | —— —— 10 —— | 138 | 0 | $11\frac{1}{2}$ |
| 17. | 147 | 8 | 6 | —— —— $12\frac{1}{2}$ —— | 165 | 17 | $0\frac{3}{4}$ |
| 18. | 680 | 12 | 8 | —— —— $7\frac{7}{8}$ —— | 734 | 4 | 7+ |
| 19. | 332 | 3 | 4 | —— —— 5 —— | 348 | 15 | 6 |
| 20. | 379 | 2 | 6 | —— —— $5\frac{5}{9}$ —— | 400 | 3 | 9 |

*Caſe* 4. To reduce Iriſh money into Engliſh at any given rate of Exchange.

*Rule.* As 100+ the given rate of Exchange is to 100l. ſo is the given ſum Iriſh to the required ſum Engliſh.

There follows a more commodious method.

* *Rule* 1. Set the farthings of the odd pence, and farthings (if any) counted as pence, which reduce to ſhillings and ſet down the odd pence.

2. To the ſhillings ſo found add twice the farthings; the whole count as ſhillings, which reduce to pounds, and write down the odd ſhillings.

3. The pounds ſo found, add to five times the ſaid ſhillings of the Multiplicand, the whole write down as pounds, if under 10 ſet a cypher to the left in the place of pounds, and to the left thereto write the pounds of the Multiplicand, and divide by 100+ the Exchange. An example will more plainly ſhew the difference of both Rules.

† In 138l. 6s. $11\frac{1}{2}$d. Iriſh money, how much Engliſh, Exchange at 10 per cent?

138

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* As 100 Farthings make two ſhillings, and one penny and 100 ſhillings make five pounds ſterling; therefore the rule is evident.

† In the courſe of Exchange between England and Ireland, we may obſerve, that the lower the rate per cent. the greater the advantage in remitting or ſending caſh from Ireland to England, and the leſs the advantage

in

£. s. d.
138 6 11½

11 ) 13834 15 10 — 140+10=110=11×10

10) 1257 14 2

£. 125 15 5 Engliſh. *Or thus by the common way.*

If 110 : 100 :: 138 6 11½
20 — 20
2200 — 2766
12 — 12
26400 — 33203
2 — 2
52800 — 66407
100

528|00 ) 66407|00 ( £. 125 s. 15 d. 5 *Anſwer, the ſame as before.*
528
1360
1056
3047
2640
407
20
8140

) 8140 ( 15*s.*
528
2860
2640
220
12
) 2640 ( 5*d.*
2640

| | *Iriſh.* £. | s. | d. | | *Exchange.* | | *Engliſh.* £. | s. | d. |
|---|---|---|---|---|---|---|---|---|---|
| 2. | 348 | 15 | 6 | at | 5 | per cent. | 332 | 3 | 4 |
| 3. | 235 | 11 | 0 | | 5 | | 224 | 6 | 8 |
| 4. | 220 | 14 | 4½ | | 5 | | 210 | 4 | 2 |
| 5. | 174 | 16 | 11¼ | | 5 | | 166 | 10 | 5 |

6. 400.

in drawing bills, and that when the courſe is above par, Ireland loſes by remitting, and gains by drawing, the contrary of which happens England.

*N B.* As various conſiderations in trade, further the price of bills more or leſs, above or below par in this kind. Amongſt the trading places therefore, a premium is paid for bills at ſight, at a variable rate per cent; or the bills are taken after ſo many days after date to make them at par.

| | *Iriſh.* | | | | | | *Engliſh.* | | |
|---|---|---|---|---|---|---|---|---|---|
| | £. | s. | d. | | *Exchange.* | | £. | s. | d. |
| 6. | 400 | 3 | 9 | ——— | $5\frac{5}{9}$ | —— | 379 | 2 | 6 |
| 7. | 272 | 18 | $6\frac{1}{2}$ | ——— | $5\frac{5}{9}$ | —— | 258 | 11 | 3 |
| 8. | 231 | 4 | $6\frac{1}{4}$ | ——— | $5\frac{5}{9}$ | —— | 219 | 1 | $1\frac{1}{2}$ |
| 9. | 205 | 17 | $6\frac{1}{2}$ | ——— | $6\frac{1}{4}$ | —— | 123 | 15 | 4 |
| 10. | 188 | 17 | $10\frac{3}{4}$ | ——— | $6\frac{1}{4}$ | —— | 177 | 15 | 8 |
| 11. | 178 | 13 | $2\frac{1}{4}$ | ——— | $6\frac{1}{4}$ | —— | 168 | 3 | 0 |
| 12. | 286 | 1 | $2\frac{1}{4}$ | ——— | $6\frac{1}{4}$ | —— | 269 | 4 | 8 |

*Caſe* 5. England exchanges with Holland, Flanders, and Germany.

Or to reduce pounds ſterling into Guilders.

In theſe places accounts are kept in guilders, ſtivers and pennings, or pounds, ſhillings and pence as in England. The money of Holland and Flanders is diſtinguſhed by the name of Flemiſh, and the Exchange by the pound ſterling. There are two ſorts of money, viz. Bank and Currency, between which there is a ſmall difference called Agio, which runs between three and ſix per cent. in favour of the Bank.*

*The Table.*

| | | | |
|---|---|---|---|
| 8 | Pennings | make | 1 Groat. |
| 2 | Groats or | | |
| 16 | Pennings | —— | 1 Stiver or 2 pence. |
| 6 | Stivers | —— | 1 Shilling. |
| 20 | Stivers | —— | 1 Florin or Guilder. |
| $2\frac{1}{2}$ | Florins | —— | 1 Rix Dollar. |
| 6 | Florins | —— | 1 Pound Flemiſh. |
| 5 | Guilders | —— | 1 Ducat. |

Exchange with Britain is from 34 to 37 Schillings groſs per cent. Flemiſh per pound ſterling. The par is at 36s. 7d. current, and therefore the value of a Guilder is about 21$\frac{7}{8}$d.

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* Current, or currency money, is the real coin which circulates in the country, but bank money is only imaginary, being the property which a perſon poſſeſſes in the treaſury lodged in the bank of Amſterdam, which lodgment cannot be drawn out again, but thoſe who pay it have credit on the books which they can transfer, ſo that it circulates in payments, and anſwers all the purpoſes of caſh. No money is received in the bank, that is in the leaſt defective in weight and ſtandard, and therefore bank money is ſuperior in value to the current coin, which commonly is not ſo pure, ducats excepted, which are generally 23 carats fine.

Before the late alteration, the Dutch gave a fluctuating number of ſhillings and pence for the pound ſterling.

Day of grace are ſeldom taken either at Amſterdam or Hamburgh, but until thoſe days are expired, bills cannot be proteſted for non-payment.

To reduce Bank money into current, and the contrary.

Say, As 100 : 100+the agio : : Banco : Current, and
As 100+agio : 100 : : Current : Banco.

To reduce pounds ſterling into Guilders at a given rate of Exchange.

*Rule.* As one pound ſterling is to the rate of Exchange, ſo the given ſum Engliſh money to the Flemiſh money required. *Note.* The par of Exchange with Holland is twenty ſhillings ſterling for thirty-three ſhillings and four pence Flemiſh.

1. In 284l. 5s. ſterling, how many pounds Flemiſh, exchange at 33s. 4d. per pound?

*£.* *s.* *d.* *£.* *s.*
If 1 : 33 4 : : 284 5
20 12 20
20 400 5685
400
2|0) 227400|0
12) 113700
2|0) 947|15
*£.* 473 15 *Anſ.*

*By Practice thus,*
*£.* *s.* *s.* *d.*
284 5 at 33 4
568 10 *at* 2 *£.*
6*s.* 8*d.* *is* $\frac{1}{3}$=94 15 *Subtract.*
*Anſ.* 473 15 *as before.*

| | *£.* | *s.* | *d.* | | *s.* | *d.* | | *Anſwers.* |
|---|---|---|---|---|---|---|---|---|
| 2. | 447 | 17 | 6 | *at* | 35 | 6 | —— | 794l. 19s. 6¾d. |
| 3. | 1597 | 9 | 11 | — | 34 | 5 | —— | 16494guil. 2ſt 14pen.+ |
| 4. | 223 | 3 | 10 | — | 32 | 6 | —— | 2176guil. 2ſt. 6pen. |
| 5. | 224 | 15 | 9 | — | 33 | 4 | —— | 391l. 6s. 3d. |
| 6. | 301 | 0 | 0 | — | 33 | 6 | —— | 504l. 3s. 6d. |
| 7. | 704 | 10 | 0 | — | 33 | 8 | —— | 7115guil. 9ſt. |
| 8. | 201 | 0 | 0 | — | 33 | 9 | —— | 2035guil. 2ſt. 8pen. |

*Caſe* 6. To reduce Flemiſh money into pounds ſterling.

*Rule.* If the rate of exchange gives 1l. ſterl. what will the propoſed ſum give.

*Or,* Reduce the rate of exchange and the given ſum into one denomination; divide the latter by the former, the quotient is the anſwer, or value in pounds; the remainder, multiplied by 20, 12, &c. gives the ſhillings, &c.

1. In 166l. 13s. 4d. Flemiſh, at 33s. 4d. how many pounds ſterling?

s. d. £. £. s. d.
If 33 4 : 1 :: 166 13 4
12 20
400 3333
12
4|00) 400|00
£.100 *Anſwer.*

*or thus,*
£. s. d.
166 13 4
3
5) 500 0 0
*Anſwer,* £.100 0 0*d.*

2. In 1223l. 1s. $1\frac{1}{4}$d. Flemiſh how many pounds ſterling, exchange at 33s. 4d.? *Anſ.* 733*l.* 16*s.* $7\frac{3}{4}$*d.*+

1223 1 $1\frac{1}{4}$
3
5) 3669 3 $3\frac{3}{4}$
£.733 16 $7\frac{3}{4}$ *Anſwer.*

*or thus, by the common method;*

If 33 4 : 1 :: 1223 1 $1\frac{1}{4}$
12 20
400 24461
4 12
1600 293533
4
16|00) 1174133 ( 733*l.* 16*s.* $7\frac{3}{4}$*d.* *the ſame as before.*
11200
5413
4800
6133
4800
1333
20
26660

1600) 26660 ( 16*s.*
1600
10660
9600
1060
12
12720
Carried over.

```
1600 ) 12720 ( 7d.
       11200
       -----
        1520
           4
       -----
      ) 6080 ( 3
        4800
       -----
remainder, 1280
```

| | Anſwers. | | |
|---|---|---|---|
| | £. | s. | d. |
| 3. How many pounds ſterling can I have for 2176 guilders, 2 ſtivers, 6 pennings, at 32s. 6d. per pound ſterling? | 223 | 3 | 10 |
| 4. How many pounds ſterling are equal to 7115 guilders, 9 ſtivers, exchange at 33s. 8d. groſs per pound ſterling? | 704 | 10 | 0 |
| 5. At 33s. 9d. groſs per pound ſterling, how many pounds ſterling can be had for 2035 guilders, 2 ſtivers, 8 pennings? | 201 | 0 | 0 |

*Caſe* 7. London exchanges on Hamburg,* or to reduce ſterling money into marks.

Great-Britain exchanges with Hamburgh upon the pound divided into ſhillings and pence, as in Holland, (the par of Exchange is 35*s.* $6\frac{2}{3}$*d.* for 1 pound ſterling) often upon the mark lubb, and often upon the Rix dollar.

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* Here as in Holland the current money has been ſo much adulterated of late years, that the agio has riſen from 15 to 20, to 30, and to 40 per cent; but Bills of exchange are all paid in Bank money.

Holland exchanges with Hamburgh at 33 ſtivers Banco, for 1 dollar of Holland at par.

The monies of Hamburgh may be diſtinguiſhed into five heads. 1ſt, Bank money, which is inſcribed in the Bank books, and transferred in payment from one perſon to another. 2nd, Specie, or the hard Rix dollars, worth (full weight) about one-half per cent more than the Banco. 3d, The golden Ducat which is about 1 per cent better or worſe than the Banco, according to the price of the bullion. 4th, Light coin, compoſed chiefly of foreign monies, which loſe about 38 per cent againſt Banco. 5th, The Currency, which conſiſts of various denominations of ſilver money coined at Hamburgh ſince the year 1726. The par between Hamburgh Bank and currency is reckoned at 23 per cent agio, or 13 marks Banco for 16 marks currency.

## *Table of Hamburgh Money.*

1 Rix dollar is 3 Marks or 18 ſols lubs.
1 Mark lub is 16 ſols, or ſous or ſchillings lub.
1 Sous lubs is 12 deniers or pennings lubs.
1 Mark lubs is 32 deniers groſs.
1 Sous lubs is 2 deniers groſs, and
1 Denier lubs is $\frac{1}{6}$ of a denier groſs.

To Exchange Britiſh to Hamburgh money.

*Rule*. Say, as 1 pound Flemiſh is to 7½ ſo is the given courſe of exchange to the given number of marks in a pound ſterling. Then ſay, as 1 pound ſterling is to the marks ſo found, ſo is the given ſum Britiſh to the required number of marks.

*Or*, Multiply continually the given courſe of exchange, the given ſum Britiſh, and 7½; the product is the required number of marks.

*Or*, Reduce the ſterling money to Florins, &c. then add ¼ of the Florins thereto, and for every 1 remaining add 4 to the ſchillings, which ſchillings, or ſous muſt be added above 16.

*N.B.* 7½ marks make 1 pound Flemiſh, or 120 ſchillings lubs.

### EXAMPLES.

1. Admit a bill of exchange to be drawn on Hamburgh for 127l. 3s. 4d. ſterling at 32s. 4d. exchange groſs per pound ſterling; how many marks lubs are to be paid for that bill?

If

The par between Engliſh money and currency at 43s. Flemiſh for 1l. ſterling; but there can be no permanent par with Banco, becauſe the agio is conſtantly fluctuating. If the agio be 23, one pound ſterling equals 34s. 11¼d. Flemiſh, Hamburgh Banco; and, according to the preſent agio of 20 it equals 35s. 10d. Banco. The above ſtatements are taken from KRUSE and NELKENBRECHER, two writers on Exchange; the former, who is of the firſt authority, publiſhed a celebrated work at Hamburgh in 1782, and the latter at Berlin in 1796: theſe authors perfectly agree, but what has been advanced by French and other writers on the ſubject, is for the moſt part both vague and contradictory.

*Note*. As 40 groſs or Flemiſh pence make a guilder of Holland, and 32 a mark of Hamburgh; guilders multiplied by 5 and divided by 4 make marks, and marks multiplied by 4 and divided by 5 make guilders.

£. *marks* *s.* *d.*
If 1 : $7\frac{1}{2}$ : : 32 4
20 12

20 388
12 $7\frac{1}{2}$

240 2716
194

24|0 ) 291|0 ( 12
24

51
48

3 ) $\frac{3}{24}$ ( $\frac{1}{8}$

*or thus,*

| | £. | *s.* | *d.* | | *s.* | *d.* |
|---|---|---|---|---|---|---|
| | 127 | 3 | 4 | at | 32 | 4 |
| | | | 9 | | | 6 |
| | 1144 | 10 | 0 | *l.*9 | 14 | 0 |
| 10s. is ½ | 63 | 11 | 8 | | | |
| 4 is ⅕ | 25 | 8 | 8 | | | |
| | 1233 | 10 | 4 | Florins. | | |
| ¼ of flor. | 308 | 4 | 0 | | | |

*Marks* 1541 14 4 *lubs.*
*Anſwer, as before.*

£. *marks* £. *s.* *d.*
If 1 : $12\frac{1}{8}$ : : 127 3 4
20 20

20 2543
12 12

240 30520
$12\frac{1}{8}$

366240
3815
*mks.*
240 ) 370055 ( 1541
240

1300
1200

1005
960

455
240

215
16

1290
215
*ſous.*
240 ) 3440 ( 14
240

1040
960

80
12
*dens.*
240 ) 960 ( 4
960

2. How many marks lubs are equal to 601l. 10s. at 32s. 6d. groſs per pound ſterling? *Anſ.* 7330 marks 12 ſ. 6 den.

3. In 1500l. ſterling how many marks lubs, exchange at 32s. 3d. per pound ſterling? *Anſ.* 18140 marks 10 ſols lubs.

4. For 500l. ſterling how many marks lubs muſt be paid in Hamburgh, exchange at 35s. 6d. per pound ſterling?

*Anſ.* 6656 marks, 4 ſous.

To reduce Hamburgh money into Britiſh money.

*Rule.* Divide the number of marks (Hamburgh money) by the produce of the given courſe of exchange, and 7½ times the quotient gives the required Britiſh.

*Or thus,* Reduce the marks to florins, by deducting ⅕ of the marks, and alſo the rate by multiplying by 6, then ſay, if the florins of the rate give 1 pound ſterling, what will the given florins be.

EXAMPLE.

1. How many pounds ſterling for a bill of 1541 marks, 14 ſous 4 den. lubs of Hamburgh, at 32s. 4d. groſs per pound ſterling? *Anſ.* 127l. 3s. 6d.

| | *mar.* | *ſous.* | *den.* | | *s.* | *d.* |
|---|---|---|---|---|---|---|
| | 1541 | 14 | 4 | lubs at | 32 | 4 |
| Subtract ⅕ | 308 | 4 | | | | 6 |
| Flor. | 1233 | 10 | 4 | | 9 14 | 0 |

| | *£.* | *s.* | | *£.* | | *F.* | *ß.* | *d.* | | *£.* | *s.* | *d.* | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Then, If | 9 | 14 | : | 1 | :: | 1233 | 10 | 4 | : | 127 | 3 | 4 | *Ans.* |

*Anſwers.*

2. In 7330 marks 12 ſous 6 den lubs, how many pounds ſterling? 601 10 0

3. How much ſterling money in 1328 marks lubs, agio, at 20 per cent, exchange at 33s. 10d. per pound ſterling? ————

4. Hamburgh draws on London for 6656 marks 4 ſchellings lubs, how much ſterling money muſt there be paid in London? ————

## London Exchange with France.

Accounts are kept in France in livres, ſols and deniers, divided as the Britiſh pound ſterling, and the Exchange with England upon the crown or ecu of 3 livres or 60 ſols tournois, and as they have not any real crown of this value, they diſtinguiſh this ideal one (from the real crown or ecu d'argent) by the name of the crown of 60 ſols tournois, which at par is eſtimated at the value of 29,149 pence. Sir Iſaac Newton makes it $31\frac{7}{18}$.

France exchanges with Britain on the crown, from 28 to 34 with Holland, about 28 ſtivers banco per crown; with Hamburgh about 26 ſchilling lubs per crown.

## *The Table of French Money.**

12 Deniers=1 Sol, 20 Sols=1 Livre, 3 Livres=1 Crown.

To change French money into pounds ſterling.

*Rule.* As 1 ecu or exchange crown : is to its value in Britiſh money : : ſo is the given ſum French money, to the required ſum Britiſh. Or, if the ſum to be exchanged, be given in livres, ſols and deniers, divide them all by 3, which reduces them to crowns, ſols and deniers, or to crowns and twentieths, and two hundred and fortieths of a crown, which denominations are of the ſame form as pounds, ſhillings and pence, and therefore proportionable to them, divide theſe crowns, ſols and deniers by the parts of which 1 crown is of a pound ſterling, and the quotient or ſum of the quotients will be the ſterling money required.

1. In 5929 livres 4 ſols, how many pounds ſterling, at $32\frac{1}{2}$ per cent?

3)

---

* The following are the principal alterations recently made at Paris in the French exchanges.

10 Cents=1 Decime, 10 decimes, or 100 Cents=1 Franc, 80 Francs=81 Livres.

Thus the franc, or new livre=243 deniers, being $1\frac{1}{4}$ per cent, better than the old livre tournois, which=240 deniers, francs and cents are reduced to livres, ſous, deniers, by multiplying by 81, and dividing by 80, and livres are reduced to francs by the reverſe operation.

It is to be remarked alſo, that Paris gives

| | | |
|---|---|---|
| London, | 23 francs, 95 cents for 1 pound ſterling. | more or leſs. |
| Amſterdam, | 3 francs for 55 pence Flemiſh. | |
| Hamburgh, | 190 francs for 100 marks banco. | |
| Madrid, | 14 francs, 50 cents for 1 piſtole | |
| Leghorn, | 5 francs, 25 cents for 1 pezza di otto. | |
| Genoa | 4 francs, 95 cents for 1 pezza di lire. | |

Paris exchanges with London, by giving a variable number of livres, ſous, deniers or francs, and cents, for the pound ſterling

Before the late alteration, London exchanged with Paris, by giving an uncertain number of ſhillings and pence for the ecu of 3 livres. The livres tournois is ſtill retained in Lloyd's, but the French government moſtly reckon in francs, decimes and centimes. Francs and livres were formerly ſynonymous, but in a late coinage, the 5 livre pieces were by ſome accident made too heavy, being worth $101\frac{1}{4}$ ſous inſtead of 100; the new franc has been therefore adopted in conformity to this value.

3 ) 5929 4

1976 8 at 2 $8\frac{1}{2}$ per.

2s. 6d. is $\frac{1}{8}$ 247 1  
$2\frac{1}{2}$ is $\frac{1}{12}$ 20 11 9

£. 267 12 9

| | *Anſwers.* £. | *s.* | *d.* |
|---|---|---|---|
| 2. In 1000 crowns at $33\frac{1}{4}$d, how many pounds ſterling? | 138 | 10 | 10 |
| 3. In $1263\frac{1}{2}$ crowns at $32\frac{1}{2}$d. per crown, how many pounds ſterling? | 171 | 1 | $11\frac{3}{4}$ |
| 4. In $1037\frac{3}{4}$ crowns at 33d. per crown, how many pounds? | 142 | 13 | $9\frac{3}{4}$ |
| 5. In 1976 crowns, 1 livre, 4 ſols, at $32\frac{1}{2}$d. per ecu, how many pounds ſterling? | 267 | 12 | 9 |
| 6. In 1000 crowns, $33\frac{3}{4}$d. per crown, how many pounds? | 140 | 12 | 6 |
| 7. Suppoſe a merchant is to remit a bill of exchange from Paris, upon any town in England or Ireland, for 2762 ecus, 12 ſols, 8 deniers, at 30 pence ſterling per crown, what ſterling money does it come to? | 345 | 6 | 7 |
| 8. How many pounds ſterling are in 2822 crowns, 2 livres, 8 ſols, 4 deniers, exchange 36 pence ſterling per crown? | 423 | 8 | 5 |

### To reduce Engliſh money into French.

* *Rule.* As the rate of exchange : to 1 crown :: ſo is the ſterling money to the French required.

1. How

---

* The foregoing method is univerſal, but may be contracted in many caſes, viz. When the rate of exchange is 30 pence or 2s. 6d. per ecu, 2s. 6d. $=\frac{1}{8}$ pound, conſequently in this caſe every pound ſterling contains 8 crowns, wherefore the Engliſh money multiplied by 8, will produce the crowns required. *Note.* If the Engliſh money be pounds, ſhillings and pence, the anſwer will be the reſult, crowns, ſols, and deniers d'or, which being multiplied by 3 throughout, will give the anſwer in livres, ſols and deniers tournois.—And if the rate exceed 30 pence by an even part thereof, as $32\frac{1}{2}$d. exceeds it by $2\frac{1}{2}=\frac{1}{12}$ of 30, then $2\frac{1}{2}$ is contained juſt once more in the given

rate

1. How many crowns in 243l. 15s. at 32½d. per crown?

| | £. | s. |
|---|---|---|
| | 243 | 15 |
| $\frac{1}{13}$ | 18 | 15 |
| | 225 | 0 |
| | | 8 |
| | 1800 | 0 *Anſ.* |

*or thus,* If $32\frac{1}{2}$ d. : 1 :: 243 £. 15 s.

$$\begin{array}{rrl} 2 & 20 & \\ \hline 65 & 4875 & \\ & 12 & \\ \hline & 58500 & \\ & 2 & \\ \hline 65\,) & 117000 & (\,1800 \text{ Crowns. Anſ.} \\ & 65\cdots & \\ \hline & 520 & \\ & 520 & \\ \hline & \cdots(00 & \end{array}$$

| *When the rate is* | *deduct* |
|---|---|
| 31½ | $\frac{1}{21}$ |
| 32 | $\frac{1}{16}$ |
| 32½ | $\frac{1}{13}$ |
| 33 | $\frac{1}{11}$ |
| 33⅓ | $\frac{1}{10}$ |
| 33¾ | $\frac{1}{9}$ |
| 35 | $\frac{1}{7}$ |
| 36 | $\frac{1}{6}$ |

| | *Anſwers.* Cr. | liv. | s. | d. |
|---|---|---|---|---|
| 2. How many crowns, &c. in 423l. 8s. 5d. at 36d. per ecu? | 2882 | 2 | 8 | 4 |
| 3. How many crowns, &c. in 345l. 6s. 7d. at 30d? | 2762 | 12 | 8 | 0 |
| 4. How many crowns, &c. in 140l. 12s. 6d. at 33¾d? | 1000 | 0 | 0 | 0 |
| 5. How many crowns, &c. in 267l. 12s. 9d. at 32½d? | 1976 | 1 | 4 | 0 |

6. How

---

rate $32\frac{1}{2}$ than in the aſſumed rate 30 pence, viz. 13 times, and $32\frac{1}{2}$ is $\frac{13}{13}$ of itſelf, and 30 is $\frac{12}{13}$ of the ſame, therefore $32\frac{1}{2} : 30\,(£.) :: \frac{13}{13}$ or $1 : \frac{12}{13}$ or $1 - \frac{1}{13}$ ſo that if $\frac{1}{13}$ be deducted, the remainder will manifeſtly be in ratio to the crowns required, as $\frac{1}{8}$ thereof, and the ſame is ſhewn of any other rate which exceeds 30d. at 2s. 6d. by any aliquot part thereof, according to the following rule.—*Rule.* If the rate exceed 30d. an even part of itſelf, deduct that part of the ſterling money from itſelf, which is denominated by 1 more than the exceſs is of 30d. and multiply the remainder by 8.

In all exchanges on the foreign piece, as the French crown, milrea, piaſtre, &c. the Britiſh merchant muſt remit when the exchange is low, and draw when the exchange is high, to avail himſelf of the difference, as will be more fully explained hereafter.

| | Anſwers. Crowns. |
|---|---|
| 6. How many crowns, &c. in 142l. 13s. 9¾d. at 33d? | 1037¾ |
| 7. How many crowns, &c. in 171l. 1s. 11¾d. at 32½d? | 1263½ |
| 8. How many crowns, &c. in 138l. 10s. 10d. at 33¼d.? | 1000 |

## *London Exchanges with Spain.*[*]

The Monies of Spain are of two ſorts, the one called Plate money, often old Plate money; and the other Vellon; the former is much more valuable than the latter, being nearly in the proportion of 32 to 17, or nearly 53⅛ per cent. By the word Plate, is underſtood ſilver money, wherein ſome of the Merchants keep their accounts, and that which is diſtinguiſhed by the name of the Old Plate, is uſed in the negociation of foreign Exchanges.

But the old plate, like the French crown and the Engliſh pound ſterling is but ideal. In Madrid, Cadiz, Malaga, and all the places of trade in the Straits, Mediterranean, Africa and the Weſt Indies, the Spaniards keep their accounts generally in Piaſters, Dollars, Rials, half Rials and Quartiles, 16 Quartiles=1 Rial, 8 Rials=1 Dollar, or in Dollars, Rials and Marvadies, reckoning 34 Marvadies to the Rial, and 8 Rials to the Dollar.

*Table*

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[*] It ſo happens, that the courſe of exchange between Great Britain and Spain is almoſt always under par, however it fluctuates between 35 and 45 pence per piaſtre, Spain exchanges with London upon the piaſtre of from 35d. to 45d. par 43; Holland upon 110d. groſs per ducat more or leſs; Hamburgh upon 100d. groſs per ducat more or leſs; France 18 livres per piſtole of 8 piaſtres more or leſs; Portugal upon 630 reas per piaſtre more or leſs; Italy at ſo many marvidies per pezzo. In moſt of the inland towns of Spain, they keep their accounts in marvidies, hundreds and thouſands of marvidies, and exchange upon the ducat of 375 marvidies. In 1737, great alterations were made in the Spaniſh money, as the vellon which before was valued at ⅔ of the plate was reduced to ½ that value at the ſame time gold was lowered 6½ per cent, the proportion being 15 to 1, the ſilver money remaining nearly as before.

## *Table of Spaniſh Coins.*

1 Rial Vellon=34 Marvadies Vellon.
1 Rial=34 Marvadies Plate.
1 Quart=16 Marvadies Vellon.
1 Rial Plate=4 Quarts, or 64 Marvadies.
1 Piaſtre of Exch.=8 Rials Plate=512 Marvadies Vellon.
1 Dollar=10 Rials Plate=640 Marvadies Vellon.
1 Ducat=11 Rials 1 Marvadie Plate.
1 Piſtole of Exchange=4 Piaſtres.
1 Piſtole=4½ Piaſtres.

To reduce Britiſh money into Spaniſh.
The Rule as with France.

EXAMPLES.

1. A merchant in London remits to his correſpondent in Cadiz 1188l. 12s. to receive the ſame in dollars at 56d. each, how many ought he to receive? *Anſ.* 5094.

*d.* *Dol.* *£.* *s.*
If 56 : 1 :: 1188 12
20

23772
12
*Doll.*
56 ) 285264 ( 5094. *Anſwer* required.

2. In 739l. 17s. 5¾d. how many Dollars, Exchange at 40½d. ſterling? *Anſwer* 4384 dol. 3r. 8q.+

3. In 594l. 6s. how many Dollars at 56d. per piece? *Anſwer* 2547.

4. How many Piaſtres, &c. ſhall I receive in Spain for 510l. ſterling, exchange at 50d. ſterling per Piaſtre? *Anſ.* 2448.

5. A gentleman travelling, would exchange 60l. 14s. 7d. ſterling, for Venice Ducats at 4s. 5d. each, how many muſt he receive? *Anſwer* 275.

6. If I pay a bill here for 2500l. what Spaniſh money may I draw my bill for at Madrid, Exchange at 57½ per per piece of eight? *Anſ.* 10434 pieces 6 rials 8 mar.

7. In 349l. 6s. 3¾d. how many Piaſtres, at 38¾ per Piaſtre? *Anſwer* 2163 pias. 4r.+

## To reduce Spaniſh money into Britiſh.

1. A merchant at Cadiz remits to London 2547 pieces of eight, at 56d. per piece, how much ſterling is the ſum? *Anſwer* 594l. 6s.

```
p.  d.    pieces.                pieces.
If 1 : 56 :: 2547        or thus  2547
              56                 ------
           ------         l.    s. 636 15  at 5s.
           15282  4 is 1/15 and 1/3  42  9 Subtract by caſe
          12735                  -------     6 practice.
          ------               £. 594  6 Anſ. as before.
     12 ) 142632
          ------
    2,0 ) 1188,6
          ------
     £. 594  6 Ans.
```

2. How much ſterling money may a perſon receive in London, if he pays in Genoa 976 dollars, at 53d. each? *Anſwer* 215l. 10s. 8d.

3. A factor has ſold goods at Florence for 250 ducatdons, at 54d. each, what is the value in pounds ſterling? *Anſwer* 56l. 5s.

4. If 275 ducats at 4s. 5d. each, be remitted from Venice to London, what is the value in pounds ſterling? *Anſwer* 60l. 14s. 7d.

5. In 10434 dollars, 6 reas, 8 milreas, at 57½d. per piece, how many pounds ſterling? *Anſwer* 2500l.

6. A merchant in Oporto, remits to London 4366 Milreas, 183 Reas, at 5s. 5⅝d. Exchange per Milrea, how much ſterling muſt be paid in London for this remittance? *Anſwer* 1193l. 17s. 6½d.

Note. 1000 Reas = 1 Milrea.

### *London Exchanges with Portugal.*

They keep their accounts in Liſbon, Oporto, and in general throughout the dominions of Portugal, in Milreas and Reas, and exchange on the Milrea. Exchange with Britain 65d. to 70d. per Milrea, par 67½d. with Holland, about 50 Groſs per Cruſado; with France, about 430 Reas per Crown.

400 Reas = 1 Cruſadoe.
1000 Reas = 1 Milrea.

## To Exchange Portugal Money into Britiſh.

*Rule.* As one piece is to its value in Britiſh : : ſo is the given ſum Portugal : to the required ſum Britiſh.

*Or thus.*—Take parts out of the Milreas for the given rate, and if there be odd Reas, divide the Milreas and Reas, as one number, and the anſwer will be had in pounds, and thouſandth parts of a pound, in the uſual way of practice, which parts may be changed into ſhillings, pence and farthings.

1. How many pounds ſterling in 1000 Milreas, at 5s. 6d. per? *Anſwer* 257l.

```
              1000          or thus,        s.  d.        m.
              ----                     If 1 : 5  6 :: 1000
d. l.     s.   250  at 5s.                  12            66
6 1/40 and 1/2  25  at 6d.                  --          -----
              ----                          66      12 ) 66000
          £. 275 Anſ.                                   -----
                                                2|0 ) 550|0
                                                      -----
                                                Ans. £. 275
```

2. A merchant at Liſbon deſires to remit to his correſpondent in London 4760 Milreas, exchange at 64d. per Milrea, how much ſterling money muſt be paid in London? *Anſwer* 1269l. 6s. 8d.

3. Liſbon owes London 9784 Milreas, 515 Reas, for how much ſterling muſt London draw, Exchange at 5s. 4d. per Milrea? *Anſwer* 2609l. 4s. 0¾d.+

4. I demand how many pounds, &c will a draft of 2523 Milreas 528 Reas amount to, Exchange at 5s. 8d. per Milrea? *Anſwer* 714l. 19s. 11¾d.+

5. How much ſterling will 730 Milreas amount to, Exchange at 5s. 9½d. per Milrea? *Ans.* 213l. 2s. 8d.

6. Liſbon ſells for Dublin a conſignment of 5745 yards of broad-cloth, at 3 Milreas 570 Reas per yard, what ſterling muſt be remitted to even accounts with the conſtituents, Exchange at 65⅔d. per Milrea? *Ans.* 5611l. 13s. 4d.+

### To reduce ſterling money into Portugal.

1. In 275l. at 5s. 6d. per milrea, how many milreas? *Anſwer* 1000.

  *s. d. m. £. m.*
If 5 6 : 1 :: 275 to 1000 the *Anſ.* required.

2. In 1566l, 6s. 8d. how many milreas, exchange at 64d? *Anſ* 5873milr. 750reas.

3. In 213l. 7s. 10d how many milreas, exchange at 5s. 9½d. per? *Anſ.* 736milr. 892 $\frac{12}{39}$reas.

4. In 213l. 2s. 8d. how many milreas, at 5s. 9½d. per? *Anſ.* 730milr.

5. In 478l. 16s. 6d. how many milreas, at 5s. 5½d. per? *Anſ.* 1756milr. 473reas. X

6. At 9s. 10d. per milrea, how many milreas are in 106l, 5s. 6¼d.? *Anſ.* 364milr. 375reas.

### *London Exchanges with Italy.*

Here, as in Holland, the money is diſtinguiſhed into banco and current, the former has a conſtant ſuperiority of 20 per cent, and alſo a valuable agio. It alſo conſiſts of lire and exchange money. Accounts are kept in the former, and exchange tranſacted by the latter.

12 Piſtoli=1 Soldo, 20 Soldi=1 Lire.

Venice exchanges by the ducat banco, value 4s. 2¼d. Geneva, by the pezzo of 5¾ lires, value 4s. 2d. Leghorn, by the piaſtre of 6 lires, value 4s. 2d. Florence, by the ducat of 7½ lires, value 5s. 2¼d.

The ducat, pezzo, and piaſtre are each divided into 20 ſoldi, and theſe into 12 denari, in the ſame manner as the lire money, or as our pound ſterling.

In the pope's territories, accounts are kept in crowns and julios, exchange by the ſtamped crown, 10 julios=1 current crown; 12 julios=1 ſtamped crown, value 6s. 1⅔d.

At Naples, accounts are kept in ducats, tarins, and grains. 20 grains=1 train, 5 tarins=1 ducat, exchange by the ducat, value 3s. 4d.

To reduce Italian money into Engliſh, at any given rate of exchange.

*Rule.* As the Italian equivalent to the rate of exchange, is to the rate, ſo is the propoſed ſum Italian money to its value in Engliſh currency.

### EXAMPLES.

1. In 7456 piaſtres, 9 ſoldi, 6 deniers, how many pounds ſterling, exchange at $49\frac{7}{8}$d. per piaſtre ?

As 1 piaſtre : $49\frac{7}{8}$ : : 7456 9 6 : 1549 10 $11\frac{183}{320}$

*Or thus :*

|  |  |  |  |
|---|---|---|---|
|  | 7456 | 9 | 6 |
| $40 = \frac{1}{6}$ | 1242 | 14 | 11 |
| $8 = \frac{1}{5}$ | 248 | 10 | $11\frac{4}{5}$ |
| $1 = \frac{1}{8}$ | 31 | 1 | $4\frac{19}{40}$ |
| $\frac{4}{8}$ is $\frac{1}{2}$ | 15 | 10 | $8\frac{19}{80}$ |
| $\frac{2}{8}$ is $\frac{1}{2}$ | 7 | 15 | $4\frac{19}{160}$ |
| $\frac{1}{8}$ is $\frac{1}{2}$ | 3 | 17 | $8\frac{19}{320}$ |

£. 1549 10 $11\frac{183}{320}$ the *Anſ.*

2. Briſtol owes to Genoa 3390 piaſtres or pezzos, 16 ſoldies ; how much ſterling money muſt be remitted to even the accounts, exchange $51\frac{7}{8}$d. per piaſ. ? *Anſ.* 732l. 18s. 1¾d.

3. Leghorn remits to London 1221 piaſtres, 1 ſ. 8 den. at 54d. ſterling per piaſtre ; how much Engliſh money does the remittance amount to ? *Ans.* 274l. 14s. 10½d.

4. When the exchange is 52½d. ſterling per piaſtre, what ſterl. money will be equal to 2821 piaſ. 4 ſol. ? *Ans.* 617l. 2s. 9d.

5. A merchant in London owes to a merchant in Genoa 4862 piaſtres, 6 ſol. 11 den. ; how much ſterling money will the debt amount to, when the exchange is at 45d. per piaſtre ? *Ans.* 911l. 13s. $9\frac{9}{16}$d.

6. A merchant in Genoa draws on a merchant in London for 3000 livres ; how much ſterling money will ſatisfy this draft, exchange at 50½d. ſterling per piaſtre ? *Ans.* 126l. 5s.

To reduce Engliſh money into Italian, at any given rate of exchange.

*Rule.* As the value of 1 piece of exchange in Britiſh money : is to 1 piece of Italian : : ſo is the given ſum Britiſh : to the required ſum Italian.

1. A Leghorn merchant draws on a merchant in London for 416l. 13s. 9¾d. Britiſh money, exchange at 50d. ſterling, per piaſtre ; how many piaſtres will be ſufficient to pay this draft ? *Anſ.* 2000 piaſtres, 3 ſols, $2\frac{3}{5}$ den.

2. In 415l. 17s. 4d. ſterling, how many ducats, &c. current ; agio 20, and exchange 53d. per ducat ?

*Anſ.* 2259 ducats, 19 groſſi. +

3. How many piaſtres, &c. muſt be given at Genoa for 175l. 15s. ſterling, at 52d. per piaſtre?

*Anſ.* 811 piaſ. 3 ſoldi, $0\frac{12}{53}$ den.

4. In 510l. ſterling how many piaſtres, &c. exchange at 50d. per? *Anſ.* 2448.

5. In 617l. 2s. 9d. ſterling how many piaſtres, &c. exchange at $52\frac{1}{2}$d.? *Anſ.* 2821 piaſ. 4 ſol.

6. How many piaſtres, &c. are in 274l. 14s. $10\frac{1}{2}$d. at 54d.? *Anſ.* 1221 pia. 1 ſ. 8 d.

## *London Exchanges with Ruſſia.*

Ruſſia generally exchanges with London by way of Hamburgh or Amſterdam, at the rate of 48 to 50 ſtivers per ruble, and ſometimes directly to London from 4 to 5 ſhillings per ruble. Accounts are kept at Peterſburgh in rubles and copecs, and the exchange by the ruble.

Exchange with Britain, from 4s. to 5s. per ruble, par 4s. 6d.
——— with Holland, about 50 ſtivers per ruble.
——— with Hamburgh, about 110 copecs, per rix-dollar lubs.

3 copecs =1 altine
10 copecs =1 grevena.
25 copecs =1 polpolitin.
2 polpolitims =1 polpolitin=5 copecs.
2 politins =1 ruble=100 copecs.
2 rubles =1 ducat=200 copecs.

### EXAMPLES.

1. A merchant in Peterſburgh owes to a merchant in London, 1317 rubles, 29 copecs; how many pounds ſterling will diſcharge the debt, Exchange 4s. 8d. per ruble?

*Anſ.* 307l. 7s. $4\frac{6}{25}$d.

| | | | |
|---|---|---|---|
| | *rub.* | *cops.* | |
| | 1317 | 29 | |
| *s.* | | | |
| 4 is $\frac{1}{5}$ = | 263 | 8 | |
| 8*d.* is $\frac{1}{6}$ = | 43 | 18 | |
| 29 *copecs* = $\frac{29}{100}$ = | 1 | 4$\frac{6}{25}$ | |
| *Anſ.* £. | 307 | 7 | 4$\frac{6}{25}$ |

2. When the exchange between London and Amſterdam is 34s. 9d. per pound ſterling, and between Amſterdam and Peterſburgh is 50 florins per ruble, how many rubles, &c. will 943l. 14s. 8d. ſterling make?

*Ans.* 3935 rubles, 36+copecs.

3. In 674l. 17s. 6d. ſterling how many rubles, &c. exchange at 49 ſtivers per ruble, and the exchange between Amſterdam and London, at 33s. 9½d. Flemiſh, per pound ſterling? *Anſ.* 2792 rubles, 46+copecs.

4. A merchant in London receives from Peterſburgh, per bill of Exchange, 4675 rubles, 46 copecs, exchange 122 per rix-dollar, of 50 ſtivers Florence, and 34s. 7d. Flemiſh per pound ſter.: how many pounds ſterling will the bill amount to? *Anſ.* 923l. 9s. 1¼d.

## *Poland and Pruſſia.*

In theſe countries they keep their accounts in Florins, Pennins, Groſs. Dantzick and Konnigſberg moſtly exchange with London at ſo many pence per florin, par 2d.; or by way of Amſterdam and Hamburgh; 270 Poliſh groſch, being=1l. rix-dollar Banco of Hamburgh.

| | | | |
|---|---|---|---|
| 18 Phenningen | make one | Groſch. | |
| 3 Groſch | ——— | Dilkins. | |
| 2 Dilkins | ——— | Sexter. | |
| 3 Sexters | ——— | Tymph. | |
| 7½ Groſch | ——— | Arch d'Halbert. | |
| 4 Arch d'Halberts | ——— | Florin or Guilder. | |
| 3 Florins or Guilders | ——— | Current | Dollar. |
| 4 Guilders | ——— | Specie | |

Change 4684 florins into ſterling money, 270 groſch poli per pound Flemiſh, and 34s. 4d. per £. ſterling.

*Anſ.* 203l. 3s. 5⅔d.

Change 390l. ſterling into florins, the exchange being 30s. 4d. per £. ſterling and 270 groſch poli per £. Flemiſh.

*Anſ.* 5850 florins.

## *Sweden.*

They keep their accounts at Stockholm in copper dollars and oarts, or in ſilver dollars, and exchange by the copper; the exchange here is ſubject to great varieties, but is uſually from 46 to 50 copper dollars per £. ſterling.

| | |
|---|---|
| 8 Pennins | 1 Runſtychen. |
| 3 Runſtychens | 1 Silver or Whitton. |
| 8 Stivers | 1 Marc. |
| 10 Stivers and 2 Runſtychens, or 32 Runſtychens | 1 Copper Dollar = value 6$\frac{2}{5}$d. |
| 3 Copper Dol. and 32 ſtivers or 96 Runſtych. or 4 Marcs | 1 Silver Dollar. |
| 24 Marcs | 1 Copper rix-dollar = value 5s. |

Exchanges with Britain at ſo many copper dollars per £. ſterling; with Holland and Hamburgh, at ſo many ſilver marks, per rix-dollar of theſe places.

### EXAMPLES.

1. A merchant in London is indebted to a merchant in Stockholm 146l. 17s 6d. Britiſh money; how many copper dollars will be equal to this debt, exchange at 48½ dollars per pound ſterling? *Anſ.* 7123$\frac{7}{16}$.

2. Change 546l. 19s. 6½d. ſterling into ſilver dollars, exchange at 49½ copper dollars per pound ſterling.
*Ans.* 9025 doll. 11 r. 5 pen.

3. Change 674l. 11s. 6d. ſterling into Marcs, &c. exchange at 48 copper dollars per pound ſterling.
*Anſ.* 43172 marcs, 6 ru. 9 pen.

4. In 11676 ſilver dollars, 18 run. 7 pen. how many £. ſterling, exchange at 49 copper dollars per pound ſterling?
*Anſ.* 714l. 17s. 4½d.

## *Norway and Denmark.*

In Copenhagen, Elſineur, Bergin, &c. accounts are kept in rix-dollars, Daniſh marks and ſchillings.

1 Mark = 16 Schillings.
6 Marks = 1 Rix-dollar.
4 Marks = 1 Oort.
11 Marks = 1 Ducat of gold = value 9s. 2d.

Exchange with Britain from 4 to 5 rix-dollars, per £. ſterl.
——— with Holland about 110 rix-dollars, Daniſh per 100 rix-dollars Banco.
——— with Hamburg, about 250 marcs lubs, per 100 rix-dollars Daniſh.

1. In 8734 rix-dollars, 5 marks, how many pounds ſterl. exchange at 48d. per doll.? *Ans.* 1746l. 19s. 4d.

2. A cargo of timber was ſold at Dublin for 824l. 14s. 6d. ſterling, what will this money reckon for at Bergin, exchange at 50d. ſterling per rix-dollar?

*Anſ.* 3958 rix-doll. 4 m. $1\frac{7}{15}$ ſch.

3. A merchant in London owes to a merchant in Elſineur 867l. 15s. Britiſh money, what will this money reckon at Elſineur, exchange at 48d. ſterling per rix-dollar?

*Anſ.* 4338 rix-doll. 4 marks. 8 ſchill.

## *America and the Weſt-Indies.*

In exchange with the colonies in America and the Weſt Indies, accounts are kept and the money divided as in England and Ireland; their money is called currency: the ſcarcity of caſh obliges them to ſubſtitute paper currency for real money, to carry on their trade, which being ſubject to caſualties, a very great diſcount in their negociation in purchaſing bills of exchange, muſt be the conſequence.

Sometimes they exchange in dollars and cents, 100 cents being equal to a dollar, the par of which is 4s. 6d. ſterling; but bills of exchange are commonly negociated in pounds, ſhillings and pence; the ſums of the invoices are often made out both ways.

As the Americans and Weſt Indians exchange by their imaginary monies, it is neceſſary to enumerate the real coin they have circulating among them, which are French crowns, Spaniſh dollars, or pieces of $\frac{8}{8}$, &c.

It is computed that 5l. Engliſh is equal to 7l. Weſt India currency; then 5 : 7 :: 100 to 140 thereof, 100l. Engliſh equal to 140l. Weſt India currency, but the courſe of Exchange is generally from 25 to 50 per cent, and ſometimes exceeds 60, but in America, it fluctuates with very conſiderable irregularity, according to the ſcarcity of money.

The method of reducing ſterling money into American or Weſt India currency, and the contrary, is the ſame as reducing Engliſh into Iriſh and Iriſh into Engliſh money.

### EXAMPLES.

1. Philadelphia is indebted to London 1474l. 16s. what ſterling money may London reckon to be remitted, when the exchange is 64 per cent? *Anſ.* 899l. 5s $4\frac{1}{4}$d. $\frac{184}{328}$

2. London receives a bill of exchange from Philadelphia for 943l. 17s. 5¾d. ſterling; for how much currency was London indebted, exchange being 64 per cent?

*Anſ.* 1547l. 18s. 1½d.

3. London conſigned to Jamaica goods per invoice, amounting to 640l. 16s. 9d. which are ſold for 987l. 12s. currency, what ſterl. ought the factor to remit, deducting 5 per cent for commiſſion and charges, and what does London gain per cent upon the adventure, ſuppoſing the exchange at 30 per cent?

*Anſ.* remitted 731l. 11s. 1¾d.½; gained 14l. 3s. 1¼d.⅓ per cent.

4. Jamaica is indebted to London 1470l. 12s. 8d. ſterling, with how much currency will London be credited at Jamaica, when the exchange is 36½ per cent? *Anſ.* 2007l. 8s. 3⅕d.

5. A merchant in balancing his books, finds that there is due to him

| | £. | s. | d. | | | |
|---|---|---|---|---|---|---|
| In Maryland, | 1575 | 14 | 0 | Currency | 45 | per cent exchange. |
| Virginia, | 3479 | 10 | 0 | Do. | 36½ | —— |
| Philadelphia, | 3419 | 19 | 0 | Do. | 27 | —— |
| Jamaica, | 4547 | 18 | 6 | Do. | 38¾ | —— |

What money ſtands in in the ſterling column of his Leger?

| | | £. | s. | d. |
|---|---|---|---|---|
| *Anſwer.* | Maryland, | 1086 | 13 | 9½+ |
| | Virginia, | 2549 | 1 | $8\frac{20}{51}$ |
| | Philadelphia, | 2692 | 17 | $5\frac{97}{127}$ |
| | Jamaica, | 3277 | 15 | $8\frac{4}{37}$ |

### *Riga.*

In Riga, Revel, Narva, &c. accounts are generally kept in rix-dollar albertuſes, and groſs.

1 Florin = 30 Groſs.
1 Rix-dollar Albertus = 90 Groſs or 3 Florins.

The value of the rix-dollar is about 4s. 6d. but exchange is generally tranſacted by the way of Holland or Hamburgh, by allowing from 90 to 96 rix-dollars albertus, for 100 rix-dollars of thoſe places.

### *Turkey.*

Accounts are kept in Lions, Dollars, and Aſpers, as follows,

An aſper = $\frac{7}{16}$d. Gold chekeen = 243 aſpers = 8s. 10,3125d. Venitian chekeen = 280 aſpers = 10s. 2½d. An Hungarian chekeen from 240 to 250 aſpers, a piece of eight from 100 to 110 aſpers. A German dollar from 110 to 120 aſpers.

### *Perſia.*

10 Coz = 1 Shabee or Shahie. 2 Shahies = 1 mamooda.
4 Sha = 1 Abaſhee. 50 Abaſhees = 1 Tomond.
A Coz value ⅔d. A Shahie value 4d. A Mamooda value 8d.
An Abaſhee value 1s. 4d. A Tomond value 3l. 6s. 8d. ſterl.

### *Arabia.*

80 Caveers = 1 Dollar. 4 Caveers = 3 Comaſhes.
100 Comaſhes = 1 Sequin. 9 Sequins = 1 Tomond.
A Caveer value ,675d. A Comaſh value 9d.
A Sequin value 7s. 6d. A Tomond value 3l. 7s. 6d.

### *Morocco.*

24 Fluces = 1 Blauquil. 4 Blauquils = 1 Ounce.
54 Blauquils = 1 Xequin. A Blaquil value 2d. A Fluce value ,8333d. A Xequin value 9s. An Ounce value 8d.

### *Eaſt Indies, Madraſs.*

8 Pieces = 1 Sanam. 10 Sanams = 1 Rupee.
36 Sanams = 1 Pagoda. 4 Pagodas = 1 Gold Rupee.
A Sanam value 3d. A Piece value ,375d. A Rupee value 2s. 6d.
A Pagoda from 8s. 9d. to 9s. A Gold Rupee from 35s. to 36s.
A lack of Rupees = 100000, and is generally 12500l.

The Rupee is the ſame in all the European ſettlements in India, but is differently divided and multiplied in different places.

### *China.*

10 Caſh = 1 Candarine. 10 Candarines = 1 Mace.
10 Mace = 1 Tale. A Tale value 6s. 8d. 1 Caſh value ,08d.
A Candarine value ,8d. A Mace value 8d.

They have no coined pieces here, the above being only imaginary, but pay in gold and ſilver by weight, which they denominate by talents and meaſure.

### *Japan.*

They have a piece of gold value 6l. 11s. 3d. another of 2l. 3s. 9d. and ſeveral pieces of different weights; they have but one piece of copper value 3½ Dutch ſtivers, their gold is worth 3l. 3s. per ounce, and their ſilver 5s. per ounce.

### *Egypt.*

Accounts are kept by the natives in aſpers of $\frac{7}{16}$d. and mudens of 1 $\frac{5}{16}$d. each, and by the Chriſtians in dollars, divided into 30 aſpers; their gold coins are the Sultany, Xeriff, and Sequin eſtimated from 9s. 4d. to 9s. 6d. each.

### *United States of America.*

To reduce the currency of one State into that of another.

I. To reduce the currency of New-Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, and Virginia.

1. Into New York and North Carolina currency.

*Rule.* To the given ſum add $\frac{1}{3}$ thereof.

2. Into Penſylvania, New Jerſey, Delaware, and Maryland.

*Rule.* To the given ſum add $\frac{1}{4}$ thereof.

3. Into South Carolina and Georgia.

*Rule.* From the given ſum ſubtract $\frac{2}{9}$ thereof; or multiply it by 7, and divide the product by 9.

EXAMPLE.

Reduce 362l. 16s. 6d. of the currency of Virginia, into that of New York, Penſylvania, and Georgia. *Anſ.* New York, 483l. 14s. 8d. Penſylvania, 453l. 10s. Georgia, 283l. 3s. 6$\frac{2}{3}$d.

II. To reduce the currency of New York and North Carolina.

1. Into New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, and Virginia.

*Rule.* From the given ſum deduct $\frac{1}{4}$ thereof.

2. Into Penſylvania, New Jerſey, Delaware, and Maryland.

*Rule.* From the given ſum deduct $\frac{1}{16}$ thereof.

3. Into South Carolina and Georgia.

*Rule.* To the given ſum add $\frac{1}{6}$ thereof, then take half the total: or, multiply by 7, and divide by 12.

EXAMPLE.

Reduce 156l. 10s. 6d. of the currency of New York to that of Virginia, Penſylvania, and Georgia. *Anſ.* Virginia, 117l. 7s. 10$\frac{1}{2}$d. Penſylvania, 146l. 14s. 10$\frac{1}{8}$d. Georgia, 91l. 6s. 1$\frac{1}{2}$d.

III. To reduce the currency of Penſylvania, New Jerſey, Delaware, and Maryland.

1. Into New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, and Virginia.

*Rule.* From the given ſum deduct $\frac{1}{5}$ thereof.

2. Into New York and North Carolina.

*Rule.* To the given ſum add $\frac{1}{15}$ thereof.

3. Into South Carolina and Georgia.

*Rule.* Multiply the given ſum by $3\frac{1}{5}$ and divide the product by 5: or, multiply it by 28, and divide the product by 45.

EXAMPLE.

Reduce 216l. 7s. 6d. of the currency of Maryland to that of Virginia, New York, and Georgia. *Anſ.* Virginia, 173l. 2s. New York, 230l. 16s. Georgia, 134l. 12s. 8d.

IV. To reduce the currency of South Carolina and Georgia.

1. Into New Hampſhire, Maſſachuſetts, Rhode Iſland, Connecticut, and Virginia.

*Rule.* To the given ſum add $\frac{2}{7}$ thereof: or, multiply it by 9, and divide the product by 7.

2. Into Penſylvania, New Jerſey, Delaware, and Maryland.

*Rule.* Multiply the given ſum by $11\frac{1}{4}$, and divide the product by 7: or, multiply it by 45, and divide by 28.

3. Into New York and North Carolina.

*Rule.* From the given ſum ſubtract $\frac{1}{7}$ thereof, and double the remainder: or, multiply by 12 and divide the product by 7.

EXAMPLE.

Reduce 256l. 15s. 6d. of the currency of Georgia to that of Virginia, Penſylvania, and New York. *Anſ.* Virginia, 330l. 2s. $9\frac{3}{7}$d. Penſylvania, 412l. 13s. $5\frac{1}{14}$d. New York, 440l. 3s. $8\frac{4}{7}$d.

The par of exchange between ſterling money and the currency of Penſylvania is nearly as 21 to 36, or as 7 to 12, conſequently 100l. ſterling is $171\frac{3}{7}$l. Penſylvania currency; that is $71\frac{3}{7}$ advance, and not $66\frac{2}{3}$, as is vulgarly ſuppoſed. From what has been ſaid, the par of exchange between ſterling money and the currency of all other ſtates may be eaſily aſcertained.

## *Federal Money:*

The Congreſs of the United States, in Auguſt, 1786, reſolved that the diviſions of the Federal money ſhould be as follows.

10 Milles = 1 Cent.
10 Cents = 1 Dime.
10 Dimes = 1 Dollar.
10 Dollars = 1 Eagle.

Hence the Federal money being decimally divided, every operation therein will be performed decimally, and ſhould this money ever actually exiſt, the rules hereafter delivered in decimals may be conſulted, it may alſo be remarked that this diviſion of money, is the moſt natural and convenient poſſible, and the operations therein eaſier than any other.

## *Arbitration of Exchange.*

As the courſe of exchange between one nation and another is almoſt continually varying, either by riſing or falling, from the variations in the circumſtances and balance of trade; ſo the deſign of Arbitration is to remit or draw upon foreign places in ſuch a manner, as ſhall, if poſſible, turn out the moſt profitable.

Arbitration is generally divided into two parts, Simple and Compound:

### 1. *Simple Arbitration.*

In ſimple arbitration, when the exchanges among three places only is concerned, the par of arbitration or arbitrated price is ſuch a rate of exchange between two places, as ſhall be in proportion with the rates aſſigned between each of them and a third place. After this par of arbitration is computed, by comparing it with the preſent courſe of exchange, a perſon can judge which way to remit or draw to the moſt advantage, and determine what that advantage ſhall be.

#### EXAMPLE.

1. If the exchange between London and Amſterdam be 33s. 9d. per pound ſterl. and the exchange between London and Paris be 32d. per crown; required the par of arbitration between Amſterdam and Paris?

*Anſ.* 54d. Flemiſh per crown.

£. s. d. d. d.
*Solution.* If 1 : 33 9 :: 32 to 54 per crown.

2. If the exchange between Amſterdam and Paris be 34d. per crown, and the exchange between Amſterdam and London 33s. 9d. per pound ſterling; required the arbitrated price between Paris and London? *Ans.* 32d. per crown.

3. If the exchange from Paris and London is 32d. per crown, and to Amſterdam 414d. per pound ſterling, and if by advice from Holland or France the courſe of exchange

between Amſterdam and Paris be fallen to 52d. per crown; what may be gained per cent by London* drawing on the one place and remitting to the other?

*Anſ.* By drawing on Paris and remitting to Amſterdam, may be gained per cent 5l. 15s. $1\frac{33}{139}$d.

4. London is indebted to Peterſburgh 5000 rubles; now the exchange between Peterſburgh and England is at 50d. per ruble; between Peterſburgh and Holland, at 90d. per ruble; between Holland and England, at 36s. 4d. which is the more advantageous method for London to be drawn upon?

*Anſ.* London may gain 9l. 11s. 1½d. by making payment by way of Holland.

5. A merchant in London had 6000 guilders in the Bank at Amſterdam; was offered 22d. ſterling a piece for them, but not liking the offer, he indorſes a bill for the whole to his factor at Paris, who ſoon brought the money to France, by exchanging at 5s. 5d. Flemiſh per crown, he allows the factor ½ per cent commiſſion for his trouble, and then draws upon him for the whole, exchange at 32d. per crown; how much was this better than the offer of 22d. per guilder?

*Anſ.* 28l. 18s. $2\frac{2}{11}$d.

6. When the exchange between London and Dublin is at 6 per cent advance, and the exchange between London and Hamburgh at 33s. 4d. groſs per pound ſterling; what then is a mark worth at Dublin? *Anſ.* $20\frac{44}{123}$d.

7. When the exchange between London and Liſbon is at 5s. 6d. per milrea, and the exchange between London and Cork is at 8¾ per cent advance for Cork; how much is a milrea worth at Cork? *Ans.* 5s. $11\frac{31}{40}$d

8. What is the exchange between London and Ireland, if for 314l. 5s. Iriſh there is paid in England 300l.? *Ans.* 4¾.

---

* Did a univerſal par of the courſe of exchange ſubſiſt between the trading nations of the world, there would be no advantage in the tranſmiſſion of money in a circuitous manner, throughout them, as the direct conveyance would be preferred; but as the courſe is always fluctuating, and is often in ſuch a ſituation, that money may be drawn or remitted to more advantage when it paſſes through ſeveral places, of which the foregoing queſtions are ſufficient inſtances: from this irregularity, occaſion has been taken to form a circulation of payments by means of bills through different places, in order to which, a compariſon, as mentioned in the definition, muſt be made: for example, If the exchange between London and Amſterdam be 35s. Flemiſh per pound ſterling, and that between Paris and Amſterdam be 52½d. Flem. per crown; then it would be, as the Flemiſh money, which is = a pound ſterling is to the Flemiſh money in a French crown :: ſo is 1 £. to the correſponding

9. How much is the exchange, when for 60l. Engliſh there is paid 62l. 2s. Iriſh ? *Ans.* 3½ per cent.

10. London was ordered to remit 800 crowns to Paris, at 31⅜d. per crown, and to draw upon Amſterdam for the value, at 36s. 9d. per ; but when the order came up, bills on Paris were at 31⅝ ; what muſt be the rate of exchange with Amſterdam, to compenſate the advance on the remittance ?

*Anſw.* 32s. 6d.

11. When 2443 marks, 9⅐ ſtivers lubs are equal to 200l. 10s. ſterling, how much is the exchange between London and Hamburgh ? *Anſ.* 32s. 6d. groſs per pound ſterling.

12. London exchanges on Amſterdam at 34s. 9d. per £. ſterling, and on Liſbon at 5s. 5⅝d. per milrea ; what is the arbitrated price between Amſterdam and Liſbon ?

*Ans.* $45\frac{39}{74}$ Flemiſh per cruſadoe.

13. How many ſhillings Flemiſh are equal to 20s. ſterl. when a guilder or 20 ſtivers are equal to 2 ſhillings ?

*Anſ.* 33⅓ Flemiſh.

14. A merchant in London pays 142l. 13s. 9¾d. for 1037¾ French crowns, what then was the rate of exchange ?

*Anſ.* 33d. per crown.

15. When 150l. 10s. Engliſh is equal to 252l. 1s. 9d. Flemiſh, how much then is a pound ſterling worth at Holland ? *Anſ.* 33s. 6d. Flemiſh.

When a factor has orders from his employer to remit a certain ſum of money to a place, provided he can do it at a certain rate of exchange, and at the ſame time, to draw for his reimburſement upon ſome other place, at a certain rate. To find whether the advantage in performing one part of the commiſſion will be ſufficient to compenſate for the eventual loſs

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reſponding rate of a crown ; that is, As 35s. Flemiſh : 52½d. :: 1l. : 30d. Thus, the value of a crown or ecu, if circulated through Holland, would be 30d. which is called the arbitrated value thereof. Now if the rate of exchange between London and Paris was alſo 30d. no advantage could be gained by circulation ; but if the courſe be higher, ſuppoſe 32d. a dealer in exchange may remit 1000 crowns to Paris by way of London, and draw directly upon Paris, and then he will gain thereby 2d. per crown ; if, on the other hand, the courſe is lower than the arbitrated value (ſuppoſe 28d. per crown) he may remit the money immediately to Paris, and draw for it by way of Holland, by which he will alſo gain 2d. per crown.

In order to carry on this trade with profit, the difference between the real and arbitrated courſe of Exchange, muſt at leaſt be ſufficient to pay the commiſſion charged by the different factors, through whoſe hands the money circulates, which is generally one-half per cent to each.

loſs upon the other, ſo that in caſe he finds the negociation would be to his employer's loſs, he may write for new orders, or wait for a more favourable courſe.

*Rule.* Let the price of exchange aſſigned for remitting, be the firſt term; the price aſſigned for drawing, the ſecond term; and the real courſe of exchange, at which the remittance can be made, the third term of a ſtating in the Rule of Three; then will the fourth term be the price at which the draft ought to be made. By comparing the reſult with the price of bills upon that place on which the draft is to be made, the factor will eaſily ſee whether he ought to obey the order or not.

## EXAMPLES.

16. A factor at London receives an order to remit to Venice 1000 ducats, at 4s. each, and for the purpoſe of putting himſelf in caſh, by drawing on Spain, at 3s. 2d. per piaſtre; when this order came to hand, bills for Venice were at 50d.; at what price muſt London draw upon Spain to compenſate the advance on the remittance to Venice by the riſe of exchange?

*Solution.* Here the price aſſigned for drawing is at 4s. remittance 3s. 2d. and the real courſe at which the remittance can be made, is 50d.; then, As 48 : 38 : : 50 to 39$\frac{7}{12}$ per piaſtre in advance.*

---

* Since the factor, agreeable to the courſe of exchange at the time he received the order, is obliged to remit at 50d. which is 1d. advance upon each ducat, he muſt draw at 39$\frac{7}{12}$d. for his piaſtre; otherwiſe his employer will be at a loſs in whatever he draws below that price; but if bills on Spain had riſen more in proportion than bills on Venice; for inſtance, at 40d. per piaſtre, the order could be performed with 416d. advantage. For 1ſt, 1000 ducats at 4s.=200l. and this at 38d.=1263.158 piaſtres; that is, the factor in London muſt ſell 1263.158 piaſtres of his conſtituent's before he is put in caſh, to make the remittance of 1000 crowns; but when he finds that the courſe of exchange hath varied ſince his employer's intelligence, he may firſt find what money the purchaſe of 1000 ducats will require at 50d. and ſuppoſing the courſe with Spain had riſen to 40d. he can ſave of the money ariſing from the ſale of the piaſtres, 2l. 10s. 3d.

## *Compound Arbitration.*

Compound Arbitration reſpects the caſes in which the exchanges between three, four, or more places are concerned. A perſon who knows at what rate he can draw or remit directly, and alſo hath advice of the courſe of exchange in foreign places, may trace out a path for circulating his money, through more or fewer of ſuch places, and alſo in ſuch order as to make a benefit of his ſkill and credit; and herein lies the great art of ſuch negociations.

*Rule.* 1. Diſtinguiſh the given courſes into antecedents and conſequents, placing the former in one column on the left, and the latter, in another on the right, by way of equation.

2. The 1ſt antecedent, and laſt conſequent to which an antecedent is required, muſt always be of the ſame name.

3. The 2d antecedent muſt be of the ſame name with the firſt conſequent, and the 3d antecedent with the 2d conſequent, and ſo throughout the whole.

4. Equal numbers in both columns, may be exterminated, and commenſurable ones abridged until they are in their loweſt terms, as in abridging Fractions.

5. If any of the numbers be mixed, exterminate the fractions, by multiplying both ſides of the equation by the denominators.

6. The continual product of the conſequents divided by the continual product of the antecedents, will quote the anſwer.

### EXAMPLES.

17. When the exchange between Dublin and London is at 8 per cent; London and Holland at 36 ſchillings, 2 den. groſs per pound ſterling, what is the amount in Holland of 250l. Iriſh, remitted from Dublin by way of London, allowing the merchant in London ½ per cent for commiſſion?

*Anſ.* 2499 guilders.

*antecedent.* *conſequent.*
Iriſh, 108=100 London.
London, 1=434 Den. groſs.
How many in Holland. } 0=250 Iriſh.

108 10850000 Carried over.

10850000 Brought over.
$\frac{1}{2}$ is $\frac{1}{200}$ 54250 commiſ.

108)10795750( 4|0)99960+

the *Anſ.* 2499 guilders

18. When the exchange between London and Cork is at 8$\frac{3}{4}$ per cent advance for Cork, and the exchange between London and Rotterdam is 33s. 4d. Flemiſh per pound ſterl, how much is 12d. of Cork worth at Rotterdam? *Anſ.* 9$\frac{17}{87}$ ſtivers.

19. London remits 500l. ſterling to Cadiz, at 40d. ſterl. per piece of 8, to be remitted thence to Leghorn, at 125 pieces 8 per 100 Leghorn dollars, of 6 livres; thence to Venice, at 90 Leghorn per 100 ducats de banco; thence to Lyons, at 66 ducats de Banco per 100 ecus tour; thence to London at 33d. ſterling per ecu tour; how much will this remittance produce in London? *Anſ.* 555$\frac{4}{9}$.

20. How much would a French livre of 20 ſols. be worth at Cork, when the exchange between London and Paris, or any other part of France, is at 34d. per crown, and the exchange between London and Cork at 5 per cent advance for Cork? *Anſ.* 11$\frac{9}{19}$d.

21. When the exchange between London and Hamburgh is at 35 ſch. groſs per pound ſterling; between Hamburgh and Venice, at 90 den. groſs per duc. banco; what is the exchange per duc. banco between London and Venice? *Anſ.* 51$\frac{3}{7}$d. ſterling.

22. London remits to Spain 1000l. by way of Holland, at 36s. Flemiſh banco, per pound ſterling; thence to via Paris at 54 grotes per ecu; thence via Venice at 100 crowns per 60 ducats; thence to Spain at 360 marvadies per ducat; how many piaſtres of 272 marvadies will be received in Spain, and how much ſterling will they get credit for, exchange at 42$\frac{1}{2}$d. ſterling, per piaſtre? *Anſ.* 1124l. 19s. 3$\frac{1}{4}$d. ſterl.

23. The exchange between London and Liſbon being at 70d. ſterling per milrea; between Liſbon and Hamburgh at 50 deniers groſs per cruſade of 40 reas; what is the rate per pound ſterling between London and Hamburgh compounded of the aforeſaid rates groſs, or 35 ſch. 8$\frac{4}{7}$ den.? *Anſ.* 428$\frac{4}{7}$d.

Foreign weights, meaſures, &c. may be computed and arbitrated in the ſame manner.

24. Suppoſe 8 lb. of Amſterdam are worth 6 lb. of Bruges, and 5 lb. of Bruges equal to 4 lb. of Ghent, and 12 lb. of Ghent equal to 10 lb. of Hambro'; how many lbs. of Hambro' are equal to 720 lb. of Amſterdam?

| | *antecedent.* | *conſequent.* |
|---|---|---|
| Amſterdam | 8= | 6 Bruges. |
| Bruges | 5= | 4 Ghent. |
| Ghent | 12= | 10 Hambro'. |
| How many lb. Hambro'* | = | 720 Amſterdam. |

$$\frac{6\times\cancel{4}\times\cancel{10}\times700}{\cancel{8}\times\cancel{5}\times 12}=\frac{6\times720}{12}=6\times60=360 \text{ the } \textit{Anſ.}$$

25. If 12 yards of cloth coſt as much as 15 yards of cambrick; and 9 yards of cambrick are worth 10 yards of ſattinett, how many yards of ſattinett will pay for 54 yards of cloth? *Anſ.* 75.

26. If 6 lb. of cloves coſt as much as 12 lb. of pepper; and 5 lb. of cinnamon coſt as much as 8 lb. of cloves, and 6 lb. of cinnamon coſt 16s. how much is then the coſt of 1 lb. of pepper? *Anſ.* 10d.

27. If 45 yards in London are equal to 60 ells in Holland, and 750 ells in Holland coſt 390l. Flemiſh, how many pounds Flemiſh will 600 yards in London come to? *Anſ.* 416l.

28. If 4 lb. of A. are as heavy as 3 lb. of B. and 5 lb. B. as 4 lb. of C. and 6 lb. C. as 5 lb. of D.; how many lbs. of D. are as heavy as 240 lb. of A.? *Anſ.* 120 lb. of D.

29. If 3 yards of cloth coſt as much as 3¾ yards of ratteen, and 4½ yards of ratteen are worth 5 yards of drugget; how many yards of drugget are worth 27 yards of cloth? *Anſ.* 37½ yards.

*END OF THE FIRST VOLUME.*

A

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# ARITHMETIC,

# *&c. &c.*

* * *

## CHAP. I.

### DECIMALS.

### *Introduction to Fractions in general.*

A FRACTION ſuppoſes an unit, or one whole of any thing to be divided into a certain number of equal parts, and is intended to expreſs a part or parts of that unit ſo divided.

And Fractions are of two kind, Vulgar and Decimal.

I. A Vulgar fraction is expreſſed by two numbers placed one above the other with a line drawn between them, thus,

$\left.\begin{array}{l}\underline{3}\text{ Numerator.}\\ 4\text{ Denominator.}\end{array}\right\}$ The denominator, or number placed underneath the line, denotes how many equal parts the thing is ſuppoſed to be divided into; (being only the diviſor in diviſion.) And the numerator, or number placed above the line, ſhews how many of thoſe parts are contained in the fraction: (it being the remainder after diviſion.) In reading fractions, both Vulgar and Decimal, the numerator is firſt mentioned, then the denominator; thus $\frac{3}{4}$ is read, three fourths; $\frac{11}{12}$ is read, eleven twelfths; $\frac{9}{10}$ is read, nine tenths; $\frac{85}{100}$ is read, eighty-five hundredths; and $\frac{38}{745}$ is thirty-eight ſeven hundred and forty-fifths; or, thirty-eight in ſeven hundred and forty-five equal parts. When a whole number ſtands before a fraction, as $27\frac{5}{14}$, it is read, twenty-ſeven and five fourteenths; and $76\frac{14}{100}$ is read, ſeventy-ſix and fourteen hundredths; and $35\frac{120}{231}$ is thirty-five and a hundred and twenty parts in two hundred and thirty-one; &c.

II. A Decimal fraction is an artificial, and very commodious way of ſetting down and expreſſing a natural or vulgar fraction, as a whole number: and whereas the denominators of vulgar fractions are diverſe, the denominators of decimal fractions are always certain: for a decimal fraction hath always for its denominator an unit with as many cyphers annexed to it as there are places or figures in the numerator; and muſt therefore be 10, 100, 1000, 10,000, &c. and hence there is no occaſion for writing the denominator of a decimal; the uſual way of expreſſing a decimal being by ſetting a point before the numerator: thus, the decimal fraction $\frac{25}{100}$ is written .25, its denominator being known to be an unit with two cyphers, from what was ſaid above, becauſe there are two figures in the numerator: thus alſo $\frac{125}{1000}$ is written .125; $\frac{3575}{10000}$ is .3575; and $\frac{75}{1000}$ is written .075; and $\frac{65}{10000}$ is .0065. A whole number with a decimal fraction, as $276\frac{14}{100}$, is thus expreſſed 276.14; and $74\frac{6}{1000}$ thus, 74.006, &c. and when a whole number and a decimal are combined together, it is called a mixt number.

As whole numbers increaſe in a decuple or tenfold proportion, towards the left hand, ſo on the contrary, decimals decreaſe towards the right hand in a decuple proportion, as in the following table.

In Integers.
thus, 1 = one, or unity.
10 = ten, or ten units.
100 = one hundred.
1000 = one thouſand.
10000 = ten thouſand.
100000 = one hundred thouſand.

In Decimals.
vice verſa, .1 = one tenth part of an unit.
.01 = one hundredth part.
.001 = one thouſandth part.
.0001 = one ten thouſandth part.
.00001 = one hundred thouſandth part.
.000001 = one millioneth of an unit, &c.

Hence it appears, that as whole numbers are increaſed in a decuple proportion by affixing cyphers, ſo decimals are decreaſed in the ſame proportion by cyphers being prefixed to them; thus .25, if a cypher be prefixed to it, becomes $\frac{025}{1000}$ or .025; and .125 by prefixing two cyphers, becomes $\frac{00125}{100000}$ or .00125, and therefore, in writing a decimal fraction, whoſe denominator hath more cyphers than there are figures in the numerator, the defect muſt be ſupplied by prefixing a cypher or cyphers thereto, as the caſe may require; thus, ſuppoſe $\frac{19}{1000}$ were to be written down without its denominator; here the denominator having 1 cypher more than there are figures in the numerator, a cypher muſt be prefixed to the numerator, and the decimal expreſſed thus .019.

Again, as cyphers on the left hand of whole numbers do not alter their value, ſo cyphers being annexed to the right hand of Decimals do neither increaſe nor decreaſe the value thereof; for $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$, or .25. Hence then, cyphers may, at any time, be annexed to the right-hand of decimals, at pleaſure, if need be. In all decimal numbers, if the point of diſtinction be removed one place towards the right hand, every figure, and conſequently the whole expreſſion will be increaſed in a ten-fold proportion, as in the following numbers, 3.756, 37.56, 375.6, and 3756, which are each one, 10 times greater than the preceding one. In which proportion alſo, it is manifeſt they decreaſe in value, by removing the decimal point a place to the left hand.

## *Addition of Decimals.*

THE numbers to be added muſt be placed, units under units, and tens under tens, as in whole numbers; and the decimal points muſt ſtand under each other, with the decimals annexed; then add as in common addition, and from the ſum point off ſo many places for decimals as are equal to the greateſt number of decimal places in any of the given added numbers.

### EXAMPLES.

1. Let 7.35, 26.324, 253.4, 46.5485, and 7.58, be added together in one ſum.

7.35
26.324
253.4
46.5485
7.58

*sum.* 341.2025

2. Let 84.72, 856, 745.5, .647, and 78.0052, be added together.

84.72
856.
745.5
.647
78.0052

*sum*. 1764.8722

Theſe examples are ſo plain, that more we think are needleſs.

## *Subtraction of Decimals.*

The ſame directions being obſerved for placing the numbers in Subtraction, as in Addition, it is likewiſe performed the ſame as in whole numbers.

### EXAMPLES.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. | From | .345 | take | .219. | 4. | From | 9.77 | take | 5.999 |
| 2. | — | 4.673 | — | 2.978 | 5. | — | 37.8 | — | .0056 |
| 3. | — | 876.7 | — | 67.46 | 6. | — | .376 | — | .5 |

## *Multiplication of Decimals.**

Multiplication of Decimals differs not from Multiplication of whole numbers, only when the product is found, from the product towards the right cut off as many places for decimals as there are in both factors together.

*Note.*

---

* To prove the truth of the rule; let .9776 and .823 be the numbers to be multiplied; now theſe are equivalent to $\frac{9776}{10000}$ and $\frac{823}{1000}$ whence $\frac{9776}{10000} \times \frac{823}{1000} = \frac{8045648}{10000000} = .8045648$ by the nature of notation, which conſiſts of as many places as there are decimals, that is, of as many places as are in both the numbers, and the ſame is true of any other two numbers whatſoever. Or the reaſon may be thus demonſtrated, why ſuch a number of decimal places muſt be cut off in the product, which will be eaſily deduced from theſe examples: thus, in example 1ſt, it is evident that 3, the whole number in the multiplicand being multiplied by 2 in the multiplier, can produce but 6, viz. $3 \times 2 = 6$; ſo that, of neceſſity, all the other figures in the product muſt be decimal parts, as the rule directs.

Or

*Note.* If there are not in the product ſo many decimal places as there are in the multiplicand and multiplier reckoned together, prefix cyphers to equal that number.

2d. If the number is to be multiplied by 10, 100, 1000, &c. remove the ſeparating point in the multiplicand ſo many places towards the right hand as there are cyphers in the multiplier.

$$\begin{array}{lr}
\text{(i) Multiply} & 3.685 \\
\text{by} & 2.75 \\
\hline
 & 18425 \\
 & 25795\phantom{0} \\
 & 7370\phantom{00} \\
\hline
\textit{product.} & 10.13375 \\
\hline
\end{array}
\qquad
\begin{array}{lr}
\text{(ii) Multiply} & 672.5 \\
\text{by} & .365 \\
\hline
 & 33625 \\
 & 40350\phantom{0} \\
 & 20175\phantom{00} \\
\hline
\textit{product.} & 245.4625 \\
\hline
\end{array}$$

| | | | | | *Answers.* |
|---|---|---|---|---|---|
| 3. | Multiply | 3.024 | by | 2.23. | 6.74352. |
| 4. | —— | 43.23 | — | 32.4. | 1400.652. |
| 5. | —— | 8.04704 | — | .2575. | 2.0721128. |
| 6. | —— | 574.7 | — | .0675. | 387.9225. |
| 7. | —— | .4 | — | .2. | .08. |
| 8. | —— | .047 | — | .046. | .002162. |
| 9. | —— | .000476 | — | .00078. | .00000037128. |
| 10. | —— | .47 | — | .0008. | .000376. |
| 11. | —— | 1.076 | — | 1000. | 1076. |
| 12. | —— | .42768 | — | 10000. | 4276.8. |

*Contraction*

Or thus; from multiplication, ſuppoſe 300 were to be multiplied by 200, their product will be 60000, that is, there will be ſo many cyphers in the product as there are in both factors; now, if inſtead of thoſe cyphers in the factors, the like number of decimal parts were in each, then it follows that there ought to be the ſame number of decimal parts in the product as there were cyphers in the factors.

Again, by Vulgar Fractions, thus, let 43.23 be multiplied by 32.4 and their product will be 1400.652, as in example 4th. Now, $43.23 = 43\frac{23}{100}$ and $32.4 = 32\frac{4}{10}$ which being brought into improper fractions, per rule 3d, Reduction of Fractions will become, viz. $\frac{4323}{100}$ and $\frac{324}{10}$ then $\frac{432}{100} \times \frac{324}{10} = \frac{1400652}{1000}$ per rule 1ſt of Multiplication of Fractions; but $\frac{1400652}{1000} =$ 1400.652, the ſame as before. Any of the foregoing methods, it is preſumed, ſufficiently proves the truth of the rule.

## *Contraction in Multiplication.*

When it happens that the places of Decimals are many in both factors, and conſequently would make a very large decimal in the product in ſuch caſes the work may be contracted to as few places of decimals in the product as you pleaſe; or is ſuitable to your deſign, (three or four places being generally ſufficient) to do which, obſerve the following

*Rule.* Set the unit's place of the multiplier directly under that figure of the decimal part of the multiplicand, whoſe place you would preſerve in the product; then invert, or place all the other figures of the multiplier in a contrary order to the common way.

Laſtly, In multiplying, always begin at the figure of the multiplicand, which ſtands over the figure wherewith you are then multiplying, ſetting down the firſt figure of each particular product directly under one another. But withal take care to ſee what increaſe would ariſe from the multiplying of the next right-hand figures of the multiplicand, which you muſt conſtantly add to the firſt figure in every product; that is, if the product of the next right-hand figure (with as many units added to it as there are tens in the product of the ſecond right-hand figure) be 5, or upwards to 15, you muſt add or carry 1; if it be 15 or upwards to 25, you muſt carry 2; and if 25, or upwards to 35, you muſt carry 3, &c.

### EXAMPLES.

1. Let 8.74694 be multiplied by 4.5276, and let there be only 4 places of decimals retained in the product.

Place them as before directed, and they will ſtand

Thus: { 8.74694 The multiplicand as uſual.
{ 6725.4 The multiplier inverted.

34.9878
4.3735
.1749
612
52

39.6026

Here, as 4 decimal places are to be pointed off in the product, the unit's place of the multiplier, is placed under the 4th decimal place of the multiplicand.

The reaſon of, and how great a part of the work is ſaved by this contraction, will appear from the operation at large.

```
     8.74694
      4.5276
   ----------
     52|48164
    612|2858
   1749|388
  43734|70
 349877|6
 ------------
 39.6026|45544
```

Hence it appears that all the figures on the right-hand of the line are wholly omitted in the above contraction, where the product to 4 decimal places is the ſame with this.

14. Multiply 3.141592 by 52.7438 ſo as to have but 4 decimal places in the product. *Anſ.* 165.6994.

15. Multiply 2.38645 by 8.2175 and leave only 4 decimal places in the product. *Ans.* 19.6107.

16. Multiply 375.13758 by 16.7324, and let there be only one place of decimals in the product. *Ans.* 6276.9

17. Multiply .248264 by .725234 reſerving 6 figures, 5 figures, and 4 figures in the product.
*Anſ.* .180049, .1805, and .1800.

18. Multiply 8634.785 by 843.7527, reſerving only the integers in the product. *Anſ.* 7285699.

## *Diviſion of Decimals.*[*]

1. Diviſion of Decimals differs not from Diviſion of Whole Numbers, only care muſt be taken that there be as many decimal places in the quotient and diviſor as there are in the dividend.

2. The quotient muſt always have ſo many decimal places as the dividend has more than the diviſor.

3. If the diviſor and dividend have both the ſame number of decimal places, the quotient will be a whole number.

---

[*] The reaſon of pointing off as many decimal places in the quotient as thoſe of the dividend exceed the diviſor, will eaſily appear; for ſince the number of decimal places in the dividend is equal to thoſe in the diviſor and quotient taken together, by the nature of Multiplication; it therefore follows that the quotient contains as many as the dividend exceeds the diviſor.

4 If the dividend hath not ſo many decimal places as are in the diviſor, then ſo many cyphers muſt be annexed to the dividend as will make them equal, and the quotient will be a whole number.

5. But when if the diviſion is done the quotient has not ſo many figures as it ſhould have places of decimals, then ſo many cyphers muſt be prefixed as there are places wanting to the left hand of the quotient.

6. When decimal numbers are to be divided by 10, 100, 1000, &c. that is, when the diviſor is an unit with cyphers, diviſion is performed by removing the ſeparating point in the dividend ſo many places towards the left hand as there are cyphers in the diviſor.

### EXAMPLES.

1. Divide 1735·5 by 6.5.

*diviſ. divd. quo.*
6.5)1735·5(267 the *Anſ.*
———
435
———
455
———

| | *Anſwers.* |
|---|---|
| 2. Divide 201.93 by 14. | 14.42+ |
| 3. —— 822 by 24. | 34·25 |
| 4. —— 67 by 287. | ·2334 |
| 5. —— 476.23 by 27. | 17.6381 |
| 6. —— 2754·385 by 960. | 2.8691 |
| 7. —— 2754·385 by 20. | 1377·192 |
| 8. —— 757·35 by 4. | 189·3375 |
| 9. —— 734·6737 by 80. | 9·18342 |
| 10. —— 476.23 by 27. | 17.6381 |
| 11. —— 756.5784 by 23·4. | 32·332 |
| 12. —— 745·3678 by 27·52. | 27.084 |
| 13. —— 113·4 by ·4725. | 240 |
| 14. —— 53·2 by 2·757. | 19·2963 |

## *Contractions in Diviſion.*

There is alſo a commodious way of contracting the work, the converſe to that in page 6, by which much labour is ſaved, eſpecially when the diviſor hath many places of decimal parts in it, which is performed by the following

*Rule.* The value or denomination of the firſt figure in the quotient being determined by the firſt general rule, you may have what number of decimal places you pleaſe, by taking as many of the left hand figures of the diviſor as you intend to have figures in the quotient for the firſt diviſor, and then take as many figures of the dividend as will anſwer them; and in dividing, omit, or point off, one figure of the diviſor at each following operation, ſtill having a due regard to the increaſe [*] which would ariſe from the figure or figures ſo omitted.

### EXAMPLES.

1. Divide 721.17562 by 2.257432 and let there be but 3 places of decimals in the quotient.

```
divisor   dividend  quotient
2.257432)721.17562(319.467
 .....    6772296
          -------
           439460.
           225743
           -------
           213717..
           203169
           -------
            10548...
             9030
            -------
             1518....
             1354
             -------
              168.....
              158
              ---
                6
```

*the common way.*

```
divisor   dividend  quotient
2.257432)721.17562(319.467
         6772296
         -------
          439460|2
          225743|2
          --------
          213717|00
          203168|88
          ---------
           10548|120
            9029|728
           ----------
            1518|3920
            1354|4592
            ----------
             163|93280
             158|02024
             ----------
               5|91256
```

---

[*] In multiplying every figure left out in the diviſor, be careful to carry 1, if it be 5 or upwards to 15; if 15, or upwards to 25, carry 2; if 25, or upwards to 35, carry 3, &c. 2. When there are not ſo many figures in the diviſor as are required to be in the quotient, begin the operation with all the figures, and continue till the number of figures in the diviſor and thoſe remaining to be found in the quotient be equal, after which uſe the Contraction.

2. Divide 5171.59165 by 8.758615, and let four decimal places be in the quotient. *Anſ.* 590.4577.

3. Divide 42.1764 by 314.2543, and let there be five decimal places in the quotient. *Anſ.* .13421.

4. Divide 6109.2674 by 240.649, and let there be only four decimal places in the quotient. *Anſ.* 25.3866.

## *Reduction of Decimals.*

* To reduce a vulgar fraction to a decimal.

*Rule.* Annex cyphers to the numerator, and divide by the denominator; the quotient is the decimal fraction required.

Reduce $\frac{1}{4}+\frac{1}{2}+\frac{3}{4}+\frac{3}{8}+\frac{5}{26}+\frac{11}{14}$ of $\frac{10}{13}+\frac{1}{25}+\frac{3}{192}$ to a decimal ſeparately. *Ans.* .25+.5+.75+.375+.1923076 &c. +.6043956 &c. +.04+.015625.

To reduce coins, weights, meaſures &c. into decimals.

*Rule* 1. Reduce the given money, weights, &c. into the loweſt denomination or name mentioned, for a dividend; then reduce the integer into the ſame denomination for a diviſor, the reſult will be the decimal required.

*Rule* 2.† Write the given denomination or parts orderly under each other, the inferior or leaſt parts being uppermoſt,
let

---

* Let the given vulgar fraction, whoſe decimal expreſſion is required be $\frac{7}{13}$ now ſince every decimal fraction has 10, 100, 1000, &c. for its denominator, and if two fractions are equal, it will be, as the denominator of the one is to its numerator, ſo is the denominator of the other to its numerator; or, they are as 13 : 7 :: 100 &c. : $\frac{7000}{13}=.5384$ the numerator of the decimal required, and is the ſame as by the rule.

The following method of throwing a vulgar fraction, whoſe denominator is a prime number, into a decimal conſiſting of a great number of figures, is given by Mr. Colſon in page 162 of Sir Iſaac Newton's fluxions. EXAMPLE, Let $\frac{1}{29}$ be the fraction which is to be converted into an equivalent decimal, then by dividing in the common way till the remainder becomes a ſingle figure, we have $\frac{1}{29}=.03448\frac{8}{29}$ for the complete quotient, and this equation being multiplied by the numerator 8, will give $\frac{8}{29}=.27584\frac{64}{29}$, or rather $\frac{8}{29}=.27586\frac{6}{29}$: and if this be ſubſtituted inſtead of the fraction in the firſt equation, it will make $\frac{1}{29}=.0344827586\frac{6}{29}$. Again, let this equation be multiplied by 6, and it will give $\frac{6}{29}=.2068965517\frac{7}{29}$; and then by ſubſtituting as before, $\frac{1}{29}=.03448275862068965517\frac{7}{29}$; and ſo on as far as may be thought proper.

† The reaſon of this rule may be explained from the firſt example thus, three farthings is $\frac{3}{4}$ of a penny, which brought to a decimal is .75: conſequently

let theſe be the dividends: againſt each part on the left hand write the number thereof contained in one of its ſuperior; let theſe be diviſors: then beginning with the loweſt denomination with cyphers adding (making a dot between the cyphers and the figure) and divide, writing the quotient of each diviſor as decimal parts on the right hand of the dividend next below it, and let this mixed number be divided by its diviſor, and ſo on till all be finiſhed; the laſt quotient will be the decimal required.

To find the decimal of any number of ſhillings, pence, and farthings by inſpection.

**Rule.* Write half the greateſt even number of ſhillings for the firſt decimal figure, and let the farthings in the given pence and farthings poſſeſs the ſecond and third places, obſerving to encreaſe the ſecond place by 5 if the ſhillings are odd, and the third place by 1 when the farthings exceed 12, and by 2 when they exceed 37.

EXAMPLES.

1. Reduce 8s. 4d. to the decimal of a pound ſterling.

8 4
12

240) 1000 (.4166 &c.
*Anſ.*

400

1600

160

*Or thus,*

---

quently $9\frac{3}{4}$d. may be expreſſed 9.75d.; but 9.75 is $\frac{975}{100}$ of a penny $=\frac{975}{1200}$ of a ſhilling, brought to a decimal is .8125; and therefore 15s. $9\frac{3}{4}$d. may be expreſſed 15.8125s. In like manner 15.8125s. is $\frac{158125}{10000}$ of a ſhilling $=\frac{158125}{200000}$ of a pound, = (by bringing it to a decimal,) to .790625£. as by the rule.

* The demonſtration of this rule is as follows: as ſhillings are ſo many twentieths of a pound, half of them muſt be ſo many tenths, and conſequently take the place of tenths in decimals; but if they are odd, their half will always conſiſt of two figures, the firſt of which will be half the even number next leſs, and the ſecond a 5; and this confirms the rule as far as it reſpects ſhillings. Again, farthings are ſo many 960ths of a pound; and had it happened, that 1000 inſtead of 960 had made a pound, it is plain any number

*Or thus, by the ſecond rule.*

12 | 4
20 | 8.3333 &c.

.4166 &c. *Anſw.*

*Or thus, by the third rule.*

4
4
—
16

8 4
—
.4166 *Anſw.*

2. Reduce 16s. $7\frac{3}{4}$d. to the decimal of a pound ſterl.

16 $7\frac{3}{4}$
12
—
199
4
—
240 ) 799 (.8322916 *Anſw.*

*or thus, 2d rule.*

4 ) 3.00
12 ) 7.75
20 ) 16.64583
*Anſ.* .832291 &c.

*or thus, by the 3d rule,* 16 $7\frac{3}{4}$
—
.832 *Anſ.*

$7\frac{3}{4}$
4
—
32

| | *Anſwers.* |
|---|---|
| 3. Reduce 15s. $9\frac{1}{2}$d. to the decimal of a pound ſterl. | .7895+ |
| 4. Reduce 9s. to the decimal of a pound. | .45 |
| 5. Reduce 12 grains to the decimal of a pound troy. | .002083 |
| 6. Reduce 12 drams to the decimal of a lb. avoirdupois. | .046875 |
| 7. Reduce 2qrs. 14lb. to the decimal of a cwt. | .625 |
| 8. Reduce 2 furlongs to the decimal of a league. | .0833 |
| 9. Reduce 2 quarts 1 pint to the decimal of a gallon. | .625 |
| 10. Reduce 4 gallons 2 quarts of wine to the decimal of a hogſhead. | .071428+ |
| 11. Reduce 52 days to the decimal of a year. | .142465+ |

To find the value of any decimal fraction in money, weight, and meaſure.

*Rule.* Multiply the decimal by the number of parts of the next inferior denomination, cutting off ſo many places for decimals to the right hand as your given decimal conſiſts of; and

number of farthings would have made ſo many thouſandths, and might have taken their place in the decimal accordingly. But 960 encreaſed by $\frac{1}{24}$ part of itſelf is =1000; conſequently any number of farthings, increaſed by their $\frac{1}{24}$, will be an exact decimal expreſſion for them in thouſandths, whence if the number of farthings be more than 12, a $\frac{1}{24}$ part is greater than $\frac{1}{2}$ and therefore 1 muſt be added, and when the number of farthings is more than 37, a $\frac{1}{24}$ part is greater than $1\frac{1}{2}$d. for which 2 muſt be added; and thus the rule is ſhewn to be right.

and thoſe on the left will be integers; then multiply the remaining decimals by the next inferior denomination, and cut off for decimals as before; thus proceed till you have brought it into the leaſt parts of the integer.

EXAMPLES.

12. What is the value of .8322916 of a pound ſterling?

$$\begin{array}{r} .8322916 \\ 20 \\ \hline 16.6458320 \\ 12 \\ \hline 7.7499840 \\ 4 \\ \hline 2.9999360 \end{array}$$

*Anſ.* 16*s.* 7½*d.*+

| | | *Anſwers.* |
|---|---|---|
| 13. | What is the value of .002084 of a lb. troy? | 12.00380grs. |
| 14. | —— ——.046865 lb. avoirdu.? | 12 drams *nearly* |
| 15. | —— —— .625 of a cwt.? | 2 qrs. 14lb. |
| 16. | —— —— .625 of a gallon? | 2quarts 1 pint |
| 17. | —— —— .071428 hhd of wine? | 4gals. 1q.999856 |
| 18. | —— —— —— .0625 barrel of beer? | 2 gals. 2quarts+ |
| 19. | —— —— —— .142465 of a year? | 51.999725days. |
| 20. | —— —— —— .3375 of a £.? | 6s. 9d. |
| 21. | —— —— —— .3375 of a ton? | 6 cwt. 3 qrs. |
| 22. | —— —— .16669 of a yard? | 6 inches+ |
| 23. | —— —— .259 of a league? | 6 furl. 8.640per. |
| 24. | —— —— .05 of an acre? | 8 perches. |
| 25. | —— —— .4765 of a day? | 11h. 26′ 9″ 36‴ |

To find the value of any decimal of a pound ſterling by inſpection.

*Rule.* Double the firſt figure, or place of tenths, for ſhillings, and if the ſecond figure be 5 or more than 5, reckon another ſhilling, then call the figures in the ſecond and third places after 5, ſo many farthings, abating 1 when they are above 12, and 2 when above 37, and the reſult is the anſwer.

EXAMPLES.

26. Find the value of .785l. by inſpection.

14 = *double of 7*
1 *for in the place of tenths*
8¾ *for 35 farthings*
¼ *deduct for the exceſs of 12*

*l.*15 8½*d. Anſ.*

27. Find

| | Anſwers. |
|---|---|
| 27. Find the value of .25 of a pound by inſpection. | 5s. 0d. |
| 28. Find the value of .972916 of a pound by inſpection. | 19s. 5½d. |
| 29. Find the value of .8322 of a pound by inſpection. | 17s. 7½d. |

# CHAP. II.

## BARTER.

IS the changing of one commodity for another, and informs us how to proportion the value of any goods, ſo that neither party may ſuſtain loſs; and if the commodities exchanged are not of equal value, the defect is ſupplied with money.

*Rule.* Find the value of that commodity whoſe quantity is given, then find what quantity of the other at the rate propoſed, you may have for the ſame money, and it gives the anſwer required.

*N. B.* This is evidently performed only by the Rule of Three Direct.

2. If the quantities of both commodities with the rate of ſelling them, be given; to find what quantity of ſome other commodity or money, muſt be given in caſe of an inequality of the amount of the firſt commodities.

*Rule.* Calculate the amount of each of the two given commodities; then their difference is the money or amount of the third commodity to be advanced, whoſe quantity from thence is eaſily found, its rate being given.

3. When in bartering, one commodity is rated above the ready money price; to find the quantity and bartering price of the other commodity.

*Rule.* Say as the ready money price of the one is to its bartering price, ſo is that of the other, to its bartering price; then the quantity of the latter commodity may be found either from the ready money or bartering prices.

The foregoing rules being well conſidered the following queſtions may be eaſily ſolved.

## EXAMPLES.

1. What quantity of chocolate at 4s. per lb. muſt be delivered in barter for 2 cwt. of tea at 9s. per lb.?

2
112
———
224
9
——— *lbs.*
4) 2016 ( 504 *Anſ.*

2. How many yards of ſhalloon may I barter for 80 yards of broad cloth, rating the broad cloth at 15s. 6d. per yard, and ſhalloon at 2s. per? *Anſ.* 620 yds.

3. A. and B. barter, A. has 50 pieces of muſlin worth one with another, 2l. 15s. 6d. per piece, B. has tea worth 8s. 4d. per lb. what quantity of tea is B. to give A. to balance the value of the muſlin? *Anſ.* 333lb. *or* 2cwt. 3qrs. 25lb.

4. Two merchants barter, A. and B. A. has ſugar worth 4l. per cwt. ready money, but in barter he will have 4l. 5s. per cwt. B. has French wine worth 45l. per pipe: how much muſt he advance his wine, to equal the advance of A's ſugar in barter? *Anſwer.* He muſt advance his wine to 2l. 16s 3d. per pipe, ſo that each pipe will be worth 47l. 16s. 3d.

5. How many dozen of candles at 5s. 2d. muſt be delivered in barter for 3cwt. 2qrs. 16lb. of tallow, at 37s. 4d. per cwt? *Ans.* 26¼ dozen.

6. A. hath 608 yards of cloth, worth 14s. per yard, for which B. gives him 125l. 12s. in ready money, and 85cwt. 2qrs. 14lb. of bee's wax; the queſtion is, what did B. reckon his wax at per cwt.? *Ans.* 3l. 10s.

7. A. and B. barter, A. hath 20cwt of prunes at 4d. per lb. ready money, but in barter will have 5d. per lb. B. hath hops worth 32s. per cwt. ready money, what ought B. to rate his hops at in barter, and what quantity muſt be given for the 20cwt. of prunes? *Anſwer.* 23cwt. 1qr. 9⅓lb. and B. riſes his hops to 40s. per cwt.

8. A. and B. barter, A. has 200 yards of ratteen which coſt him 3s. per yard, but in barter would have 3s. 6d. B.

has ſugar at 7½d. per lb. which coſt him but 6d. how many lb. of ſugar muſt be delivered to A. and who made the beſt bargain? *Ans.* B. muſt deliver 1120lb. and B. gets the beſt bargain by 2l.

9. A. and B. barter, A values his goods at 60s. though they are vendable but for 52s. beſides he would have 5 per cent. what muſt B. value a yard of cloth at, which is vendable for 8s. to make the barter equal? *Ans.* 9$\frac{9}{13}$s.

10. A. has currants worth 4d. per lb. but in truck charges 6d. and alſo requires one half in ready money, B. has candles worth 6s. 8d. the dozen, and in barter the honeſt man charges but 7s. ſhould theſe perſons deal together for the value of 20l. how much will A. get of B? *Ans.* The real value of A's currants 13l. 6s. 8d. the value of B's candles 9l. 10s. 4½d. and A. gets of B. 6l. 3s. 9½d.

11. A. lets B. have a hogſhead of ſugar, wt. 18 cwt. worth 31s. for 42s. per cwt. ⅓ of which he is to pay in cloth, B. has paper worth 14s. per ream, which it is agreed ſhall bear no more than 15s. 6d. at that rate, and barter for the reſt; how ſtood the account? *Ans.* A. advanced his ſugar 9l. 18s. and received in cloth 12l. 12s. and in paper 25l. 4s. B's advance on his paper 2l. 8s. 9¼d.+$\frac{51}{82}$ and 7l. 9s. 2½d.+$\frac{26}{31}$ in A's favour.

12. A. has kerſey at 4l. 5s. a piece ready money, in barter they are charged by him at 5l. 6s. each piece, and ½ of that required down; B. has flax at 3d. per lb. how much ought he to rate it in truck, not to be hurt by the extortion of A?

*Solution.* Firſt 5l. 6s.—4l. 5s.=1l. 1s. the gain per piece, and 5l. 6s.÷2=2l. 13s. required down; alſo 4l. 5s.—2l. 13s.=1l. 12s. the value of the remaining, then 1l. 12s.+1l. 1s.=2l. 13s. If 1l. 12s. : 2l. 13s. :: 3d. to 4¾d.+$\frac{412}{128}$ the lb. *Answer.*

13. A. and B. would barter, A. hath 40 yards of cloth at 7s. 4d. per yard, B. has 28½lb. of tea at 11s. 6d. per lb. which of them pay the balance, and how much?

*Answer,* A muſt pay 1l. 14s. 5d.

14. A. has a quantity of pepper, wt. nt. 1600lb. at 17d. per lb. which he barters with B. for two ſorts of goods, the one at 5d. and the other at 8d. per lb. and to have ⅓ in ready money, and of each ſort of goods an equal quantity, how many lb. of each muſt he receive, and how much in money? *Ans.* 1394$\frac{36}{39}$lb. and 37l. 15s. 6⅔d.

15. Two merchants, A. and B. barter thus, A. hath 86 yards of broad cloth worth 9s. 2d. per yard ready money, but in barter he will have 11s. per yard; B. hath ſhalloon worth 2s. 1d. ready money, it is required to find how many yards of the ſhalloon B. muſt give to A. for his cloth, to make his gain in the barter, equal to that of A.'s?

*Anſ* $378\frac{2}{5}$ yards.

16. A. advances his goods from 19s. 6d. to 22s. but is willing to give 4 months' time for the payment, B puts his off at 16d. with 3 months' time for the payment; now admitting the proportional advancement in this latter price, and time to the former to find the ready value of B's goods?

*Ans.* $14\frac{3}{5}\frac{4}{7}$d.

17. I have merchandize which coſt 30s. per cwt. but puts it off at 34s. A. has ſome which coſt him 25s. per cwt. but puts them off at 28s. per cwt. if we barter thus, how much per cent do our bargain differ?

*Ans.* I have the better by $1\frac{1}{3}$.

## CHAP. III.

## LOSS AND GAIN.

LOSS AND GAIN is a rule, by which we diſcover the gain or loſs by any parcel of goods, and ſo inſtructs us how to raiſe or fall the price of any commodity, in ſuch proportion, that neither our gain may be ſo exorbitant as to injure our cuſtomers, nor our loſs ſo great as to impoveriſh ourſelves, which is generally ſo much per ſhilling, per cent, &c.[*]

To make this uſeful rule as plain as poſſible, we ſhall divide it into the eight following caſes, with ſuitable notes and remarks.

---

[*] So that when the gain is 10 per cent; it it to be underſtood, that when 100l., 100s., &c. have been laid out on goods, 110l, 110s., &c. had been recovered by the ſales; and, in the ſame manner, if 100l. were laid out and 90 received, then it is underſtood, that 10 per cent was loſt by ſuch goods.

### *Caſe* I.

Given the buying price and ſelling price, to find the gain or loſs upon the whole, or proportionable part thereof,

*Rule.* If the ſelling price exceeds the buying, the difference ſhews the gain upon the whole; but if the buying exceeds the ſelling price, their difference ſhews the loſs.

#### EXAMPLES.

1. A merchant bought 2 bags of hops, each weighing 1 cwt. 2 qrs. 10 lb for which 8l. 10s. per cwt. is paid, and ſold them by retail in his ſhop at 6d. per lb. what did he gain or loſe?

*Solution.*

1 cwt. 2 qrs. 10 lb.×2=3 cwt. 0 qrs. 20 lb. at 8l. 10s. per cwt.=27l. 0s. 4$\frac{2}{7}$d. and 3cwt. 0qrs. 20lb. at 6d. per lb.= 8l. 18s.

| | *£.* | *s.* | *d.* | |
|---|---|---|---|---|
| | 27 | 0 | 4$\frac{2}{7}$ | |
| | 8 | 18 | 0 | |
| *Anſ.* | *£.*18 | 2 | 4$\frac{2}{7}$ | loſs. |

2. If 840 eggs are bought at the rate of 10 a penny; and 240 more at the rate of 8 a penny, and ſold all at the rate of 18 for 2d. I demand the profit or loſs? *Ans.* 6d. profit.

3. Bought 18 cwt. of butter, at 28s. per cwt. which I ſell out at the rate of 3½d. the lb. what is the profit upon the whole? *Anſ.* 4l. 4s.

4. If I bought 436 yards of cloth, at 8s. 6d. per yard, and ſold it at the rate of 10s. 4d. per yard; required the whole profit? *Anſ.* 39l. 19s. 4d.

5. If 980 lb. of tea be bought for 61l. 5s. and ſold for 69l. 8s. 4d. required the profit upon each lb.? *Anſ.* 2d.

6. Paid 69l. for 1 tun of ſteel, which is retailed at 6d. per lb. what is the profit or loſs upon 14 tun, at that rate? *Anſ.* 182l. loſs.

### *Caſe* II.

Given the buying and ſelling prices, to find the gain or loſs per cent.

*Rule.* As the buying price is to the difference between the buying and ſelling : : ſo is 100l. (100s. &c.) to the gain or loſs per cent.

EXAMPLES.

### EXAMPLES.

7. Suppoſe I bought tea at 9d. per lb. and ſell it again at 15d. per lb. what do I gain per cent? *Anſ.* 66l. 13s. 4d.

The difference is 6d. per lb. profit: then,
As 9d. : 6 : : 100l. to 66l. 13s. 4d.

8. Suppoſe I buy tea at 17s. 6d. per lb. and ſell it again at 21s. 6d. what do I gain per cent (or in laying out 100l.)?
*Ans.* 22l. 17s. 1 5/7d.

9. A merchant bought a puncheon of rum for 66l. 13s. 4d.; it contained 150 gallons, he retailed it at 12s. 6d. per gallon; whether did he gain or loſe, and at what rate per cent?

*Anſ.* He gained by the whole 27l. 1s. 8d. per cent 40 5/8.

### *Caſe* III.

When the buying price and the propoſed rate per cent profit are given, to find the advanced price. added.

*Rule.* As 100 : is to 100 more the given rate : : ſo is the buying price to the advanced price.

### EXAMPLES.

10. If a tun of tallow coſt 35l. and is ſold at the rate of 10 per cent profit, I demand how much it was ſold for?

*Solution.*

As 100 : 110 : : 35 to 38l. 10s. the *Anſ.*

11. At what rate muſt I ſell 1 cwt. of madder, which coſt 3l. 15s. to gain 20 per cent? *Anſ.* 4l. 10s.

12. Bought 40 gallons of brandy, at 3s. per gallon, but by accident, 6 gallons of it were loſt, at what rate muſt I ſell the remainder per gallon, to gain upon the whole prime coſt 10 per cent? *Anſ.* 3s. 10½d.+

13. A merchant in Dublin receives from his correſpondent in London 314 cwt. 2 qrs. 17 lb. of hops, the amount of invoice in Iriſh currency is 6l. 10s. per cwt. the duty, charges, &c. at the quay, amount to 1l. 0s. 10d. per cwt. I demand at what rate I muſt ſell it per cwt. to gain 20 per cent on the whole? *Anſ.* 9l. 1s.

### *Caſe* IV.

Given the advanced price and rate per cent gained, to find the prime or firſt coſt.

*Rule.* As 100 + the given rate : is to 100 : : ſo is the advanced price to the firſt coſt.

#### EXAMPLES.

15. When I ſell my cloth at 5s. per yard, I gain 8 per cent, I demand what a yard coſt me at that rate?

*Solution.*

As 108 : 100 : : 5 : 4s. 7½d.+

16. When I ſell a hogſhead of wine for 20l. I gain 15 per cent, I demand what the hogſhead coſt me?

*Ans.* 17l. 7s. 9¾d.+

17. If when I ſell goods for 13½d. the lb. I find I gain 35 per cent, I demand what the goods were bought for?

*Anſ.* 10d.

18. A merchant ſells a parcel of jewels, which coſt him 250l. ready money, for 550l. payable at the end of 6 months; the queſtion is, (this ſecurity being good) what was his gain ready money, rebate at the rate of 6 per cent per annum?

*Ans.* 300l.

### *Case* V.

Given the reduced price and loſs per cent, to find the firſt coſt.

*Rule.* As 100 leſs the loſs : is to 100 : : ſo is the reduced price to the firſt coſt.

#### EXAMPLES.

19. If on the ſale of rum, at 7s. 6d. per gallon, there was a loſs of 30 per cent, I demand the firſt coſt?

*Solution.*

As 70 : 100 : : 7½s. to 10s. 8$\frac{4}{7}$d.

20. Suppoſe I ſold cloth at 6s. 2¼d. per yard, and find that I was at a loſs of 17½ per cent; I demand the firſt coſt?

*Anſ.* 7s. 6d.

### *Case* VI.

By having the given price of a commodity, the rate per cent thereon gain or loſs, and advanced or reduced price of the ſame commodity, to find the rate per cent.

*Rule.* With the given rate and price find the firſt coſt, then ſay, as the firſt coſt is to the advanced or reduced price : : ſo is 100 more or leſs the required rate, according as the price is advanced or reduced.

*Or thus*, As the price, whoſe rate is known, is to that whoſe rate is required : : ſo is 100 more or leſs to the given rate : to 100 more or leſs, the required rate.

EXAMPLES.

21. Having ſold a yard of cloth for 11s. 6d. I gained at the rate of 15 per cent; but if I had ſold it for 12s. what ſhould I gain per cent?

*Solution.*

115 : 100 : : 11s. 6d. to 10s. then from 12s. take 10s. remains 2s. then, As 10s. : 2s. : : 100 to 20s. the anſwer required.

22. A merchant ſold certain merchandizes, at 8s. per lb. upon which he gained 15 per cent; what will be his gain when he ſells the ſame goods at 9s. the lb.? *Anſ.* $29\frac{3}{8}$.

23. Suppoſe a merchant bought a parcel of merchandizes for 457l. 9s. by which he cleared 18 per cent; the purchaſer ſold them immediately for 500l.; what would be the merchant's gain, had he gone to the ſame market?

*Anſ.* 28l. 19s. $6\frac{3054}{9149}$d.

24. Sold linen at 13d. per yard; in the houſe of a neighbour, who bought on the ſame terms with me, the ſame linen, ſold for 14d.; now ſuppoſing my profit to be $12\frac{1}{3}$ per cent; what is his? *Anſ.* 20l. 19s. $5\frac{9}{13}$d.

25. A merchant ſold certain goods at 2s. 10d. per lb. on 3 months' credit, he found that he had gained 25 per cent per annum; what did the goods coſt him per lb.?

*Anſ.* 2s. 8d. firſt coſt.

26. A merchant ſells certain goods, at 2s. 8d. per lb. ready money, for how much may he ſell it a pound, on 3 months' credit, to gain 25 per cent per annum?

*Ans.* 2s. 10d. ſelling price.

27. Bought a tun of wine for 50l. ready money, and ſold it for 54l. 10s. payable in 8 * months' time; what is my gain thereon per cent per annum? *Anſ.* $13\frac{1}{2}$ per cent per ann.

28. Having

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* All queſtions in Loſs and Gain, wherein time is concerned, may be ſolved by the following proportions—To find the rate per cent per annum:

As

28. Having bought goods for 18l. and ſold them immediately for 25l. on 4 months' credit, what rate per cent per annum is gained?

*Solution.*

Here 4 m.$=\frac{1}{3}$ of a year$=$time and gain$=$7l. As $\frac{1}{3}\times\frac{18}{1}$ : 100 :: 7 or, As $\frac{18}{3}$ : 100 :: 7 to $116\frac{2}{3}-100=16\frac{2}{3}$ per cent per annum, *Anſ.*

29. Upon the ſame goods (at 4 months' credit) which coſt me 18l. my gain was $16\frac{2}{3}$ per cent per annum; what price did I ſell it at? *Anſ.* 25l.

30. Upon the ſale of goods for 25l. which coſt me 18l. $16\frac{2}{3}$ per cent per annum was gained; what length of time credit did I give? *Anſ.* 4 months.

31. A merchant ſells goods at 4 months' credit, for 25l. thereby gaining $16\frac{2}{3}$ per cent per annum, at what price did he ſell it? *Anſ.* 18l.

The five following queſtions are wrong ſolved by their reſpective authors, which is the reaſon of our inſerting them here; the error conſiſts in ſtating the queſtion, by making the gain or loſs of 100 the ſecond term, inſtead of its amount.

32. If by ſelling cloth at 5s. per ell, I gain 8 per cent, what ſhall I gain per cent if I ſell the ell at 6s. 3d.? (*Webſter*, 2d ed. p. 32.)

*Solution.*

If 5s. : 108 :: 6s. 3d. : 135, then $135-100=35$ *Mr. Webſter's* anſwer is 10.

33. At 5s. per doben I gain 7l. 10s. per cent, how much ſhall I gain per cent if I ſell the doben at 5s. 9d. (*Stonehouſe*, 2d ed. p. 103.)

*Solution.*

As the product of the prime coſt and the given time : is to 100 :: ſo is the gain or loſs ſuſtained in the given time : to the required rate per cent per annum. To find the prime coſt:

*Rule.* As the given rate per cent per annum : is to the gain or loſs ſuſtained in the given time :: ſo is 100 to the product of the firſt coſt and time, divdie this product by the time, the quotient gives the firſt coſt.

Given the prime coſt, the time and rate per cent per annum, to find the gain or loſs.

*Rule.* As 100 : is to the product of the firſt coſt and time :: ſo is the given rate per cent per annum : to the given gain or loſs on the propoſed time.

*Solution.*

As 5s. : 107l. 10s. :: 5s. 9d. : 123l. 12s. 6d. then 123l. 12s. 6d.—100=23l. 12s. 6d. the *Anſ.* *Mr. Stonehouſe's* anſwer is 8l. 12s. 6d.

34. A Mancheſter tradeſman going to a fair, ſold fuſtians for 11s. 6d. the end, wherein was gained 15l. per cent, but ſeeing no other tradeſman had ſo good, raiſed them at the latter end of the fair to 12s. the end; I demand what he gained per cent by this laſt ſale? (*Hill*, page 289).

*Solution.*

If 11s. 6d. : 115 :: 12s. : 120 per cent, therefore 120—100=20 the *Anſ.* *Mr. Hill's* 15l. 13s. 0$\frac{1}{2}$d. $\frac{2}{23}$.

35. Suppoſe I ſell 500 deals at 15d. per piece and at 9 per cent loſs, what do I loſe by the whole quantity? (*Dilworth*, 2d ed. p. 73.)

*Solution.*

500 at 1s. 3d.=31l. 5s. 100—9=91 As 91l. : 100l. :: 31l. 5s. : 34l. 6s. 9$\frac{1}{4}$d. $\frac{6}{187}$ amt. Therefore, 34l. 6s. 9$\frac{1}{4}$d. $\frac{3}{91}$—31l. 5s.=3l. 1s. 9$\frac{1}{4}$d. $\frac{1}{9}$ the *Ans.* *Mr. Dilworth* makes it 2l. 16s. 3d.

36. Suppoſe I ſell 1 cwt. of hops for 6l. 15s. and gain 25l. per cent, what would have been the gain per cent, if I had ſold them for 8l. per cwt.? *Walkingham*, 3d ed. p. 70.

*Solution.*

As 6l. 15s. : 125 :: 8l. : 148l. 2s. 11$\frac{1}{2}$d.$\frac{2}{9}$ amount per cent; then, 148l. 2s. 11$\frac{1}{2}$d.$\frac{2}{9}$—100=48l. 2s. 11$\frac{1}{2}$d.$\frac{2}{9}$, the *Ans.* *Mr. Walkingham*'s anſwer is 29l. 12s. 7$\frac{8}{9}$d.

*Queſtions*

The gain or loſs, the firſt coſt, and the rate per cent per annum being given to find the time.

*Rule.* As the rate per cent per annum : is to 100 :: ſo is the gain or loſs : to the product of the time and firſt coſt, divide this product by the firſt coſt, the quotient gives the required time.

*Demonſtration of the foregoing Rules:*—Let T=the time in years or fractions of a year; P=the prime coſt; R=rate per cent per annum; G=gain or loſs in the given time; then, T : 1 :: G : $\frac{G}{T}$=the gain or loſs for one year; alſo, P : 100 :: $\frac{G}{T}$ : $\frac{100\times G}{P\times T}$ but 100×G=R×P×T by turning this equation into an analogy, we have As P×T : G :: 100 : R, whence the rate per cent is manifeſt, and R : 100 :: G : P×T ∴ $\frac{P\times T}{T}$=P,

Q. E. D.

### *Queſtions for Exerciſe.*

37. Received from London 11300 lbs. of pewter, which coſt 706l. 5s. Engliſh money; the charges here are 10l. 6s. 8d.; at what rate muſt I ſell it a lb. to gain 15 per cent, having remitted the price of the pewter, when the exchange between London and Dublin was at 5 per cent?

*Ans.* 1s. $6\frac{42430}{45[illegible]00}$d.

38. A merchant in Dublin receives from Liſbon 180 caſks of raiſins, each weighing 80lb. trucks them againſt other merchandizes, at 28s. per cwt. and finds to have made 25 per cent profit; I demand what each caſk ſtood him in?

*Ans.* 16s.

39. If by ſelling hops at 3l. 10s. per cwt. the planter clears 30 per cent; what was his gain per cent when the ſame goods ſold for 4l. 5s.? *Anſ.* 57l. 17s. 1½d. $\frac{6}{7}$.

40. A merchant ſends to Barbadoes 300 firkins of butter, wt. nt. 175 cwt. 2 qr. 19 lb. at 18s. 8d. per. cwt.; pays for duty and other charges, 11l. 0s. 10d.; his correſpondent at Barbadoes ſells the butter at 6d. per lb. (weight as above) pays for duty, &c. 24l. 17s. 1d. takes 5 per cent for his commiſſion; I demand the loſs or gain, if 135l. Barbadoes is worth but 100l. in Ireland? *Anſ.* 152l. 14s. 5$\frac{1}{2}\frac{9}{7}$d. profit.

41. Sold a repeating watch for 52l. 10s. and by ſo doing, loſt 17 per cent, whereas, I ought, in dealing, to have cleared 20 per cent; then how much was it ſold under the juſt value? *Ans.* 23l. 8s. 0¼d. $\frac{49}{83}$.

## CHAP. IV.

### SINGLE FELLOWSHIP;

### *Or Fellowſhip without Time.*[*]

IS when two or more perſons join their ſtocks and trade together; to determine each perſon's particular ſhare of the gain

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[*] 1. The Contractions in the Rule of Three are the beſt for working queſtions in this rule; becauſe the two firſt terms of all the ſtatings being the ſame, there will be had a conſtant multiplier or diviſor for every third term.

2. When two or more of the terms to which all the required ones muſt be proportionable, are equal, ſo many operations will be as there are equalities.

3. That

gain or loſs, in proportion to his principal part in the ſtock.—Obſerve the following

*Rule.* As the ſum of the ſeveral ſtocks : is to the gain or loſs : : ſo is each perſon's ſhare in the ſtock to his ſhare of the gain or loſs. *Or*, divide the whole gain or loſs by the total ſtock, and multiply the quotient by each man's particular part of the ſtock, and the ſeveral products will be each man's particular part of the gain or loſs reſpectively. By this rule are adjuſted the gain or loſs, or charges of merchants in company, the effects of bankrupts, legacies, in caſe of a deficiency in aſſets, &c.

*Proof.* Add all the ſhares together; if the ſum is equal to the whole gain or loſs, the work is right.

EXAMPLES.

1. Two merchants, A. and B., join in partnerſhip: A. lays in 80l., B. 60l., and they gain 28l. what is each man's ſhare of the gain? *Anſ.* A. 16l. and B. 12l.

$$80+60=140 \text{ their ſtock; then,}$$

$$\text{As } 140 : 28 :: \begin{cases} 80 : 16 \text{ A.'s gain.} \\ 60 : 12 \text{ B.'s gain.} \end{cases}$$

2. A. and B. in trading, gained 364l.; A. put into ſtock 600l. and B. 800l.; what is each perſon's ſhare of the profit? *Anſ.* A. 156l. and B. 208l.

3. A bankrupt owes to A. 500l.; to B. 900l. and his effects amount to no more than 1100l.; how much is each perſon's ſhare? *Anſ.* A. $392\frac{6}{7}$l. and B. $707\frac{1}{7}$l.

4. Divide 120l. between three perſons, ſo that their ſhares ſhall be to each other as 1, 2, and 3 reſpectively? *Anſ.* 20, 40, and 60.

5. Suppoſe the money and effects of a bankrupt amounted to 2420l. 17s. 6d. and is indebted as follows: to A. 1000l. to B. 640l.; to C. 900l.; and to D. 842l. 16s.; how muſt his effects be divided amongſt them? that is, what muſt each have?

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3. That the gain or loſs in this rule is in proportion to the reſpective ſtocks, is evident; for as the time the ſtocks are in trade is the ſame, the gains or loſſes muſt be regulated by the ſtocks; ſo if a perſon put in half the ſtock, he is entitled to half the gain, or liable to pay half the loſs; and in general, whatever part of the ſtock he contributes to, ſuch is the part of the gain or loſs which will fall to his ſhare.

*Anſ.* A. 715l. 12s. 10d., B. 458l. 0s. $2\frac{1}{3}$d., C. 644l. 1s. 6d., D. 603l. 2s $10\frac{1}{4}$d.; remainders, A.'s 45504, B.'s 45360; C.'s 360; D.'s 44088.

6. A. and B. venturing equal ſums of money, cleared by joint trade 154l. by agreement, A. was to have 8 per cent, (becauſe he ſpent his time in the execution of his object) and B. was to have only 5 per cent, what was allowed for his trouble? *Ans.* 35l. 10s. $9\frac{3}{13}$d.

7. Three merchants, A. B. and C. freight with 340 tun of wine, whereof A. loaded 110 tuns, B. 91, and C. the reſt, in a ſtorm the ſeamen were obliged to throw over board 85 tuns, how much muſt each ſuſtain of the loſs?

*Anſ.* A. $27\frac{1}{2}$, B. $22\frac{51}{68}$, and C. $34\frac{51}{68}$.

8. Three merchants joined 2640l. in company; now A. gets three times as much as C. and B. twice as much as C. how much is each to get?

*Anſ.* A. 1320l. B. 880l. and C. 440l.

9. A bankrupt's effects amount to 2080l. and he is indebted to A. 1440l. to B. 1920l. and to C. 2880l. how much of the effects belong to each of them?

*Anſ.* A. muſt have 480l. B. 640l. and C. 960l.

10. Three merchants, A. B. and C. have entered into partnerſhip, the two firſt put in a ſum yet unknown, and C. 585l. the gain or loſs 232l. 10s. whereof A's part is 61l. 10s. and B's 73l. 10s. to find the ſtock of A. and B. and gain or loſs of C. *Anſ.* Stock of A. is 396l. of B. 441l. and the gain or loſs of C. 97l. 10s.

11. A father portioned off his three daughters with 5600l. for every 13l. with Pamela he gave 17l. with Amelia, and 15l. to Charlotte, how much was the portion of each?

*Anſ.* Pamela 1617l. 15s. $6\frac{2}{3}$d. of Amelia 2115l. 11s. $1\frac{1}{3}$d. and of Charlotte 1866l. 13s. 4d.

CHAP.

# CHAP. V.

## DOUBLE FELLOWSHIP;

### *Or Fellowſhip with Time.*

WHICH is ſo called from proportioning the gain or loſs among partners according to the ſtock and time of each.

The rule of partition to be given in this caſe, is founded upon this principle, viz. that when ſtocks continue unequal times in company, the partners' ſhares of the gain or loſs ſhould be ſo proportioned to each other, as thoſe ſums of intereſt which at any rate per cent per annum, might be gained by the ſeveral ſtocks, during the times of their continuance in the common ſtock.

* *Rule.* Make all the ſtocks of one denomination, as alſo the times; then multiply each partner's ſtock by his time, and add theſe products together, then proceed as in Single Fellowſhip.

#### EXAMPLES.

1. Three merchants, A. B. and C. enter into partnerſhip, thus A. puts into the ſtock 240l. for 4 months, B. 120l. for 6 months, and C. 200l. for 8 months, with this joint ſtock they traffic, and gain 260l. it is required to find each perſon's ſhare of the gain proportionable to his ſtock and time of employing it?

$$\textit{Solution.}\quad \begin{array}{r} £.\\ 240\\ 4\\ \hline 960 \end{array}\qquad \begin{array}{r} £.\\ 120\\ 6\\ \hline 720 \end{array}\qquad \left.\begin{array}{r} £.\\ 200\\ 8\\ \hline 1600 \end{array}\right\}\qquad \begin{array}{r} £.\\ 960\\ 720\\ 1600\\ \hline 3280 \end{array}$$

$$\textit{As}\ \begin{array}{c} £.\\ 3280 \end{array} : \begin{array}{c} £.\\ 260 \end{array} :: \left\{\begin{array}{rcrrr} £. & & £. & s. & d.\\ 960 & : & 76 & 1 & 11\frac{1}{4}\\ 720 & : & 57 & 1 & 5\frac{1}{2}\\ 1600 & : & 126 & 16 & 7\frac{1}{4}\\ \cline{3-5} & & \multicolumn{3}{l}{} \end{array}\right.$$

$$\textit{Proof}\ £\ 260\quad 0\quad 0$$

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* Mr. Ward has given an analytical inveſtigation of this rule. Mr. Malcolm has given the reaſon of it in a manner evident enough, but probably the moſt general and elegant manner of its proof is thus—When the times are equal, the ſhares of the gain or loſs are evidently as the ſtocks, as in ſingle fellowſhip, and when the ſtocks are equal, the ſhares are as the times, therefore when neither are equal, the ſhares muſt be in the compound ratio of both, that is as their products.

2. A. B. and C. rent a piece of land, for which they pay 40l. per annum, A. puts in 60 oxen for 4 months, B. 40 oxen for 5 months, and C. 30 oxen for the remainder of the year; what muſt each perſon pay of ſaid rent?

|  | £. | s. | d. |  |  |  |
|---|---|---|---|---|---|---|
| *Anſ.* | 18 | 2 | 3 | *remd.* | $\frac{9}{53}$ | *A's.* |
|  | 15 | 1 | 10½ | *remd.* | $\frac{34}{53}$ | *B's.* |
|  | 6 | 15 | 10 | *remd.* | $\frac{10}{53}$ | *C's.* |

3. Two merchants enter into partnerſhip, A. puts into ſtock 2000l. for 4 months, and B. puts in 1000l. for 2 months, they gained by trade 250l. what is the ſhare of each? *Ans.* A. gained 200l. and B. 50l.

4. Three merchants, A. B. and C. made a ſtock together, A. put 120l. for 11 months, B. put in 144l. for 15 months, and C. put in 460l. for 9 months, and they gained by trade 269l. 19s. I demand each man's ſhare of the gain, according to the ſum put in and time of continuance in company? *Ans.* A's ſhare of the gain 46l. 15s. 5¼d. B's 76l. 10s. 5d. $\frac{15}{127}$, and C's 146l. 13s. 3d. $\frac{99}{127}$.

5. Three merchants, A. B. and C. in partnerſhip together for a year, put into one common ſtock as follows, viz. A. put in 400l. and at 6 months' end withdraws 200l. B. put in 360 and at 7 months' end 100l. more, but at the end of 9 months he takes out 120l. C. put in 190l. and at 8 months' end 110l. more, but at the end of 10 months he takes out 100l. they gain 460l. what is each man's ſhare? *Ans.* A's 126l. 10s. 5d. remainder 920, B's 193l. 18s. 3d. remainder 552. and C's 109l. 11s. 3½d. remainder 644.

6. A. B. and C. in company, A. put in his ſhare of the ſtock for 5 months, and laid claim to $\frac{1}{5}$ of the profit; B. put in his for 8 months; C. advanced 400l. for 7 months, and required on the balance $\frac{2}{3}$ of the gain, the ſtock of the other adventurers is ſought. *Ans.* A's ſtock 168l. and B's ſtock 70l.

7. A ſhip's company take a prize, value 4000l. which they agree to divide amongſt them according to their pay and time they have been on board; now the officers and midſhipmen have been on board 4 months, and the ſailors 3 months; the officers have 50s. a month, the midſhipmen 40s. and the ſailors 28s. moreover there are 4 officers, 8

midſhipmen, and 120 ſailors? I demand what each perſon's ſhare is of the ſaid prize? *Ans.* an officer 65l. 15s. $9\frac{1}{4}$d. a midſhipman 52l. 12s. $7\frac{1}{2}$d. and a ſailor 27l. 12s. $7\frac{1}{2}$d. *Note.* Fractions are rejected.

8. Four perſons hired a boat for 19s. to go 24 miles, but being come within 9 miles of the place where they intended to go, there came 2 perſons who deſired to come into the boat, with condition to pay proportionally; tell me how much each of them is to pay?

*miles. m. m. m. s. m. s. d.*

*Solution.* $24-9=15$ *then as* $24 : 19 :: 15 : 11\ 10\frac{1}{2}$ the ſum the four firſt perſons are to pay before they meet the other two, then 19s.—11s. $10\frac{1}{2}$d.=7s. $1\frac{1}{2}$d. ∴ as 6 perſons : 7s. $1\frac{1}{2}$d. :: 2 perſons : 2s. $4\frac{1}{2}$d. the ſum the two laſt perſons are to pay; and the four firſt perſons are to pay 11s. $10\frac{1}{2}$d.+4s. 9d.=16s. $7\frac{1}{2}$d. for 19 miles, therefore 16s. $7\frac{1}{2}$d.+2s. $4\frac{1}{2}$d.=19s. proof.

*Or thus,*

$$\begin{array}{rrrl}
\text{m.} & & & \qquad\qquad\qquad\qquad\quad \text{s.} \\
24 & 9 & 96 & \text{As } 114 : 19 :: \left\{\begin{array}{rcr} 96 & : & 16 \\ 18 & : & \underline{3} \end{array}\right\} \text{Ans.} \\
\underline{4} & \underline{2} & \underline{18} & \\
96 & 18 & 114 & \qquad\qquad\qquad\qquad\quad 19 \text{ proof.}
\end{array}$$

9. Four perſons hired a coach to go 50 miles for 40s. now when they were gone 20 miles, they met 2 perſons who deſired to come into the coach, with condition to pay proportionably, how much is each of them to pay?

*Solution.* As $50 : 40 :: 20 : 16$ then as 6 perſons : $40-16=24 :: 2 : 8$ which is the money the laſt perſons ſhould pay, then $40-8=32$ the money paid by the four firſt perſons ∴ $32+8=40$ *Proof.*

*Or thus.*

$$\begin{array}{rrl}
 & & \qquad\qquad\qquad\qquad\qquad\qquad \text{s. d.} \\
30 & 50 & \\
\underline{2} & \underline{4} & \therefore \text{As } 260 : 40 :: \left\{\begin{array}{rcrl} 60 & : & 9 & 2\frac{10}{13} \\ 260 & : & 30 & 9\frac{3}{13} \end{array}\right. \text{Anſ.} \\
60 & 200 & \qquad\qquad\qquad\qquad\qquad \overline{\text{40ſ. 0 Proof.}} \\
 & \underline{60} & \\
 & 260 &
\end{array}$$

We have taken the two laſt queſtions from Voſter's Arithmetic, and have given our own ſolution and his, and leave the reader to judge which of the two is the more equitable.

CHAP.

# CHAP. VI.

## ALLIGATION.

ALLIGATION is either Medial, Alternate, Partial, or Total.

### *Alligation Medial,*

Is when the price and quantities of ſeveral ſimples are given to be mixed, to find the mean price of that mixture.

*Rule.** As the whole compoſition : is to its total value :: ſo is any part of the the compoſition : to its mean price.

*Proof.* Find the value of the whole mixture at the mean rate, and if it agrees with the total of the ſeveral quantities at their reſpective prices the work is right.

EXAMPLES.

1. A malſter mixes 40 buſhels worth 5s. per buſhel, with 72 buſhels at 3s. per buſhel, and 80 at 2s. per buſhel; I demand what a buſhel of this mixture is worth? *Anſ.* 3s.

40 × 5 = 200
72 × 3 = 216 *buſhels. s. buſh. s.*
80 × 2 = 160 Then, As 192 : 576 :: 1 : 3 *Anſ.*
192 576

2. A vintner mixes 214 gallons of canary at 7s. 6d. per gallon, with 416 gallons at 5s. 4d. with 312 gallons at 9s. 5d. with 145 gallons at 8s. 7d. and 255 gallons at 9s. 2d. I demand what is this mixture worth per gallon? *Anſ.* 7*s.* 8*d.*+

3. A tobacconiſt would mix 40lb. of tobacco at 9d. per lb. with 6lb. at 12d. with 40lb. at 18d. and with 12lb. at 2s. per lb; the queſtion is what a pound of this mixture is worth? *Anſ.* 1*s.* $4\frac{3}{13}$*d.*

4. A farmer

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* The truth of this rule is too evident to need a demonſtration. *Note.* If an ounce or any other quantity of pure gold be reduced into 24 equal parts, theſe parts are called carats: but gold is often mixed with ſome baſer metal, which is called alloy, and the mixture is ſaid to be ſo many carats fine, according to the proportion of pure gold contained in it; thus if 22 carats of pure gold, and 2 of alloy are mixed together, then this gold is ſaid to be 22 carats fine. 2d. If any of the ſimples be of little or no value with reſpect to the reſt, its rate is ſuppoſed to be nothing, as water mixed with wine, and alloy with gold.

4. A farmer would mix 5 buſhels of wheat at 6s. per buſhel, with 12 buſhels of rye at 4s. per buſhel, 8 buſhels of beans at 5s. and with 18 buſhels of barley at 2s. 6d. the buſhel; it is required to know the price of 1 buſhel of this mixture? *Anſ.* 3s. $9\frac{1}{2}$*d. nearly.*

5. A goldſmith melts 8lb. $5\frac{1}{2}$oz. of gold bullion of 14 carats fine, with 12lb. $8\frac{1}{4}$oz. of 18 carats fine: how many carats fine is this mixture? *Ans.* $16\frac{51}{125}$.

6. A refiner melts 10lb. of gold of 20 carats fine with 16lb. of 18 carats fine, how much alloy muſt be put to it to make it 22 carats fine? *Ans.* Not fine enough by $3\frac{3}{13}$ carats, ſo that no alloy muſt be put to it, but more gold.

## *Alligation Alternate;*

Is when the rates of ſeveral commodities are given, to find ſuch quantities of them as, being mixed together, ſhall bear a mean price.

*Rule.** 1. The rates (if not already) muſt all be reduced to one denomination. 2. Set down the rates, or prices, in a column under one another, and the mean on the left hand. 3. Connect together the ſeveral rates, ſo that every one leſs than the mean be linked with ſome one greater, and every greater with one leſs, or with as many as you pleaſe, each leſs than the mean rate. 4. Take the difference between each rate and the mean rate, and ſet them down alternately, and if only one difference ſtand againſt any rate, it will be the quantity belonging to the ſaid rate; but if there are ſeveral, then their ſum will be the quantity, which quantities are the anſwers to thoſe rates againſt which they ſtand.

EXAMPLES.

7. A merchant hath ſpices, ſome at 9d. the lb. ſome at 12d. ſome at 24d. and ſome at 30d.; how much of each ſort muſt he take that he may ſell a lb. for 20d.?

20

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* It appears from the rule, that many queſtions of this caſe will admit of various anſwers, but from an algebraic proceſs, it appears that they will admit of an infinite variety of anſwers: after one or more anſwers are found by the rule, as many more as you pleaſe are found by increaſing or decreaſing the quantities in any proportion, or by only increaſing or decreaſing the any one or more ſingle pair of yoke-fellows in any proportion, and leaving the other rates as they are: but as that anſwer is commonly deſired which gives the rates in the leaſt integers poſſible, and thoſe the neareſt to each other, we have to each of the following queſtions ſet down ſuch anſwers as we found by linking the rates together the moſt poſſible, and then where no limitation was propoſed, dividing the reſulting quantities by the common meaſure.

20 { 9 — 10lb. at 9d.
12 — 4lb. at 12d.
24 — 8lb. at 24d.
30 — 11lb. at 30d.

*Or thus*, 20 { 9 — 4lb. at 9d.
12 — 10lb. at 12d.
24 — 11lb. at 24d.
30 — 8lb. at 30d.

8. A merchant hath wines at 24d. the quart, ſherry at 16d. the quart, and malaga at 12d. the quart; how much of each ſort muſt be taken to ſell a quart for 18d.? *Anſ.* 8 quarts at 24d. 6 quarts at 16d. and 6 quarts at 12d.

9. A grocer would mix a quantity of ſugar at 10d. with other ſugars at 7½d. 5d. and 4½d. per lb. intending to make up a quantity worth 6d. per lb. in what proportion is he to take of thoſe ſugars? *Ans.* 1lb. at 10d. 1½lb. at 7½d. 4lb. at 5d. and ½lb. at 4½d.

10. A merchant would mix wines at 17s. 18s. and 22s. per gallon, ſo that the mixture may be worth 20s. the gallon? *Ans.* 2 gallons at 17s. 2 at 18s. and 5 at 22s.

11. A goldſmith has gold of 17, 18, 22, and 24 carats fine, how much muſt he take of each to make it 21 carats fine? *Ans.* 3 of 17, 1 of 18, 3 of 22, and 4 of 24 carats.

*Note.* Sometimes one or more of the ingredients, and ſometimes the whole compoſition is limited to a certain quantity; which we divide into the three following caſes or limitations.

*Limitation 1ſt.* When the whole compoſition is limited to a certain quantity, and that quantity is not found from the method of linking and taking the differences, then you may augment or diminiſh the quantity of each ingredient in the ſame proportion as the given quantity is greater or leſs than the total quantity found from the linking, by ſaying, as the total quantity ſo found is to the given quantity, ſo is the quantity of each ingredient, found by linking, to the required quantity of each.

### EXAMPLES.

12. How much wine at 4s. at 5s. at 5s. 6d. and at 6s. per gallon, muſt be mixed together, to form a compoſition of 18 gallons worth 5s. 4d. per gallon? *Ans.* 3 gallons at 4s. 3 gallons at 5s. 6 gallons at 5s. 6d. and 6 gallons at 6s.

13. How much gold of 15, of 17, of 18, and of 22 carats fine, muſt be mixed together, to form a compoſition

of 40 ounces of 20 carats fine? *Anſ.* 5 oz. each of 15, of 17, and of 18, and 25 oz. of 22 carats fine.[*]

*Limitation 2d.* When one of the ingredients is limited to a certain quantity, and that quantity is not found by the method of linking; you muſt either augment, or diminiſh the quantities of all the reſt in the ſame proportion as the given quantity is greater or leſs than the quantity of the limited ſimple found by linking; by ſtating as in the firſt limitation. *Or*,[†] you may only augment, or diminiſh, in the above proportion, that part of the quantity of the ingredients with which the limited one is linked, which is the difference of the mixture rate and the rate of the limited ſimple, and add the reſulting quantity to the other parts, inſtead of the ſaid difference; keeping the quantities of the other ſimples unaltered.

### EXAMPLES.

14. How much wine at 5s. at 5s. 6d. and 6s. per gallon, muſt be mixed with 3 gallons at 4s. per gallon, that the mixture may be worth 5s. 4d. per gallon?

*Anſ.* 3 gallons at 5s. 6 at 5s. 6d. and 6 at 6s. by proportioning all the quantities: or, by proportioning only the difference of the mixture and limited rates, 10 gallons at 5s. 8$\frac{4}{5}$ at 5s. 6d. and 8$\frac{4}{5}$ at 6s.

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[*] To this caſe belongs the queſtion concerning king Hiero's crown, which the workmen had debaſed with ſilver or copper; to find what quantity of gold and copper was in it, the famous Archimedes is ſaid to have made two other crowns of the ſame weight with the former, the one of gold, and the other of ſilver or copper, and by putting each into a veſſel full of water, the quantity of water expelled by them determined their ſpecific bulk; from which and their given weight it is eaſier to determine the quantities of gold and copper in the crown by this caſe of alligation than by an algebraic proceſs. We ſhall aſſume the ſame numbers which Ronayne has in his algebra; thus, ſuppoſe the weight of each crown to be 10lb. and that the water expelled by the copper or ſilver one was 92lb. by the gold one 52lb. and by the compound crown was 64lb. that is, their ſpecific bulks were 92, 52, and 64.

Here then the rates of the ſimples are 92 and 52, and of the compound 64; whence,

$$64\begin{cases}92 & 12 \text{ of copper.}\\ 52 & 28 \text{ of gold.}\end{cases}$$

The ſum of theſe is $12+28=40$, which ſhould have been but 10, wherefore by our rule,

$$\text{As } 40 : 10 :: \begin{cases}12 : 3 \text{ of copper.}\\ 28 : 7 \text{ of gold.}\end{cases}$$

[†] Hence we may obſerve that Mr. Malcolm inadvertently gives a rule in page 569 of his arithmetic, for queſtions of this ſort, when the limited ſimple is only once linked, which will not always give true anſwers; he ſays

15. How much gold of 15, of 17, and of 22 carats fine, muſt be mixed with 5 ounces of 18 carats fine, that the compoſition may be 20 carats fine? *Anſ.* 5 oz of 16 carats, 5 oz. of 17, and 25 oz. of 22 carats, by proportioning all the quantities; but by proportioning only the difference of the mixture and limited rates, the anſwer is, 2 oz. of 15 carats, 2 oz. of 17, and 13 oz. of 22 carats fine.

*Limitation* 3*d*. If more than one of the ſimples be limited, but in a proper ratio; find (by limit. 2.) what will be the rate of a mixture made up of the given quantities of the limited ſimples only; then conſider this as the rate of a limited ſimple, whoſe quantity is the ſum of the firſt given limited ſimples, from which, and the rates of the limited ſimples, (by the ſecond limitation) calculate the quantity of each.

16. Required to make 100l. by 100 pieces of gold, viz. moydores at 30s. guineas at 23s. and piſtoles at 18s. 6d.?

*Solution*, by comparing them by couples.

$$20\left\{\begin{matrix}30\\18\frac{1}{2}\end{matrix}\right)\begin{matrix}1\frac{1}{2}\times 4=6\text{ moydores,}\\10\times 4=40+36=76\text{ piſtoles,}\end{matrix}$$

$$20\left\{\begin{matrix}23\\18\frac{1}{2}\end{matrix}\right)\begin{matrix}1\frac{1}{2}\times 12=18\text{ guineas,}\\3\times 12=\underline{36}\end{matrix}$$

*Proof*, 100

*or thus*,
$$20\left\{\begin{matrix}30\\23\\18\frac{1}{2}\end{matrix}\right)\begin{matrix}1\frac{1}{2}\\1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}=4\frac{1}{2}\\3+3+3+10=\underline{19}\\25\end{matrix}$$

$$\text{Then, As }25:100::\left\{\begin{matrix}\text{——}\ 1\frac{1}{2}:6\\\text{——}\ 4\frac{1}{2}:18\\\text{——}\ 19:76\end{matrix}\right\}\textit{Anſwers.}$$

*Proof*, 100

16. How

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" If the ſimple whoſe quantity is limited, is only once linked, we need do " no more than raiſe or diminiſh the quantity of that one ſimple with which " it is linked, and leave the reſt as they are." Inſtead of which, if he had conſidered that the ſimple with which the limited one is linked, may be alſo linked with ſome one or more of the reſt, we apprehend he would have ſaid " Raiſe or diminiſh that part of the ſimple with which the limited one is " linked, which is the difference betwixt the mixture rate and the limited " ſimple."

17. How much wine at 5s. 6d. and at 6s. per gallon, muſt be mixed with 3 gallons at 4s. and 3 gallons at 5s. per gallon, that the mixture may be worth 5s. 4d. per gallon? *Anſ.* 6 gallons at 5s. 6d. and 6 gallons as 6s. per gallon.

18. How much gold of 15 and 17 carats fine, muſt be mixed with 5 ounces of 18, and 13 ounces of 22 carats fine, that the compoſition may be of 20 carats fine?

*Anſ.* 2 ounces of each ſort.

## *Alligation Partial,*

Is when the price of each ſimple is given, alſo the quantity of one of them, and the mean rate, to find the ſeveral quantities of the reſt in proportion to that given.

*Rule.* Take the difference between each price and the mean rate as before in the laſt rule; then ſay, as the difference of that ſimple whoſe quantity is given, is to the known quantity :: ſo is any other difference to the quantity of its oppoſite name.

EXAMPLES.

19. How much tea at 6s. 6d. 7s. 6d. and 9s. per lb. muſt be taken to be mixed with 36lb. at 12s. per lb. that the mixture may be worth 8s. per lb.?

$$96\left\{\begin{array}{l} \text{d.} \\ 144 \\ 108 \\ 90 \\ 78 \end{array}\right. \begin{array}{r} \text{dif.} \\ 6 \\ 18 \\ 48 \\ 12 \end{array}\Bigg\} \qquad \text{If } 6 : 36 \begin{array}{l} \\ 18 : \text{to } 108 \text{ at } \\ 48 : \text{to } 288 - \\ 12 : \text{to } \;\;72 - \end{array} \begin{array}{l} \text{Ans. s. d.} \\ 9\;\;0 \\ 7\;\;6 \\ 6\;\;6 \end{array}\Bigg\} \begin{array}{l} \\ \textit{per} \\ \textit{lb.} \end{array}$$

20. A tobacconiſt has by him 120lb. of fine Oroonoko tobacco, worth 2s. 6d. a lb. with this he would mix York river ditto, at 20d. and other inferior tobaccoes, at 18d. and 15d. a pound, as will make up a mixture anſwerable to 2s. a pound, what will this parcel weigh? *Anſ.* $233\frac{13}{19}$lb.

21. How much alloy muſt be put to bullion of $10\frac{4}{5}$oz. fine, to bring it to $7\frac{1}{4}$oz. fine? *Ans.* $5\frac{127}{145}$oz.

22. How much alloy muſt be put to gold of $21\frac{5}{6}$ carats fine, to bring it to $17\frac{1}{2}$ carats fine? *Ans.* $5\frac{33}{35}$ carats.

23. How much water muſt be mixed with 63 gallons of brandy at 5s. 5d. the gallon ſo that it may ſtand in 4s. 6d. the gallon? *Anſ.* $12\frac{5}{6}$ gallon.

24. How much braſs of 14d. and pewter of 10½d. the pound muſt I melt with 50lb. of copper worth 16d. the lb. ſo that the whole mixture may ſtand in 1s. the lb.?

*Ans.* 200lb. at 10½d. and 50 at 14d.

25. A vintner would mix 8 gallons of wine at 9s. 4d. per gallon, with ſome at 6s. per gallon, with ſome at 6s. 8d. with ſome at 8s. per gallon, ſo that when mixed, it may be worth 7s. 4d. per gallon, how much muſt be taken of each ſort? *Ans.* 8 gallons at 6s. 24 at 6s. 8d. 16 gallons at 8s. and 8 at 9s. 4d.

26. A diſtiller would mix 40 gallons of French brandy at 12s. per gallon, with Engliſh at 7s. and ſpirits at 4s. per gallon, what quantity of each ſort muſt he take to afford it for 8s. per gallon? *Ans.* 40 gallons of French, 32 Engliſh, and 32 of ſpirits.

## *Alligation Total,*

Is, when the price of each ſimple is given, as alſo the mean rate and the quantity of the compound, to find how much of each ſort will make that quantity.

*Rule.* Take the difference between each price, and the mean rate as before. 2. Say, as the ſum of theſe differences : is to the whole quantity of the mixture :: ſo is each particular difference : to its particular quantity.

EXAMPLES.

27. A druggiſt has by him 4 ſorts of medicines, viz. of 5s. 6s. 8s. and 9s. the lb. out of theſe he is inclined to mix up a canniſter, containing net a hundred and a half, ſo as to make the medicine worth 7s. the lb. in what proportion muſt thoſe be taken?

*s.* *dif.*
*s.* 5 — 1
7 6 — 2
8 — 2
9 — 1
*Sum* 6

*Sum.* *lb.* *lb.* *s.*
*As* 6 : 168 :: ——1 : *to* 28 *at* 5 *per*
——2 : —56— 6 *lb.*
——2 : —56— 8 *Ans.*
——1 : —28— 9

28. A grocer has four ſorts of ſugar, viz. 12d. 10d. 6d. and 4d. per lb. and would make a compoſition of 144lb. worth 8d. per lb. I deſire to know what quantity of each he

muſt take? *Anſ.* 48lb. at 12d. 24 at 10d. 24 at 6d. and 48lb. at 4d.

29. A grocer having four ſorts of tea, 5s. 6s. 8s. and 9s. per lb. would have a compoſition of 87lb. worth 7s. the lb. what quantity muſt there be of each? *Ans.* 14½lb. of 5s. 29lb. of 6s. 29lb. of 8s. and 14½lb. of 9s.

30. A groeer has currants at 4d. 6d. 9d. and 11d. per lb. and he would make a mixture of 240lb. ſo that it might be aſſorted at 8d. per lb. how much of each ſort muſt he take?

*Ans.* 72lb. at 4d: 24 at 6d. 48 at 9d. and 96 at 11d.

31. A goldſmith has two ſorts of ſilver bullion the one of 10 oz. and the other of 5 oz. fine, and has a mind to mix a lb. of it, ſo that it ſhall be 8 oz. fine, I demand how much of each he muſt take? *Ans.* 4⅘ of 5 oz. and 7⅕ of 10 oz. fine.

## CHAP. VII.

## SQUARE ROOT.

EXTRACTING the Square Root, is to find out ſuch a number as, being multiplied into itſelf, the product will be equal to a given number.

*Rule.* Firſt point the given number, beginning at the units' place, then to the hundreds, and ſo upon every ſecond figure throughout.

2. Seek the greateſt ſquare number in the period towards the left hand, placing the ſquare number under the firſt period, and the root thereof in the quotient; ſubtract the ſquare from the firſt period, and to the remainder bring down the next, and call that the reſolvend.

3. Double the quotient, and place it for a diviſor on the left hand of the reſolvend; ſeek how often the diviſor is contained in the reſolvend (reſerving always the units' place) and put the anſwer in the quotient, and alſo on the left hand of the diviſor; then multiply by the figure laſt put in the quotient, and ſubtract the product from the reſolvend; bring

down the next period to the remainder (if any) and proceed as before.

| Root, | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Square, | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |

1. Extract the ſquare root of 119025

11·90·25 ( 345 the *Anſ.*
9

64 ) 290
256

685 ) 3425 (
3425

|  |  |  | *Anſwers.* |
|---|---|---|---|
| 2. | What is the ſquare root of | 5499025? | 2345. |
| 3. | —— | —— 106029? | 327. |
| 4. | —— | —— 2268741? | 1506.23.+ |
| 5. | —— | —— 7596796? | 2756.228.+ |
| 6. | —— | —— 36372961? | 6031. |
| 7. | —— | —— 22071204? | 4698. |
| *8. | —— | —— 3.1721812? | 1.78106 &c. |
| 9. | —— | —— 21035 800? | 145.037237. |
| 10. | —— | 820967940.529? | 90607.21792.+ |
| 11. | —— | —— 2.2710957? | 1.50701.+ |
| 12. | —— | 36.00000625? | 6.0000005.+ |

To extract the ſquare root of a Vulgar Fraction,

*Rule.* Reduce the fraction to its loweſt terms, then extract the ſquare root of the numerator for a new numerator, and the ſquare root of the denominator for a new denominator.

*Note* If the fraction be a ſurd *(i. e.)* a number whoſe root can never be extracted, reduce it to a decimal, and extract the root of that decimal.

### EXAMPLES.

|  |  |  | *Anſwers.* |
|---|---|---|---|
| 13. | What is the ſquare root of | $\frac{4}{2}$? | 1.414. |
| 14. | —— | —— $\frac{9}{45}$? | $\frac{3}{7}$. |

15. What

---

* When the given number conſiſts of a whole number and decimal together; make the number of decimals even, if not ſo, by adding a cypher to them: ſo that there may be a point on the unit's place of the whole number.

| | | *Anſwers.* |
|---|---|---|
| 15. | What is the ſquare root of $\frac{9}{12}$? | .866025+ |
| 16. | —— —— $\frac{5}{12}$? | .645497+. |
| 17. | —— —— $\frac{2304}{5184}$? | .666, &c. |

To extract the ſquare root of a mixt number,

*Rule.* Reduce the fractional part of the mixt number to its loweſt term; and then the mixed number to an improper fraction.

2. Extract the root of the numerator and denominator, for a new numerator and denominator.

*Note.* If the mixed number given be a ſurd, reduce the fractional part to a decimal, annex it to the whole number, and extract the ſquare root therefrom.

### EXAMPLES.

| | | *Anſwer.* |
|---|---|---|
| 18. | What is the ſquare root of $5\frac{21}{25}$? | 2.416+ |
| 19 | —— —— $17\frac{3}{8}$? | 4.168333.+ |
| 20. | —— —— $6\frac{2}{5}$? | 2.5298.+ |
| 21. | —— —— $27\frac{9}{16}$? | $5\frac{1}{4}$. |
| 22. | —— —— $8\frac{5}{7}$? | 2.9519+ |
| 23. | —— —— $6\frac{2}{3}$? | 2.5819.+ |

## *The Uſe of the Square Root.*

To find a mean proportional between any two given numbers.

*Rule.* Multiply the two given numbers together, and extract the ſquare root of the product, which root will be a mean proportional.

### EXAMPLES.

1. What is the mean proportional between 4 and 9? *Anſ.* 6.

2. What is the mean proportional between 4276 and 842? *Anſ.* 1197.4.+

To find the ſide of a ſquare, equal in area to any given ſuperficies.

*Rule.* The ſquare root of the content of any ſuperficies, is the ſide of the ſquare ſought.

EXAMPLES.

### EXAMPLES.

3. If the content of a given circle be 160, what is the ſide of a ſquare, equal in area thereto? *Anſ.* 12.64911.

4. Suppoſe I have an elliptical or irregular fiſh-pond, containing in ſurface 9 acres, 2 roods, 15 perches, and would have a ſquare one of the ſame content; I deſire to know how many yards each ſide muſt be?

*Ans.* 215 yards, 1 foot, 5.467 inches.

Having the area of a circle, to find the diameter,

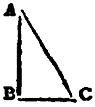
*Rule.* As 355 : 452 or as 1 : 1.273239 :: ſo is the area to the ſquare of the diameter; or multiply the ſquare root of the area by 1.12837, and the product will be the diameter.

### EXAMPLES.

5. In the midſt of a meadow, well ſtored with graſs,
I took juſt two acres to tether my horſe;
How long will the cord be, that, feeding all round,
He mayn't graze leſs or more than theſe 2 acres of ground?

*Ans.* 55½ yards, the length of the cord.

Any two ſides of a right-angled triangle, A, B, C, being given, to find the remaining ſide: 1ſt, The baſe and perpendicular being given, to find the hypothenuſe.



*Rule.* Square each ſide, add the ſquares together, and the ſquare root of their ſum gives the hypothenuſe required.

2. If the hypothenuſe and one leg be given, to find the other ſide.

*Rule.* From the ſquare of the hypothenuſe ſubtract the ſquare of the given leg or ſide; the ſquare root of the remainder gives the ſide required.

6. There are two pillars in a ſtraight line, perpendicular to the plane of the horizon, whoſe diſtance aſunder is 180 feet; the one is 60 and the other 40 feet high: quere in what part of the line of diſtance a ladder may be fixed, ſo as to reach the top of each pillar, without removing it at bottom, alſo the length of the ladder?

*Rule.* Divide the difference of the ſquares of the pillars by twice the line of diſtance; to the quotient add half the line of diſtance, which will give the diſtance from the leſſer pillar, to where the ladder is to be fixed; then by the foregoing rule the length of the ladder is found.

*Ans.* the ladder is to be fixed 95.55 feet from the leſſer pillar; and the length of the ladder 103.44 feet.

7. A ladder, 103.44 feet, may be ſo planted, that it ſhall reach a window 40 feet from the ground, on one ſide of the ſtreet, and without removing it at the foot, will do the ſame by a window 60 feet high on the other ſide; the breadth of the ſtreet is required? *Ans.* 180 feet nearly.

8. An unknown number of men went to drink, and ſpent 30s. 1d.; each man paid one penny for himſelf and for each in company: quere the number of men in company?
*Anſ.* 19.

9. Divide 6l. 13s. 4d. between an unknown number of men, and give each man as many groats as there are men in company: quere the number of men in company? *Anſ.* 20.

10. How often will a coach-wheel turn from Dublin to Limerick, ſuppoſe the diſtance 92 miles, allowing it to turn as often in a mile as the circumference contained inches?
*Anſ.* 21157.32.

11. A caſtle wall there was, whoſe height was found
To be an hundred feet from th' top to th' ground;
Againſt the wall a ladder ſtood upright,
Of the ſame length the caſtle was in height:
A waggiſh youth did the ladder ſlide,
(The bottom of it) ten feet from the ſide:
Now I would know how far the top did fall,
By pulling out the ladder from the wall?
*Anſ.* 6 inches, nearly.

12. Suppoſe a light-houſe built on the top of a rock; the diſtance between the place of obſervation and that part of the rock level with the eye and directly under the building is given 310 fathoms; the diſtance from the top of the rock to the place of obſervation is 423 fathoms; and from the top of the building, 425; the height of the edifice is required? *Anſ.* 287.8 fathoms height from the rock; 2.93156 fathoms height of the light-houſe.

13. Two ſhips ſailed, one due eaſt, 50 leagues, the other due north 84; how far are they aſunder? *Ans.* 97.75 leagues.

14. The height of an elm, growing in the middle of a circular iſland, 30 feet in diameter, plumbs 53 feet; and a line ſtretched from the top of the tree, ſtraight to the hither edge of the water, 112 feet; what then is the breadth of the moat, ſuppoſing the land on the other ſide of the water to be level? *Anſ.* $83\frac{2}{3}$ feet.

15. Required the length of a ſhore that being to ſtretch 11 feet from the upright of a building, will ſupport a jamb, 23 feet, 10 inches from the ground? *Anſ.* 26 feet, 9 inches.

Any number of men being given to form them into a ſquare battle, to find the number of rank and file.

*Rule.* Extract the ſquare root of the number of men given, which will give the number of men either in rank or file.

### EXAMPLE.

16. A general diſpoſing his army into a ſquare battle, finds he has 23716 men; required the number in rank and file? *Anſ.* 154 men.

## *Effects of Light and Heat.*

The effects or degrees of light, heat, and attraction, are reciprocally proportional to the ſquare of their diſtances, from the centre whence they are propagated.

17. Suppoſe that in a room where two men, A. and B. are ſitting, there is a fire, from which A. is 3 feet, and B. is 6 feet diſtance; it is required to find how much hotter it is in A.'s ſeat than B.'s? *Anſ.* A.'s is 4 times as hot as B.'s.

18. Suppoſe the earth to be 81 millions of miles diſtant from the ſun; I would know at what diſtance from him another, B. muſt be placed, ſo as to receive light and heat double to that of the earth? *Anſ.* 57275649 miles.

19. The diſtance between the earth and the ſun is accounted 81 millions of miles; the diſtance between Jupiter and the Sun, 424 millions of miles; the degree of light and heat received by Jupiter, compared with that of the earth, is required?

*Anſ.* The Sun's influence on the Earth, to that on the planet Jupiter is, as $27\frac{4}{10}$ to 1.

20. Mercury, the neareſt of all the planets to the Sun, is about 32 millions of miles from him; Saturn is diſtant about 777 millions of miles; what proportion is there between the ſolar influence on theſe two bodies?

*Anſ.* The ſolar influence on Mercury to that of Saturn is at $589\frac{1}{2}$ to 1, nearly.

21. Suppoſe, with Dr. Keil, the diſtance of the ſun to be from us, 115 of his diameters; how much hotter is it at the ſurface of the ſun than under our equator?

*Anſ.* 13225 degrees hotter.

### *Velocities acquired by heavy Bodies falling.*

The velocity acquired by heavy bodies falling nearly to the ſurface of the earth, is $16\frac{1}{2}$ feet in the firſt ſecond; and as $16\frac{1}{2}$ are to the ſquare of one ſecond or 1, ſo is the given diſtance to the ſquare of the ſeconds required; or, on the contrary, to determine what ſpace a heavy body has paſſed in any time given.

*Rule.* Multiply $16\frac{1}{2}$, the deſcent of a heavy body in one ſecond of time by as many of the odd numbers, beginning from unity, as there are ſeconds in the given time, viz. by 1 for the firſt; 3 for the ſecond; 5 for the third; 7 for the fourth, &c. the ſum total will give the ſpace it hath paſſed.

22. Suppoſe a ſtone let fall into an abyſs, ſhould be ſtopped at the end of the eleventh ſecond, after its delivery; what ſpace would it have gone through? *Anſ.* 1946.083 feet.

23. A ball deſcending by the force of gravity, from the top of a tower, was obſerved to fall half the way in the laſt ſecond of time; required the tower's height, and the whole time of deſcent?

*Anſ.* $\begin{cases} 187\frac{41}{25} & \text{tower's height.} \\ 3\frac{2}{3} & \text{seconds, the time of descent.} \end{cases}$

24. What is the difference between the depth of two wells, in each of which, ſhould a ſtone be dropped at the ſame inſtant, one will meet with the bottom at 6 ſeconds, the other at 10? *Anſ.* 1029.3 feet.

25. If a ſtone be $19\frac{1}{2}$ ſeconds in deſcending from the top of a precipice to the bottom, what is the height of the ſame?

*Anſ.* 1019 fathoms, 1 foot, $8\frac{1}{4}$ inches.

26. In what time would a muſket ball, dropped from the top of a ſteeple 400 feet high, be at the bottom?

*Anſ.* 5 ſeconds, nearly.

27. If a hole could be bored through the centre of the earth, in what time would a heavy body, let fall from its ſurface, arrive at the centre?

*Ans.* 18 min. 55 ſecs. 33 thirds.

### *Vibration of Pendulums.*

It hath been found by experiment, that a pendulum, 39.2 inches long, in our latitude, vibrates 60 times in one minute; and that the length of the pendulums are to one another reciprocally, as the ſquares of the number of their vibrations, made in the ſame ſpace of time.

28. What difference will there be in the number of vibrations made by a pendulum of 6 inches long, and another of 12 inches long, in an hour's time? *Anſ.* 2695.08 difference.

29. What difference is there in the length of two pendulums, the one ſwinging 3 times, the other 100 times in an hour? *Anſ.* difference 42806.4 feet.

30. Required the length of pendulums, that will ſwing once in a third, once in a minute, once in an hour, once in a day?

*Anſ.* 10108 inches, the length of that which vibrates thirds; $2\frac{5}{22}$ miles, the length of a pendulum which vibrates once in a minute; $8018\frac{9}{11}$ miles, the length of the pendulum which vibrates once in an hour; $4618188\frac{1032}{1760}$ miles the length of the pendulum which vibrates once in a day.

The leſs porous a body is, the greater is its denſity. The compactneſs or denſity of the moon is to that of the earth as $132\frac{1}{2}$ to 100.

31. What proportion then is there between the quantity of matter in the earth and that in the moon, ſince the earth's diameter is 7970 miles, and that of the moon 2170?

*Anſ.* The earth contains 41 17 times more matter than the moon.

# CHAP. VIII.

## EXTRACTION OF THE CUBE ROOT.

TO extract the Cube Root is to find a number which, being multiplied into itſelf, and then into that product produceth the given number.

*Rule.* 1. Point every third figure of the cube given, beginning at the units' place; ſeek the greateſt cube in the firſt period, and ſubtract it therefrom; put the root in the quotient, and bring down the figures in the next period to the remainder for a reſolvend.

2. Find a diviſor, by multiplying the ſquare of the quotient by 3, ſee how often it is contained in the reſolvend,

rejecting the units and tens, and put the anſwer in the quotient.

3. To find the ſubtrahend: 1ſt, cube the laſt figure in the quotient. 2d, Multiply all the figures in the quotient by 3, except the laſt, and that product by the ſquare of the laſt. 3d, Multiply the diviſor by the laſt figure, add theſe products together, gives the ſubtrahend, which ſubtracted from the reſolvend, to the remainder bring down the next period, and proceed as before.

| Root. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Cube, | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 |

### EXAMPLES.

1. What is the cube root of 99252847?

99252847 ( 463
64=cube of 4.
*divis.*
ſquare of 4=16×3=48 ) 35252 reſolvend.

216=cube of 6.
432 =4×3×ſquare of 6.
288 =diviſor×6.

33336 ſubtrahend.
*divis.*
ſquare of 64×3=6348 ) 1916847 reſolvend.

27=cube of 3.
1242 =46×3×by the ſq. of 3.
19044 =the diviſor×3.

1916847 ſubtrahend.

.......

| | *Anſwers.* |
|---|---|
| 2. What is the cube root of 18812904? | 234. |
| 3. —— —— 7584? | 19.64.+ |
| 4. —— —— 614125? | 85.+ |
| 5. —— —— 46656? | 36. |
| 6. —— —— 146363183? | 527. |
| 7. —— —— 48228544? | 364. |

8. What

| | | *Answers.* |
|---|---|---|
| *8. | What is the cube root of $\frac{8}{27}$? | $\frac{2}{3}$. |
| 9. | ——— ——— $\frac{80}{405}$? | $\frac{4}{7}$.+ |
| 10. | ——— ——— $\frac{250}{1500}$? | $\frac{2}{3}$.+ |
| 11. | ——— ——— $\frac{1520}{5130}$? | $\frac{5}{8}$.+ |
| 12. | ——— ——— 28[illegible]? | 3.06.+ |
| 13. | ——— ——— $2\frac{10}{27}$? | $\frac{4}{3}$ or $1\frac{1}{3}$. |
| 14. | ——— ——— $7\frac{1}{5}$? | 1.930979. |
| 15.† | ——— ——— 12.977875? | 2.33. |
| 16. | ——— ——— 36155.02756? | 33.06.+ |
| 17. | ——— ——— 15926.972504? | 25.16.+ |
| 18. | ——— ——— .0531573[illegible]? | .376. |
| 19. | ——— ——— $1953\frac{1}{8}$? | 12.5. |
| 20. | ——— ——— $9\frac{1}{6}$? | 2.092.+ |
| 21. | ——— ——— $8\frac{5}{7}$? | 2.057.+ |
| 22. | ——— ——— $405\frac{83}{125}$? | $7\frac{2}{5}$. |
| 23. | ——— ——— $31\frac{15}{343}$? | $3\frac{1}{7}$. |

## *The Uſe of the Cube Root.*

1. There are two numbers, whereof the greater is 2579890752; their difference is 1152; what is the leſs, and what is the cube root of their ſum?

*Anſ.* Leſs, 2579889600, and the cube root of their ſum is 1728.

2. There are three boxes, the content of one is 10000 ſolid inches; of another, 16656; and of the third, 20000; required the ſide of a cubical box, that ſhall contain as much as all the three? *Anſ.* 36 inches, or 3 feet.

3. Required the ſide of a cubical veſſel, that will contain 80 wine gallons? *Anſ.* 26.43 inches.

4. A braſs bullet, of 5 inches diameter, weighs 20lb.; what is the diameter of another of the ſame metal, that weighs 160lbs.? *Anſ.* 10 inches.

CHAP.

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* To extract the cube root of a Vulgar Fraction.

*Rule.* Reduce the fractional part to its loweſt terms, and then the mixt number to an improper fraction, extract the cube roots of both the numerator and denominator, for a new numerator and denominator; but if the mixed number given be a ſurd, reduce it to a decimal, annex it to the whole number, and extract the cube root therefrom.

† When the given number conſiſts of a whole number and decimal together, make the number of decimals to conſiſt of 3, 6, 9, &c. places, by adding cyphers thereto, ſo that there may a point fall under the unit's place of the whole number.

# CHAP. IX.

## BIQUADRATE ROOT.

TO extract the Biquadrate Root, is to find out a number which, being involved into itſelf ſucceſſively, will produce the cube into the root of the given number.

*Rule.* Firſt, extract the ſquare root of the given number, and then extract the ſquare root of that ſquare root, and it will give the biquadrate root required.

### EXAMPLES.

| | | *Answers.* |
|---|---|---|
| 1. | What is the biquad. root of 5308416? | 48. |
| 2. | —— —— 97.41? | 3.1415999.+ |
| 3. | —— —— 2? | 1.189207. |
| 4. | —— —— 3336217176? | 76. |
| 5. | —— —— 5715140625? | 275. |

### *A general Rule for extracting the Roots of all Powers.**

1. Prepare the number given for extraction, by pointing off from the unit's place, as the root required directs.

2. Find the firſt figure in the root by the Table of Powers which ſubtract from the given number..

3. Bring down the firſt figure in the next point to the remainder, and call it the dividend.

4. Involve the root into the next inferior power to that which is given; multiply it by the given power, and call it the diviſor.

5. Find a quotient figure by common diviſion, and annex it to the root, and call that the ſubtrahend.

6. Then ſubtract that number from as many points of the given power as is brought down, beginning at the lower place, and to the remainder bring down the firſt figure of the next period for a new dividend.

7. Find a new diviſor, and proceed in all reſpects as before.

### *A ſhort Method of extracting the Cube Root.*

1. By trials, take the neareſt rational cube to the given cube or number, whether it be greater or leſs, and call it the aſſumed cube.

---

* Let G be the given power or number in the Index of the power; A, the aſſumed; r, its root; R, the required root of G: Then, as the ſum of

n+1

2. Then the ſum of the given number and double the aſſumed cube, will be to the ſum of the aſſumed cube, and double the given number, as the root of the aſſumed cube is to the root required, nearly; or as the firſt ſum is to the difference of the given and aſſumed cube, ſo is the aſſumed root to the difference of the roots nearly.

3. Again, by uſing, in like manner, the cube root, laſt found, as a new aſſumed root, another root will be got ſtill nearer, and ſo on, as far as you pleaſe, uſing always the cube of the laſt found root, for the aſſumed cube.

### EXAMPLES.

Find the cube root of 21035.8. Here we find that the root lies between 20 and 30, and then between 27 and 28, taking therefore 27, its cube is 1968, the aſſumed cubed.

$$\left.\begin{array}{rr} 19683 & 21035.8 \\ 2 & 2 \\ \hline 39366 & 42071.6 \\ 21035.8 & 19683 \\ \hline 60401.8 & 61754.6 \end{array}\right\}$$

Then, As 60401.8 : 61754.6 :: 27 : 27.6047 the root nearly like proceſs purſued the root= 27.604910834, root required.*

CHAP.

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$\overline{n+1}$ times, A and $\overline{n-1}$ times G is to the ſum of $\overline{n+1}$, G and $\overline{n-1}$ times A aſſumed, ſo is the aſſumed root *r*, to the required root *r*, to the difference between the true and aſſumed roots, which difference added or ſubtracted, gives the true root nearly, that is, $\overline{n+1}.A+\overline{n-1}.G:\overline{n+1}.G+\overline{n-1}.A::r:R$. or $\overline{n+1}.\frac{1}{2}A+\overline{n-1}.\frac{1}{2}G : A \backsim G :: r : R \backsim r$ and the operation may be repeated as often as we pleaſe, by uſing always the laſt found root for the aſſumed root, and its *n*th, the power for the aſſumed power A.

* The uſual method for extracting the cube root, is ſo exceedingly tedious and difficult to beginners to be remembered, that various other approximating rules have been invented by Newton, Raphſon, Hally, De Lagny, Simpſon, Emerſon, and ſeveral other mathematicians; but no one that we have yet ſeen, ſo well adapted for general uſe, as that above given; this rule, as far as we have learned, came firſt from Mr. James Dodſon, and is the ſame in effect as Dr Hally's Rational Formula; but ſomewhat more commodiouſly expreſſed; but the learned Dr. Hutton has brought it to its *ne plus ultra*.

# CHAP. X.

## PROGRESSION

### *Conſiſts of Two Parts,*

### ARITHMETICAL AND GEOMETRICAL.

ARITHMETICAL PROGRESSION is when a rank or ſeries of numbers increaſe or decreaſe, by a common difference, or by a continual adding or ſubtracting of the ſame number.

$$\text{As}\begin{cases}1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8\\ 8,\ 7,\ 6,\ 5,\ 4,\ 3,\ 2,\ 1\end{cases}\text{Here the common difference is 1;}$$

Or, 1, 3, 5, 7, 9, 11, 13; here the com: dif. is 2:
Alſo, 35, 30, 25, 20, 15, 10, 5; here the com. dif. is 5.

1. In any ſeries of numbers in Arithmetical progreſſion, when the number of terms are even, as 1, 3, 5, 7, 9, 11, or the like; the ſum of the two extremes will be equal to the ſum of any two means that are equally diſtant from the extremes.

Viz. 1, 3, 5, 7, 9, 11 $1+11=5+7=3+9=12$.

2. When the number of terms are odd, as 2, 4, 6, 8, 10; double the middle number or term will be equal to the ſum of the extremes, or to any two means equally diſtant from the middle term, viz. 2, 4, 6, 8, 10.

$$6+2=4\times2=8, \text{ and } 2+10=6\times2=12=4+8.$$

In Arithmetical Progreſſion, there are five things to be obſerved, viz.—

1. The firſt term.
2. The laſt term.
3. The number of terms.
4. The common difference.
5. The aggregate, or ſum of all the terms.

Any three of which being given, the other two may be found.

*Propoſition* 1. When the two extremes and number of terms are given, to find the ſum of all the ſeries or terms.

*Rule.* Multiply the ſum of the two extremes into the number of terms and divide the product by 2, the quotient will be the ſum of all the ſeries; *or*, multiply the ſum of the two extremes by half the number of terms.

### EXAMPLES.

1. How many ſtrokes do the clocks at Venice (which go on to 24 hours) ſtrike in the compaſs of a natural day?

*Ans.* 300.

*Solution.* 1+24=25 ſum of the extremes, then, per rule, 25×24÷2=300 ſtrokes, the *Anſ.*; *or*, per rule 2d, 25×12 =300.

2. How many ſtrokes does the hammer of a clock ſtrike in 12 hours? *Anſ.* 78 ſtrokes.

No of terms 94

3. The length of my garden is 94 feet; now if eggs be laid along the pavement a foot aſunder, and be fetched up ſingly to a baſket removed one foot from the firſt; how much ground does he travel that does it? 8930 *Anſ.* [illegible] feet.

4 Suppoſe 100 ſtones were placed in a right line, a yard diſtant from one another, and the firſt ſtone was one yard from a baſket, I demand how many miles he muſt travel that gathers them ſingly into a baſket?

*Anſ.* 5 miles, 5 furlongs, 36 perches, 2 yards, Engliſh meaſure, or 4 miles, 4 furlongs, 2 per. $5\frac{4}{7}$ yds. Iriſh meaſure.

5. A butcher bought 100 ſheep, and gave for the firſt ſheep, 1s. and for the laſt, 9l. 19s.; I demand what he gave for the 100 ſheep? *Ans.* 500l.

*Propoſition* 2. When the two extremes and number of terms are given, to find the common difference.

*Rule.* Difference of two extremes divided by the number of terms leſs an unity or 1, the quotient will give the common difference.

### EXAMPLES.

6. One had 20 children that differed alike in their ages; the youngeſt was 5 years old; the eldeſt 43; what was the difference of their ages, and the age of each?

*Solution.* Here 43—5=the difference of the extremes, and 20—1=the number of terms—1, then 38÷19=2, the common difference, which, added to the age of the youngeſt, and ſo on to the reſt, will give their ages, viz. 5+2=7 age of the ſecond; and 7+2=9, the age of the third, and ſo on with the reſt.

7. A traveller (for a wager) is to travel from Dublin to a certain place, in 19 days, and to go but 6 miles the firſt day, encreaſing every day's journey by an equal exceſs, ſo that

the laſt day's journey be 60 miles: I demand each day's journey and the diſtance of the place he goes to?

*Ans.* Each day's journey will be as follows: $6+9+12+15+18$, &c. to the 19th day, and 627 the diſtance.

8. A debt is to be diſcharged at 10 different payments, in Arithmetical Progreſſion; the firſt payment is to be 5l. and the laſt 50l.; what is the whole debt, and what muſt each payment be?

*Ans.* Each payment will be as follows: $5+10+15+20$, &c. the whole debt, 275l.

*Propoſition* 3. When the two extremes and common difference are given, to find the number of terms.

*Rule.* Divide the difference of the two extremes by the common difference or exceſs; add unity or 1 to the quotient, and the ſum will be the number of terms.

### EXAMPLE.

9. A man being aſked how many children he had, anſwered, my youngeſt is 5 years old, and the eldeſt 43, and that he had increaſed one in his family every two years; how many children had he? *Anſ.* 20.

*Propoſition* 4. When the laſt term, the common difference, and the number of terms are given, to find the firſt term.

*Rule.* Multiply the number of terms, leſs unity or 1, by the common difference, the product ſubtracted from or added to the laſt term, leaves the firſt; that is, ſubtract in an increaſing progreſſion, but add in a decreaſing one.

### EXAMPLES.

10. A perſon takes out of his pocket, at 10 different times, ſo many different numbers of guineas, every one exceeding the former by two, the laſt was 23; what was the firſt?

*Solution.* $10-1=9$ number of terms, and $9\times2=18$, then $23-18=3$ the *Ans.*

11. A man in 19 days went from Dublin to a certain place; every day's journey was greater than the preceding one, by 3 miles; his laſt day's journey was 60 miles; what was the firſt? *Ans.* 6 miles.

*Propoſition* 5. When the number of terms, common difference, and the ſum of all the terms are given, to find the firſt or leaſt term.

*Rule.* Divide the ſum of all the terms by the number of terms, and from the quotient ſubtract half the product of the

the common difference, multiplied by the number of terms leſs one, gives the firſt term.

EXAMPLES.

12. A perſon is to receive 275l. at 10 different payments, each payment to exceed the former by 5l. he is willing to beſtow the firſt payment on any one that can tell him what it is he muſt give the arithmetician for his pains. *Ans.* 5l.

*Solution.* 275÷10=27l. 10s. and 10—1=9 alſo 9×5 =45÷2=22l. 10s. then from 27l. 10s. take 22l. 10s. remains 5l. Q. E. F.

13. Suppoſe it is 100 leagues between Dingle and Derry, two couriers ſet out from each place on the ſame road, that from Dingle towards Derry travelling every day two leagues more than the day before; that from Derry to ſet off one day after the other, travelling every day three leagues more than the preceeding one, and that they meet exactly half way; the firſt at the end of five days, and the other at the end of four days, how many leagues did each travel per day? *Ans.* He that travelled from Dingle, 6 firſt day 6+2 =8 ſecond day, 8+2=10 the third, 12 the fourth, 14 fifth, the ſum=50; alſo 8 his firſt that travelled from Derry, 11 ſecond, 14 third, 17 fourth=50.

*Propoſition 6th.* When the firſt or leaſt term, number of terms, and the common difference are given to find the laſt term.

*Rule.* Subtract the common difference from the product of the number of terms multiplied by the common difference, the remainder added to the firſt term will give the laſt, *or greateſt.*

EXAMPLES.

14. What is the laſt term of an Arithmetical progreſſion, beginning at 6 and continuing by the increaſe of 3 to 19 places? *Ans.* 60.

15. What is the [illegible] laſt term of an Arithmetical progreſſion, beginning at [illegible] and continuing by the increaſe of [illegible] to 1[illegible] places? *Anſ.* 199.

*Propoſition 7th.* The firſt term, common difference, and number of terms given to find the ſum of all the ſeries.

*Rule.* From the product of the number of terms into the common difference, ſubtract the common difference, and to the remainder add double of the firſt term; half the product of that ſum multiplied by the number of terms, gives the ſum of all the ſeries.

### EXAMPLES.

16. A gentleman bargains with a bricklayer to ſink a well 30 yards deep, upon theſe terms, viz. to pay him 3 ſhillings for the firſt yard, 5 for the ſecond, 7 for the third, &c. raiſing two ſhillings for every yard; what will be due to the bricklayer for completing the ſame? *Anſ.* 48l.

*Propoſition 8th.* The firſt term, the number of terms, and ſum of all the terms being given, to find the common difference.

*Rule.* Divide double the ſum of all the ſeries by the number of terms, and from the quotient ſubtract double the firſt term; divide the remainder by the number of terms, leſſened by unity, the quotient will be the common difference.

### EXAMPLES.

19. A gentleman travelled 200 miles in 8 days, and every day he travelled equally farther than the preceding day, it is known that the firſt day he travelled 6 miles; how many miles did he travel each of the other days?

*1ſt. 2nd. 3rd. 4th. 5th. 6th. 7th. 8th.*
*Anſ.* 6 $11\frac{3}{7}$ $16\frac{6}{7}$ $22\frac{2}{7}$ $27\frac{5}{7}$ $33\frac{1}{7}$ $38\frac{4}{7}$ 44, in all 200 *Proof.*

20. A perſon travelled from Cork to Belfaſt, being 200 miles, in 9 days, and every day travelled equally farther than the preceding day, it is known that the firſt day he travelled 4 miles, how many miles did he travel each of the other days?

*1ſt. 2nd. 3rd. 4th. 5th. 6th. 7th. 8th. 9th.*
*Ans.* 4, $8\frac{5}{9}$, $13\frac{1}{9}$, $17\frac{6}{9}$, $22\frac{2}{9}$, $26\frac{7}{9}$, $31\frac{3}{9}$, $35\frac{8}{9}$, $40\frac{4}{9}$, in all 200. *Proof.*

*Propoſition 9th.* When one perſon or thing moves with an equal motion, and another the ſame way by a progreſſive motion, to find in what time the firſt will be overtaken by the laſt.

*Rule.* Double the ſpace gone each day by the purſued, add the common difference of the purſuer's day's journey, from that ſum ſubtract double the ſpace he travelled the firſt day, and divide the remainder by the common difference, the quotient will give the number of days in which the purſued will be overtaken by the purſuer.

### EXAMPLES.

21. A noted highwayman having committed a robbery, not ſuſpecting a purſuit fled northward at the rate of nine leagues a day, one of Sir John Shea's men upon the ſcent, follows him in a progreſſive motion, only 3 leagues the firſt day, five the ſecond, ſeven the third, and ſo on encreaſing every day's journey two leagues, in how many days will he overtake the highwayman? *Anſ.* In 7 days.

22. Suppoſing a preſs-gang having a warrant to preſs for 30 days: preſſes the firſt day 300, and every ſucceeding day 10 more than the former, how many will they raiſe in the 30 days? *Anſ.* 13350.

23. A merchant hires a clerk by covenant, for 14 years, to give him 5l. the firſt year, and raiſe his ſalary 40s. per year during the term; the queſtion is to diſcover how much he paid him one year with another on an average? *Anſ.* 18l.

*Note.* By the foregoing problems and examples may moſt queſtions in annuities in arrear be ſolved. For the ſeveral yearly or half yearly intereſts, form an Arithmetical progreſſion, of which the laſt intereſt due may be taken as the firſt term and common difference, and the number of years, &c. leſs 1, the number of terms, becauſe there is no intereſt due upon the laſt payment, it being only now due.

### EXAMPLES.

24. If an annuity of 70l. be forborn 5 years, what will be due at the end of that term for principal and intereſt, intereſt being computed for every annuity, from the time it became due, at 5 per cent per annum ſimple intereſt?

*Anſ.* 385l.

*Solution.* As 100 : 5 :: 70 : to 3l. 10s. a year's intereſt; then 3l. 10s.×4=14l. the laſt term; then 14l. +3l. 10s.=17l. 10s. the ſum of the firſt and laſt, which multiplied by 2, half the number of terms=35 amount of intereſt; alſo 70l.×5=350 amount of annuity, therefore 35+350=385 the *Anſ.* required.

25. If an annuity payable by quarterly payments were unpaid 5 years, what will it amount to in that time, ſimple intereſt being computed at 5 per cent? *Anſ.* 391l. 11s. 3d.

CHAP.

# CHAP. XI.

## GEOMETRICAL PROGRESSION.

ANY ſeries of numbers which gradually increaſe or decreaſe by a conſtant multiplication, or diviſion of the number, are ſaid to be in geometrical progreſſion; thus 2, 4, 8, 16, &c. and 486, 162, 54, 18, 6, &c. are in infinite geometrical proportion, the one increaſing by a conſtant multiplication by 2, and the other decreaſing by a conſtant diviſion by 3; the number by which the ſeries are conſtantly increaſed or diminiſhed, is called the ratio.

Any three of the following terms being given, the reſt may be found.

1. The firſt Term, } called the Extremes.
2. The laſt Term, }
3. The Number of Terms,
4. The Ratio.
5. The Sum of all the Terms.

Any three numbers in geometrical progreſſion will form an analogy, by making the conſequent of the firſt ratio the antecedent of the ſecond ratio, thus the numbers 2, 4, 8, will form this analogy, as 2 : 4 : : 4 : 8, hence the ſquare of the mean is equal to the product of the extremes; and from this the method of finding a mean proportional between two numbers, is manifeſt; thus let it be required to find a mean proportional between 2 and 32, then the ſquare root of 32×2=8 the mean proportional required; for 32 : 8 : : 8 : 2 or 2 : 8 : : 8 : 32.

In any geometrical progreſſion as 3, 6, 12, 24, 48, 96, &c. the product of the two extremes equally diſtant from the two other extremes; or the product of any two terms of the ſeries is equal to the product of two other terms, which are equally diſtant from them.

Thus in the foregoing ſeries 3×96; or 24×12=6×48, for ſince all the terms are continully proportional the matter is ſufficiently manifeſt.

If a geometrical progreſſion beginning with unity, then the ſecond term will be equal to the ratio, the third the ſquare of the ratio, the fourth the cube of the ratio, &c. if therefore, in a geometrical progreſſion beginning with unity, we place 1 over the ſecond term, 2 over the third, 3 over the fourth, and ſo on in an arithmetical progreſſion, whoſe common difference is unity; then the terms of the arithmetical ſeries will expreſs the powers of the ratio, which their correſponding terms in the geometrical ſeries exhibit; the terms of the arithmetical ſeries may be called the indices or exponents of their correſponding terms in the geometrical expreſſion.

Thus { 0 1 2 3 4 5 are Indices.
1 2 4 8 16 32 may be called powers.

When the firſt term is equal to the ratio, then every ſucceeding term of the geometrical progreſſion or ſeries is the ſame power of the ratio, as the order of its place exhibits; then writing the arithmetical ſeries, 1 muſt be put over the firſt term, 2 over the ſecond, 3 over the third, &c. as in the following.

1 2 3 4 5 6 7 Indices.
2 4 8 16 32 64 128 Powers.

In any other geometrical progreſſion beginning with unity, the ſum of the indices of any two terms of the progreſſion will be the index of the product of the ſaid two terms; and if the index of any term be doubled, it will be the index of the ſquare of ſaid term.

The ſquare of any term in geometrical progreſſion not proceeding from unity, divided by the firſt or laſt term, will give a term of the ſame progreſſion, doubly diſtant from the firſt.

*Propoſition* 1*ſt*. When the firſt is an an unit, the ratio and and number of terms being known, to find the laſt or any one remote term.

*Rule*. Find a few of the leading terms, over which place their indices as before directed, then find what figure of the indices, which added together, will give the index of the term wanted; multiply the number ſtanding under ſuch indices into each other, and the laſt product will be the ſum required. *Note*. When indices begin with a cypher, the ſum of the indices made choice of, muſt be always one leſs than the number of terms given in queſtion, as 1, in the indices ſtand over the ſecond term.

EXAMPLES.

### EXAMPLES.

1. A boy agrees for 16 apples, to pay only the price of the laſt, reckoning a farthing for the firſt, an halfpenny for the ſecond, &c. doubling the price to the laſt, how much did he give for them? 34.2.8. *Anſ.* ~~30l. 2s. 6d.~~

2. A man bought a horſe, and by agreement was to give what the laſt nail would come to, at a farthing for the firſt, 2 for the ſecond, 4 for the third, &c. there were 4 ſhoes and 9 nails in each; I demand the price of the horſe?

*Anſ.* 35791394l. 2s. 8d.

*Propoſition 2nd.* In any ſeries not proceeding from unity the ratio and firſt term being given to find any remote term without producing all the intermediate terms.

*Rule.* Proceed as in the laſt propoſition, only obſerve to divide every product by the firſt term, and the quotient will be the term required.

### EXAMPLES.

3. A perſon dying, left eleven children, to whom and to his executor he bequeathed in the manner following, viz. to his executor for ſeeing his will performed 10l. the youngeſt child to have 30l. and ſo on, every child to exceed the next younger, in triple proportion, what will be the ſhare of the eldeſt? *Ans.* 1771470l.

4. A nobleman dying, left 10 ſons, to whom he left a certain ſum of money to be divided among them, viz. the youngeſt ſon to have 500l. the ſecond to have as much and half as much, and ſo on, every one to exceed the next younger in the ſame ratio of 1½, what is the ſhare of the eldeſt? *Anſ.* 19221l. 13s. 7d.+125.

*Propoſition 3rd.* When the firſt term, ratio and number of terms are given to find the ſum of all the terms.

*Rule.* Find the laſt term as before, from which take the firſt, divide the remainder by the ratio leſs one, and to that quotient add the laſt term, which gives the ſum required.

### EXAMPLES.

5. On New-year's day a gentleman married, and received of his father-in-law 1l. 1s. on condition that he was to have a preſent on the firſt day of every month for the firſt year,

which ſhould be doubled ſtill, to what he had the month before, what was the young lady's portion ?

*Anſ.* 3780 guineas Iriſh, or 4095 guineas Engliſh money, or 4299l. 15s.

6. One at a country fair had a mind to a ſtring of 20 fine horſes; but not caring to take them at 20 guineas Engliſh per head, the jockey conſented that he ſhould, if he thought good, pay but a ſingle farthing for the firſt, doubling it only to the nineteenth, and he would give the twentieth into the bargain: this being preſently accepted, how were they ſold per head? and what did the whole amount to?

*Anſ.* 27l. 6s. 1½d.$\frac{7}{10}$ each; the whole amounted to 546l. 2s. 7¾d.

## *Muſical Proportion.*

There is alſo a kind of proportion, called Muſical, which being but of little or no common uſe, a very ſhort account of it may here ſuffice.

Muſical Proportion is when of three numbers, the firſt hath the ſame proportion to the third as the difference between the firſt and ſecond hath to the difference between the ſecond and third: as theſe three, 6, 8, 12; where 6 : 12 :: 8—6 : 12—8, that is, as 6 : 12 :: 2 : 4. When four numbers are in Muſical proportion, then the firſt has the ſame proportion to the fourth as the difference between the firſt and ſecond hath to the difference between the third and fourth, as in theſe, 6, 8, 12, 18, where 6 : 18 :: 8—6 : 18—12, that is 6 : 18 :: 2 : 6. When numbers are in muſical progreſſion, their reciprocals are in arithmetical progreſſion; and the converſe, that is, when numbers are in arithmetical progreſſion, their reciprocals are in muſical progreſſion; ſo in theſe muſicals 6, 8, 12, their reciprocals $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, are in arithmetical progreſſion for $\frac{1}{6}+\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$; and $\frac{1}{8}+\frac{1}{8}=\frac{2}{8}=\frac{1}{4}$, that is, the ſum of the extremes is equal to double the mean, which is the property of arithmeticals.

The method to find out numbers in muſical proportion is beſt expreſſed by letters in algebra.

CHAP.

# CHAP. XII.

## PERMUTATION,

IS the changing or varying the order of things.

*Rule.* Multiply all the given terms one into another, and the laſt product will be the number of changes required.

### EXAMPLES.

1. How many changes may be rung upon 12 bells? and how long would they be ringing but once over, ſuppoſing 10 changes might be rung in 1 minute, and the year to contain 365 days 9 hours?

*Solution.* 1×2×3×4×5×6×7×8×9×10×11×12= 479001600 changes, which divided by 10=47900160 minutes, and if reduced =91 years, 3 weeks, 5 days, and 6 hours.

2. A young ſcholar coming into town for the convenience of a good library, demands of a gentleman with whom he lodged, what his diet would coſt for a year, who told him 10l. but the ſcholar not being certain what time he ſhould ſtay, aſked him what he muſt give him for ſo long as he ſhould place his family (conſiſting of 6 perſons beſides himſelf) in different poſitions every day at dinner, the gentleman thinking it would not be long, tells him 5l. to which the ſcholar agrees; what time did the ſcholar ſtay with the gentleman? *Anſ.* 5040 days, or 13 years, 19 hours, and 23 minutes, which will hardly amount to 7s. 3d. per annum.

3. Four gentlemen that were travelling, met together at a certain inn upon the road, where they were ſo pleaſed with their landlord, and each others company, that in a frolic they made a contract to ſtay at that place ſo long as they, together with their landlord, could ſit every day in a different order or poſition at dinner; required the time they ſtaid? *Anſ.* 120 days.

# CHAP. XIII.

## EQUATION OF PAYMENTS

IS the finding a time to pay at once ſeveral debts, due at different times, ſo that no loſs ſhall be ſuſtained by either party.

* *Rule.* Multiply each payment by the time in which it is due, then divide the ſum of the products by the ſum of the payments, and the quotient will be the time required.

Or thus: firſt find the preſent worth of each payment for its reſpective time, thus $\frac{S}{tr+1}=P$. Second, add all the preſent worths together, and call that ſum P; then will $S-P=D$ the rebate. Third, and $\frac{d}{Pn}=E$ the true equated time required.

1. Suppoſe I owed as follows, viz. 100l. payable in 6 months, 30l. payable in 7 months, 50l. payable in 5 months, and 80l. payable in 4 months, what is the equated time to pay the whole in one ſum? *Anſ.* $5\frac{4}{13}$ months.

2. A. owes

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* No rule in arithmetic has been the occaſion of ſo many diſputes, as that of Equation of payments, almoſt every writer upon this ſubject, has endeavoured to ſhew the fallacy of the methods made uſe of by others, and to ſubſtitute a new one in their ſtead, but the only true one is that by Mr. MALCOLM, in page 621 of his arithmetic, the principles of which are derived from the conſideration of intereſt and diſcount, the rule is, to the ſum of both payments, add the continued product of the firſt payment, the rate, or intereſt of 1l. for 1 year, and the time between payments, and call this the firſt number. 2. Divide the firſt number by twice the product of the firſt payment and the rate, and call the quotient the ſecond. 3. Divide the product of the ſecond payment, and the time between the payments by the product of the firſt payment and the rate, and call the quotient the third number. 4. From the ſquare of the ſecond number take the third and call the ſquare root of the difference the fourth number; then the difference of the ſecond and fourth number will be the equated time after the firſt payment is due. The firſt rule is not accurately true, but anſwers very near the truth. Suppoſe a ſum of money due immediately, and another ſum at the expiration of a certain time afterwards, and that it is propoſed to find a time to pay the whole at once, ſo that neither party ſhall ſuſtain loſs;

it

2. A. owes B. 52l. 7s. 6d. to be paid in $4\frac{1}{2}$ months, 80l. 10s. to be paid in $3\frac{1}{2}$ months, and 76l. 2s. 6d. to be paid in 5 months; what is the equated time to pay the whole debt? *Anſ.* $4\frac{1241}{4308}$ months.

3. A debt is to be paid in the following manner, viz. $\frac{1}{4}$ in 2 months, $\frac{1}{8}$ in 3 months, $\frac{1}{8}$ in 4 months, $\frac{1}{8}$ in 5 months, and the reſt in 7 months; what is the equated time to pay the whole debt? *Ans.* $4\frac{5}{6}$ months.

4 A. owes B 200l. to be paid in 9 months, and 600l. to be paid in a year and a half, at what time may he pay the whole without any detriment to himſelf? *Anſ.* $15\frac{3}{4}$ months.

5. A debt of a 1000l. to be paid in the following manner, viz. $\frac{1}{2}$ at 8 months, $\frac{1}{3}$ at 12 months, and the reſt at 18 months; what is the equated time to pay the whole in one payment? *Ans.* 11 months.

6. A debt of 500l. is to be diſcharged thus, 100l. at preſent, 300l. at 4 months, and the reſt at 6 months; what is the equated time to pay the whole in one ſum? *Anſ.* $3\frac{3}{5}$ months.

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it is plain, that the equated time muſt fall between the two payments and what is got by keeping the firſt debt after it is due ſhould be equated to what is loſt, by paying the ſecond before it is due, but the gain ariſing from keeping of a ſum of money before it is due, is evidently equal to the intereſt of the debt for that time, and the loſs which is ſuſtained by the paying of a ſum of money before it is due, is evidently equal to the diſcount of the debt for that time. Now to find the equated time, let $a =$ firſt payment, $b =$ the ſecond, $t =$ the time between the payments. $r =$ ratio or intereſt of 1l. for 1 year, and $x =$ the equated time of the firſt payment; then $arx =$ intereſt of $a$ for the time $x$ and $\overline{btr} - \overline{btr} \div \overline{r+tr+rx} =$ diſcount of $b$ for the time $\overline{t-x}$, therefore $arx = \overline{btr} \div \overline{br \div r + bt - rx}$: now let $\overline{a+b} \div ar = n$, $bt \div ar = m$ and $t+n = w$, then $x = \frac{w}{2} \pm \sqrt{w^2-4m}$, but of theſe two values of $x$ $\frac{w}{2} \pm \frac{1}{2}\sqrt{w^2-4m}$ is the whole value which anſwers the condition of the queſtion.

CHAP.

# CHAP. XIV.

## POSITION;

## *Or the Rule of Falſe.*[*]

IS ſo called, becauſe we uſe aſſumed numbers in order, thereby reaſoning from them, according to the nature of the queſtion we do, by thoſe ſuppoſed numbers find the true number ſought. This rule is divided into two parts, commonly called the Single and Double rules.[†]

*Rule.* Make choice of your poſition, work with that, according to the nature of the queſtion, as if it were the true number, and if you find (after ordering your poſition) the reſult either too much or two little; you may then find the true anſwer by this proportion, viz. as the reſult of your poſition : is to the poſition : : ſo is the *given number* : to the number ſought.

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[*] This rule has in ſeveral late arithmetical authors been omitted, becauſe, ſay they, all queſtions ſolvable by it, may be more ſpeedily, and better anſwered by a common ſmattering of algebra. But this, though true, does not ſeem to be dealing fairly with their pupils, who may a great many of them perhaps never be acquainted with that ſcience, and, if ſo, they would certainly thank their inſtructor for giving them ſomething which might in a little degree ſupply its place, and contribute in like manner to their amuſement. For the ſciences ſhould be made in ſome ſort entertaining, or elſe very few will be tempted to purſue them; and this is tacitly agreed to by the practice of writers claſſed far above arithmeticians. Beſides this, to have dealt conſiſtently, other rules, that might have been better effected by algebra, ſhould have been omitted, and Progreſſion (not to mention ſome other articles,) as not very uſeful where it is taught, would have gone along with it. But it may be anſwered, that ſeveral of the particulars there laid down lead to what is very uſeful in the mathematics, and conſequently a hint of their nature can never be unſeaſonably thrown in. I agree to it, and would have this another reaſon for retaining Poſition, as it may be applied ſometimes to the ſolution of *adfected* and *exponential* equations better than the method of ſeries. Several other approximations it is very uſeful in, and if great names, and great ſubjects, may raiſe it in the opinion of the learner, we ſhall obſerve to him, that no leſs a man than Dr. *Gregory*, in his aſtronomy, has applied it in determining, by approximation, the trajectory of a comet, *&c &c.*

[†] Thoſe queſtions in which the reſults are proportional to their ſuppoſitions, belong to ſingle poſition, ſuch are thoſe which require multiplication or diviſion of the number ſought, by any number, or in which it is to be increaſed or diminiſhed by itſelf any number of times, or by any part or parts of it, but thoſe in which the reſults are not proportional to their poſitions belong to the double rule; are thoſe in which the number ſought is increaſed or diminiſhed by ſome given number, which is no known part of the number required.

*Proof.* Add the ſeveral parts of the ſum together, and if the ſum agrees with that given, it is right.

EXAMPLES.

1. A ſchool-maſter being aſked how many ſcholars he had, ſaid, if I had as many, half as many, and one quarter as many more I ſhould have 88, how many had he? *Anſ.* 32.

*Solution.*

| | | | |
|---|---|---|---|
| Suppoſe he had | 40 | *then as* 110 : 88 :: 40 : | 32 *Anſ.* |
| As many | 40 | | 32 |
| Half as many | 20 | | 16 |
| ¼ as many | 10 | | 8 |
| | 110 | | 88 *Proof.* |

2. A perſon having about him a certain number of crowns, ſaid, if he had the third, fourth, and ſixth of them, they would make 54; I want to know how many he had? *Ans.* 72.

3. A gentleman bought a chaiſe, horſe, and harneſs, for 60l. the horſe came to twice the price of the harneſs, and the chaiſe to twice the price of the horſe and harneſs; what did he give for each? *Ans.* Horſe 13l. 6s. 8d. harneſs 6l. 13s. 4d. chaiſe 40l.

4. A. B. and C. being determined to buy a quantity of goods, which would coſt them 120l. agreed among themſelves that B. ſhould have a third part more than A. and C. a fourth part more than B. I deſire to know what each man muſt pay? *Ans.* A. 30l. B. 40l. and C. 50l.

5. A perſon delivered another a ſum of money unknown, to receive intereſt for the ſame, at 6 per cent per annum, ſimple intereſt, and at the end of 10 years received for principal and intereſt 300l. what was the ſum lent. *Ans.* 187l. 10s.

6. A man overtaking a maid driving a flock of geeſe, ſaid to her, how do you do, ſweet-heart, where are you going with theſe 40 geeſe? No, ſir, ſaid ſhe, I have not 40, but if I had as many more, and half as many, and ten geeſe beſides, I ſhould have 40; how many had ſhe? *Ans.* 12.

7. I have a ciſtern with three unequal cocks, containing 60 pipes of water the greateſt cock will empty the ciſtern in 1 hour, the ſecond in 2, and the third in 3; in what time will they empty the ciſtern, ſuppoſing they be all ſet open at once? *Anſ.* 32 minutes $43\frac{7}{11}$ ſeconds.

CHAP.

# CHAP. XV.

## DOUBLE POSITION,

IS when two ſuppoſitions are uſed; and if we miſs in both (as it generally happens) obſerve the nature of the errors, whether they be greater or leſs than the given number, and accordingly they muſt be made uſe of thus;

**Rule*. Set down your errors and poſitions. 2. Multiply each of the errors by the contrary ſuppoſition, and find the ſum and difference of the products. 3. If the errors are alike, divide the difference of the products by the difference of the errors, and the quotient will be anſwer. 4. If the errors are unlike, divide the ſum of the products by the ſum of the errors, and the quotient will be the anſwer.

*Note*. The errors are ſaid to be alike when they are both too great or too little; and unlike when the one is too great and the other too little, or inſtead of the above rule you may uſe the following proportion; as the difference of the errors if alike, or their ſum if unlike : is to the difference of the poſitions :: ſo is either error : to a fourth number, which fourth number, added to, or taken from the poſition producing the error, gives the number ſought.

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* The rule is founded on this ſuppoſition; that the firſt error is to the ſecond as the difference between the true and firſt ſuppoſed number is to the difference between the true and ſecond ſuppoſed number; when that is not the caſe, the exact anſwer to the queſtion cannot be found by this rule.

That the rule is true, according to the ſuppoſition, may be thus demonſtrated; let A. and B. be any two numbers produced from $a$. and $b$. by ſimilar operations it is required to find a number which is produced, by a like operation; put $X=$ number required, and let $N-A=r$ and $N-B=S$ then according to the ſuppoſition on which the rule is founded, as $r : S :: X-a : X-b$ whence by multiplying the means and extremes $rX-sb=sX-sa$ and by tranſpoſition $rX-sX=rb-sa$ and by diviſion $X=\frac{rb-sa}{r-s}$ the number ſought. Again if $r$ and $s$ be both negatives, we ſhall have $-r : -s :: X-a : X-b$, and therefore $-rX+rb=-sX+sa$ and $rX-s=rb-sa$ from whence $X=\frac{rb-sa}{r-s}$ as before. In like manner if $r$ or $s$ only be negative we ſhall have $X=\frac{rb+sa}{r+s}$ working as before, which is the rule. *Note*. It will be often advantageous to make 1 and 0 the ſuppoſition. Alſo in ſingle poſition 1 may be made a conſtant ſuppoſition in all queſtions, and in moſt caſes it is better than any other number.

1. A gentleman hath two horſes, and a ſaddle worth 50l. which if ſet on the back of the firſt horſe, will make his value double that of the ſecond; but if ſet on the back of the ſecond horſe, will make his value triple that of the firſt horſe: I demand the value of each horſe?

*Solution.*

Suppoſe the value of the firſt to be 20l.
To which add the value of the ſaddle, 50

2 ) 70 double the value of the 2d.

Then the value of the ſecond is 35
To which add the value of the ſaddle, 50

85=25 too much.

The ſum ſhould have been triple the value of the firſt horſe. £.
Again, Suppoſe the firſt horſe to be worth 24
Then proceeding as before, 50

2 ) 74

Alſo the value of the ſecond is 37
50

*Suppoſitions.* 87=15 too much.

20 $\times$ 25 then 25—15=10 difference of errors.
24 15

600 300 alſo, 600—300=300 difference of the products.

∴ 300÷10=30 the value of the firſt horſe.

30l.+50l. (the value of the ſaddle)=80l. double the value of the 2d horſe, which divided by 2=40l. his value; then 40+50=90=30×3=90 proof; *or thus*, As 10 diff. er. : 4 diff. ſup. :: 25 to : 10 then 20+10=30 the value of the firſt horſe, and ſecond horſe will be 40l. from the queſtion.

2. A man had two ſilver cups of an unequal weight, having one cover to both, of 5 ounces; now if the cover is

put on the leſs cup, it will double the weight of the greater cup; and ſet on the greater, it will be thrice as heavy as the leſs cup; what is the value of each cup?

*Ans.* 3 oz. the leſs; 4 oz. the greater.

3. There is a fiſh, whoſe head is 9 inches long, and his tail is as long as his head and half his body, and his body is as long as his tail and his head; what is the whole length of the fiſh? *Anſ.* 3 feet.

4. Double my money for me ſaid A. to B. and I will give thee 6d. out of the ſtock; with the remainder he applied in like manner to C. with equal ſucceſs, and gave him alſo 6d.: he repeated this propoſal to D. and then 6d. was all he had to give; pray what ſum had he to begin with? *Anſ.* 5$\frac{1}{4}$d.

5. A gentleman had a houſe with a garden, and a horſe in the ſtable for 500l.; now he paid 4 times the price of the horſe for the garden, and 5 times the price of the garden for the houſe; what was the value of the houſe, garden and horſe ſeparately?

*Anſ.* Horſe, 20l.; garden, 80l.; houſe, 400l.

6. Three gentlemen, A., B., and C. playing at hazard together, the money ſtaked was 112 guineas; but diſagreeing, each ſeized as many as he could; A. got a certain quantity; B. as many as A. and 16 more; but C. got only a 6th part of their ſum; how many had each got?

*Anſ.* A. 40; B. 56; and C. 16.

7. A gentleman going in a garden, meets with ſome ladies and ſays to them, "Good morning to you 10 fair maids:" "Sir, you miſtake," anſwered one of them, "we are not ten; but if we were twice as many more as we are, we ſhould be as many above 10, as we are now under," how many were there. *Anſ.* 5.

8. Two perſons, A. and B. lay out equal ſums of money in trade; A. gains 126l. and B. loſes 87l. and A.'s money is now double of B.'s; what did each lay out? *Ans.* 300l.

9. A labourer was hired for 40 days, upon this condition, that he ſhould receive 20d. for every day he wrought, and forfeit 10d. for every day he was idle: now he received at laſt 2l. 1s. 8d.; how many days did he work, and how many was he idle? *Anſ.* 30, and was idle 10.

10. A ſon aſked his father how old he was; his father replied, your age is now $\frac{1}{5}$ of mine, but 4 years ago, your age was only $\frac{1}{7}$ of what mine is now; what were their ages?

*Anſ.* 70 years the father's age, and ſon's age 14.

11. One being aſked the hour of the day, anſwered that the time paſſed from noon was equal to $\frac{3}{18}$ of the time remaining from midnight; I demand what o'clock?

*Ans.* 36 minutes paſt 1.

12. A gentleman finding ſeveral beggars at his door, gave to each fourpence, and had ſixpence left; but if he had given to each ſixpence, he would have wanted twelvepence; how many beggars were there? *Ans.* 14.

13. To find a number, which, if added to itſelf, and the ſum multiplied by the ſame, and the ſame number ſtill ſubtracted from the product; and laſtly, the remainder divided by the ſame, that it may produce 13? *Ans.* 7.

14. Find a number, which, being multiplied by 3, ſubtract 5 from the product; and the remainder divided by 2, if the number ſought be added to the quotient, that ſum will be 40? *Ans.* 17.

15. Two companions have got a parcel of guineas; ſays A. to B. if you will give me one of your guineas, I ſhall have as many as you have left: no, replies B. if you will give me one of your guineas, I ſhall have twice as many as you have left; How many guineas had each of them?

*Anſ.* B. 7, and A. 5.

16. When firſt the marriage knot was tied,
Betwixt my wife and me,
My age did her's as far exceed,
As three times three does three;
But when ten years, and half ten years,
We man and wife had been,
Her age came up as near to mine,
As eight is to ſixteen:
——————— Now tell me pray,
What was our ages on the wedding day?

*Ans.* His age 45, and her age 15.

17. A father dying, left to his three ſons, A., B. and C. his eſtate in money, dividing it as follows, viz. to A. he gave the half, wanting 44l.; to B. he gave a third, and 14l over; and to C. he gave the remainder, which was 82l. leſs than the ſhare of B.; what was the ſum left, and what was each of the ſon's ſhare?

*Ans.* The ſum left was 588l. whereof A. had 250l., B. 210l., and C. 128l.

18. What number is it that, being multiplied by 6, the product increaſed by 18, and the ſum divided by 9, the quotient will be 20? *Anſ.* 27.

19. Suppoſe

19. Suppoſe the number 50 was to be divided into two parts, ſo that the greater part being divided by 7, and the leſſer multiplied by 3, the ſum of this product, and the former quotient, may make the ſame number propoſed, which was 50; I demand the parts? *Anſ.* 35 and 15.

20. A young gentleman, at the age of 21 years, was told by his guardian, that his fortune conſiſted in caſh, to the amount of 7400l., and that his father died when he was but 10 years old; and for the money your father left, ſaid the guardian, I have allowed you 5 per cent per annum, ſimple intereſt, only I have deducted 100l. per annum for your education, &c.; what was the ſon's fortune that was left by the father? *Ans.* 5483l. 17s. 5$\frac{2}{31}$d.

*APPENDIX*

# APPENDIX

# OF

# *ALGEBRA.*

ALGEBRA is a kind of ſpecious arithmetic, in letters; and is that ſcience which teaches in a general manner the compariſon of abſtract quantities; by means whereof ſuch queſtions are reſolved, whoſe ſolutions would be ſought in vain from common arithmetic: in which the operator generally makes uſe of ſymbols or characters inſtead of numbers.

It is adviſed that every one, who would make himſelf a complete arithmetician, ſhould aſſiduouſly cultivate this art: it explores all the hidden and combined powers of numbers, it will greatly aſſiſt the judgment, and give the young ſtudent a habit of cloſe and infallible reaſoning on the various ſubjects to which it may be applied, and therefore it can never be too warmly recommended.

## *Definitions.*

1. *Like quantities*, are thoſe that conſiſt of the ſame letters; as $a$, $4a$, $-3a$. Alſo $bb$, $3bb$, $-11bb$; alſo $2abc$, $15abc$, $-abc$; &c.

2. *Unlike quantities*, are thoſe conſiſting of different letters, or of the ſame letters differently repeated; as $a$, $b$, $2c$, $-3d$ Alſo $a$, $2aa$, $-5aaa$.

3. *Given quantities*, are thoſe whoſe values are known.

4. *Unknown quantities*, are thoſe whoſe values are not known.

5. *Simple quantities*, are thoſe conſiſting of one term only, as $5b$, $3a^2c$, $13dcc$, &c.

6. *Compound quantities*, are thoſe conſiſting of ſeveral terms, as $a+b$, $2a-3c$, $a+2b-3d$, &c.

7. *Poſitive quantities*, are thoſe to be added.

8. *Negative quantities*, are thoſe to be ſubtracted.

9. *Like ſigns*, are either all +, or all —, ſee the characters.

10. *Unlike ſigns*, are + and —.

11. *The Coefficient*, is the number prefixed to any letter or letters in any term. As 3 is the coefficient of $3aa$. If no number be prefixed, then 1 muſt be underſtood, as $aa$ ſignifies $1aa$.

12. A *Binomial quantity*, is one conſiſting of two terms, as $2a+3b$. A *Trinomial*, one of three terms, as $\overline{a+b-c}$. A *Quadrinomial*, of four, &c. A *Reſidual* is a binomial, where one of the quantities is negative.

13. *Power* of a quantity, is its ſquare, cube, biquadrate, &c.

14. An *Equation*, is the mutual comparing of one thing with another, by the ſign of equality put between them.

15. A *dependent Equation*, is an equation which may be deduced from ſome others.

16. An *independent equation*, is one that cannot, by any means, be produced from the others.

17. A *pure equation*, is an equation containing but one power of the unknown quantity; as a *ſimple equation*, a *pure quadratic*, a *pure cubic*, &c.

18. An *affected equation*, is that which contains ſeveral powers of the unknown quantity; and is denominated according to the higheſt power in it; as an *affected quadratic*, an *affected cubic*, an *affected fourth power*, &c. Thus a ſimple equation contains only the ſimple quantity itſelf; a quadratic, a quantity of two dimenſions; a cubic, a quantity of three dimenſions; a biquadratic, of four dimenſions; &c.

19. *Index* or *exponent*, is the number ſet over a letter, ſhewing what power it is: as $a^3$; here 3 ſhews it is the third power; or that $a^3$ is equivalent to $aaa$. And thus $a^4$ is the ſame as $aaaa$; $a^5$ the ſame as $aaaaa$; &c. the index always ſhewing how often the letter is repeated.

20. A *Fraction*, conſiſts of two quantities placed one above another, with a line between them, as $\frac{a}{b}$; the upper ($a$) is called the numerator, the lower ($b$) the denominator.

21. A *Surd*, is a quantity that has not a proper root, as ſquare root of $a$ ($\sqrt{a}$), cube root of $bb$ ($\sqrt[3]{bb}$), &c. Roots of compound quantities that contain other ſurds, are called *univerſal ſurds*.

22. A *rational quantity*, is a quantity that has no radical ſign.

## *Axioms.*

1. If equal quantities be added to equal quantities, the ſums will be equal.

2. If equal quantities be taken from equal quantities the remainders will be equal.

3. If equal quantities be multiplied by equal quantities, the products will be equal.

4. If equal quantities be divided by equal quantities, the quotients will be equal.

5. The equal powers or roots of equal quantities, are equal.

6. If to or from equal quantities, unequal ones be added or ſubtracted, the ſums or remainders will be unequal.

7. If equal quantities be multiplied or divided by unequal quantities, the products or quotients will be unequal.

8. Quantities ſeverally equal to a third, are equal to one another.

9. The whole is equal to all the parts taken together.

10. If a quantity be added, and the ſame quantity ſubtracted, they deſtroy one another, and are both reduced to nothing.

## *Explanation of the characters uſed in Algebra.*

$+$ *Plus*, or *more*, the ſign of addition, as $6+2$ are 8.
$-$ *Minus*, or *leſs*, the ſign of ſubtraction, as $6-2$ are 4.
$\times$ *Multiplied into* or *by*, the ſign of multiplication, as $6\times2$ are 12. *Note*, $ab$ is the ſame as $a\times b$, and $3abc$ is the ſame as $3\times a\times b\times c$; &c.
$\div$ *Divided by*, the ſign of diviſion, as $6\div2$ are 3.
$=$ *Equal to*, the ſign of equality, as $6+2=8$.
: *is to*, : : *ſo is*, the ſigns of proportion, as $2 : 4 :: 6 : 12$.
$\surd$ The ſign of the ſquare root; the ſquare root of $9=3$ is $^2\sqrt{9}=3$, and the cube root of $8=2$ is $^3\sqrt{8}=2$
$\overline{6-4\times9}=18$ ſignifies that $\overline{6-4\times9}=18$.

*The learner ought firſt of all to be made acquainted with theſe ſigns and their names.*

## *Addition.*

*RULE.* 1. When the quantities are alike and the ſigns alike; then to the ſum of the coefficients of the like quantities, join the like quantity, prefixing the common ſign. 2. When the quantities are alike and the ſigns unlike; then to the difference of the coefficients join the like quantity, prefixing the ſign of the greater coefficient. 3. When the quantities are unlike; they are added by ſetting them down in order as they ſtand with their proper ſigns prefixed.

I. To add quantities when they are alike and the ſigns alike.

EXAMPLES.*

$$\begin{array}{lr} \text{1. Add} & +5a \\ & +7a \\ \hline \text{Total} & +12a \\ \hline \end{array} \qquad \begin{array}{lr} \text{2. Add} & -16ab \\ & -5ab \\ & -2ab \\ \hline \text{Total} & -23ab \\ \hline \end{array} \qquad \begin{array}{lr} \text{3. Add} & +4a-3x \\ & +5a-x \\ & +a-5x \\ \hline \text{Total} & +10a-9x \\ \hline \end{array}$$

4. Add $3dc+2dc+5by+by-c^2-4c^2+3d^2+d^2$ together.
*Anſ.* $5cd+6by-5c^2+4d^2$.

5. Add $2b-2c+5y-3p+b-5c+y-2p$ together.
*Anſ.* $3b-7c+$[illegible]$-5p$.

6. Add $5b-2c+5d-y+b-3c+d-2y$ together.
*Anſ.* $6b-5c+6d-3y$.

7. Add $3d^2-5yz+2c^2+2d^2-2yz+4c^2$ together.
*Anſ.* $5d^2-7yz+6c^2$.

II. To add quantities when they are alike and the ſigns unlike.

EXAMPLES.

$$\begin{array}{lr} \text{8. Add} & +6a \\ & -3a \\ \hline \text{Total} & +3a \\ \hline \end{array} \qquad \begin{array}{lr} \text{9. Add} & -16d \\ & +3d \\ \hline & -13d \\ \hline \end{array} \qquad \begin{array}{lr} \text{10. Add} & 3a-7b \\ & -3a+8b \\ \hline & 0+b \\ \hline \end{array}$$

11. Add $7b-2y+5x-3m-2b+7y-2x+m$ together.
*Anſ.* $5b+5y+3x-2m$.

12. Add $3z-2y+4x-y-z+4y-x+y$ together.
*Ans.* $2z+2y+3x$.

13. Add $7z-5b+3cz-2y-z+b-cz+4y$ together.
*Anſ.* $6z-4b+2cz+2y$.

III. When the quantities are unlike.

EXAMPLES.

14. Add $3c+5b$. *Anſ.* $3c+5b$. 15. Add $5c+7y$. *Anſ.* $5c+7y$.
16. Add $3c+2x-3y+2b-y$. *Ans.* $3c+2x-4y+2b$.

Examples

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* When a leading quantity has no ſign before it, + is always underſtood; and a quantity without any coefficient prefixed, is ſuppoſed to have 1 or unity for its coefficient; alſo quantities with the ſign + prefixed are called affirmative ones, and thoſe with the ſign — negative.

### Examples wherein all the three caſes are ſhewn.

17. Add $2b-3z+4d-2y+3bc-2+y-2bc+5+d-2z+3b+3z-12+4bc-5b+4y-3d+3x-2y+5bc-12+4b-6z$ together. Anſ. $4b-8z+2d+y+10bc-26$.

18. Add together $3zy-2bx+4cz-2dy+8+3dy+2cz-2zy+4bc-9+5bx+2zy-3cz+dy-3dy-14+2bx-7cz$. Anſ. $3zy+5bx-4cz-dy-15$.

## *Subtraction.*

SUPPOSE the ſigns of all the quantities (that are to ſubtracted) changed, that is, make the affirmative ſigns negative, and the negative ones affirmative; and then add them, ſo changed, to the other quantities; obſerving the rules in Addition.

### EXAMPLES.

| | | | |
|---|---|---|---|
| 1. From | $5z-7b$ | 2. From | $3z-5p+8y^2$ |
| Take | $2z-3b$ | Take | $-2z+2p-2y^2$ |
| Remains | $3z-4b$ | Remains | $5z-7p+10y^2$ |

3. From $3cd-2bx+4cy-2b^2$ take $cd-3bx+2cy-4b^2$. Anſ. $2cd+bx+2cy+2b^2$.

4. From $6bd-2cy+5p^2$ take $5bd+3cy+2p^2$. Anſ. $bd-5cy+3p^2$.

5. From $5n^2-6cy+2dp-d^2$ take $4n^2+2cy-3d-5d^2$. Anſ. $n^2-8cy+2dp+3d+4d^2$.

6. From $5p^2-3cdq+4my$ take $2p^2-3cdq+3my$. Anſ. $3p^2+my$.

7. From $7bd-2cy+4cx$ take $-bd-4cy+4cx$. Anſ. $8bd+2cy$.

8. From $7cy-3pd+5xz-y^2$ take $-cy+4pd-2xz-4y^2$. Anſ. $8cy-7pd+7zx+3y^2$.

9. From 0 take $2cx-2cy-d^2$. Anſ. $-2cx+2cy+d^2$.

## *Multiplication.*

*CASE* 1ſt. When both factors are ſimple quantities.

*Rule.* Multiply the coefficients of the two terms together, and to the product affix all the terms, and the reſult will be the whole product required.

*Caſe* 2d. When one of the factors is a compound quantity.

*Rule.* Find the product of the multiplier, and every term of the multiplicand ſeverally, and place them one after another with their proper ſigns, and the reſult will be the whole product together.

*Caſe* 3d. When both factors are compound quantities.

*Rule.* Multiply every term of the multiplicand by every term of the multiplier, and add the products together.

*Note.** Like ſigns produce +, and unlike ſigns —.

EXAMPLES.

| | |
|---|---|
| 1. Multiply $5z+2y-3d+2x$ | 2. Mult. $2m-3c-4x+3y$ |
| by $3c$ | by $-3p$ |
| *Anſ.* $15cz+6cy-9dc+6cx$ | *Anſ.* $-6pm+9cp+12px-9py$ |

3. Multiply $7bc-3cd+qs$ by $r$. *Anſ.* $7bcr-3cdr+qrs$.

4. Multiply $cdz-cdx-pzy+c^3$ by $-c^2$. *Anſ.* $-c^3dz+c^3dx+c^2pzy-c^5$.

5. Mult. $5cy-2cd+3c^2x$ by $bd$. *Anſ.* $5cybd-2cd^2b+3c^2xbd$.

6. Multiply $2cz-bd-cd+qs$ by $-cs$. *Anſ.* $-2c^2zs+bdcs+c^2ds-qcs^2$.

The 1ſt and 2nd caſes being nearly alike, we ſhall proceed to caſe 3d.

EXAMPLES.

| | | |
|---|---|---|
| 1. Multiply $a+b$ | 2. Multiply $a-b$ | 3. Multiply $3a-2b$ |
| by $a+b$ | by $a-b$ | by $5a+4b$ |
| $ab+b^2$ | $-ab+b^2$ | $12ab-8b^2$ |
| $a^2+ab$ | $a^2-ab$ | $15a^2-10ab$ |
| *Anſ.* $a^2+2ab+b^2$ | $a^2-2ab+b^2$ | $15a^2+2ab-8b^2$ |

4. Multiply $c+x$ by $c-x$. *Anſ.* $c^2-x^2$.

5. Multiply $m+x-z$ by $x-z$. *Anſ.* $xm+x^2-2xz-mz+z^2$.

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* That like ſigns make more and unlike ſigns leſs, in the product may be ſhewn thus: when $+a$ is to be multiplied by $+b$ this implies that $+a$ is to be taken as many times as there are units in $b$; and ſince the ſum of any number of affirmative terms is affirmative, it plain that $+a\times+b=+ab$. 2: If ſeveral quantities are to be multiplied together, the reſult will be exactly the ſame, in whatſoever order they are placed, for $a$ times $b$ is the ſame as $b$ times $a$; and therefore when $-a$ is to be multiplied by $+b$, or $+b$ by $-a$, this is the ſame as taking $-a$ as many times as there are units in $+b$, and ſince the ſum of any number of negative terms is negative, it is evident that $-\times+b$, or $+\times-a=-ab$. 3. When $-a$ is to be multiplied by $-b$, here $a-a=0$, therefore $+a-a\times-b$ is alſo $=0$, becauſe 0 multiplied by any quantity produces 0; and ſince the firſt term of the product, or $a\times-b$ is, by caſe 2, $=-ab$ the laſt term, or $-a\times-b$ muſt be $=+ab$ in order to make the ſum $-ab+ab=0$, conſequently $-a\times-b=+ab$.

6. Multiply $c+y+z$ by $y+z$. *Anſ.* $cy+y^2+2yz+cz+z^2$.

7. Multiply $3c+5y$ by $3c+5y$. *Anſ.* $9c^2+30yc+25y^2$.

8. Multiply $5x-7c$ by $5x+7c$. *Ans.* $25x^2-49c^2$.

9. Multiply $x+z-y$ by $x+z-y$.
*Ans.* $x^2+2xz+z^2-2xy-2xz+y^2$

10. Multiply $z+y-b+c$ by $y-b+c$.
*Ans.* $zy+$[illegible]$^2-2yb+2yc-zb+b^2+zc-2bc+c^2$.

11. Multiply $3x+2z-5d$ by $2z-5d$.
*Anſ.* $6zx+4x^2-20dz-15xd+25d^2$.

12. Multiply $4dz+5x-2y$ by $4z+5x-2y$.
*Ans.* $16dz^2+20zx+20dx-8yz+25x^2-8dzy-20xy+4y^2$.

13. Multiply $3x+2y+5z$ by $3x+2y+5z$.
*Anſ.* $9x^2+12xy+30xz+4y^2+20yz+25z^2$.

14. Multiply $30z+2cx-3cy$ by $3cz+2cx-3cy$.
*Anſ.* $90cz^2+6c^2xz+6cxz-9c^2y+4c^2$[illegible]$-9czy-12c^2xy$ $+9c^2y^2$.

15. Multiply $5cz+2bx+3cy-2bd$ by $2bx+3cy-2bd$.
*Ans.* $10czbx+4b^2x^2+12cybx-8b^2xd+15c^2xy-10czbd$ $-12bcdy+4b^2d^2+9c^2zy^2$.

16. Multiply $2c-3d+5y-4n$ by $2c-3d+5y-4n$.
*Ans.* $4c^2-12cd+20cy+9d^2-16cn-30dy+24dn+25y^2$ $-40ny+16n^2$.

17. Multiply $3x+2m-5n+2z-5p$ by $3x+2m-5n+2z-5p$.
*Anſ.* $9x^2+12mx-30xm+12xz+4m^2-20mn-30xp$ $+8mz+25n^2-20mp-20nz+50np+4z^2-20pz+25p^2$.

17. Multiply $d+x-y$
by $x$

$dx+x^2-xy$

$dx=72\times38=2736$
$x^2=38\times38=1444$

4180
$xy=38\times12=$ 456

Proof, 3724

$d=72$
$x=38$
$y=12$

$d=72$
$x=38$

110
$y=12$

98
$x=38$

784
294

3724 product

18. Multiply $b+c-d$ by $b$. *Anſ.* $b^2+cb-db=864$ $b=24$ $c=18$ and $d=6$.

19. Multiply $2+3d-z$ by $3c=6c^2+9cd-3z=6048dc=24$, $d=16$and $z=12$.

20. Multiply $3cz-4dx$ by $3cz-4dx$. *Anſ.* $9c^2z^2-24dxcz+16d^2x^2=746496$ ($c=24$, $d=12$, $x=6$, $z=16$).

21. Multiply $3bc-4cz$ by $2cz$. *Anſ.* $6bc^2z-8c^2z=23836$ ($b=36$, $c=14$, $z=8$).

22. Multiply $4bx+3bz-2cx-3cz$ by $4bx+3bz-2cx-3cz$. *Anſ.* $16b^2x^2+24b^2zx-16bcx^2+9b^2z^2-36bxcz-18bz^2c+4c^2x^2+12c^2xz+9c^2z^2=3444736$ ($b=24$, $x=16$, $z=12$, $c=8$).

## *Diviſion*

IS the direct contrary to that of Multiplication, and conſequently performed by direct contrary operations.

*Rule.* 1. When the quantities in the dividend have like ſigns of thoſe in the diviſor, and no coefficient in either; caſt off all the quantities in the dividend that are like thoſe in the diviſor, and ſet down the other quantities with the ſign + for the quotient. *

2. When the quantities in the dividend have unlike ſigns to thoſe in the diviſor, then ſet down the quotient quantities found as in the laſt rule with the ſign — before them.

3. If the quantities in the diviſor cannot be exactly found in the dividend, then ſet them both down like a vulgar fraction, and find all the quantities of the ſame letters that are in the dividend and diviſor, and proceed with the coefficients as in the 1ſt volume, page 145.

### EXAMPLES.

$$\begin{array}{rll}
\textit{diviſ.} & \textit{dividend.} & \textit{quotient.} \\
2bz\,) & 6bcdz+4bzd^2-2b^2z^2 & (\,3cd+2d^2-bz \\
 & 6bcdz & \qquad\qquad 2bz \\
\cline{2-2}\cline{3-3}
 & \quad 4bzd^2 & \text{the divid. proof } 6bcdz+4bzd^2-2b^2z^2 \\
 & \quad 4bzd^2 & \\
\cline{2-2}
 & \quad -2b^2z^2 & \\
 & \quad -2b^2z^2 & \\
\cline{2-2}
\end{array}$$

2. Divide

---

* That the like ſigns give + and unlike ſigns — as in Multiplication may be thus proved:

The diviſor multiplied by the quotient muſt produce the dividend; therefore, 1ſt, When both the terms are +, the quotient is + becauſe + × + produces + in the dividend.

2. When

2. Divide $3c^2dy-4c^2y^2+5cdy^2-cy^3$ by $cy$.
*Anſ.* $3cd-4cy+5dy-y^2$.

3. Divide $9c^2z^2-12bz^2c+3bzdc$ by $3cz$. *Anſ.* $3cz-4bz+bd$.

4. Divide $b^3x-3bx^3+2b^2x^2-5bcx^2$ by $bx$.
*Anſ.* $b^2-3x^2+2bx-5cx$.

5. Divide $5bx^3-2b^2x^2+5b^3x-7bx$ by $7bx$.
*Anſ.* $\frac{5}{7}x^2-\frac{2}{7}bx+\frac{5}{7}b^2-1$.

6. Divide $y^4-x^4$ by $y-x$. *Anſ.* $y^3+xy^2+x^2y+x^3$.

7. Divide $9c^2z^2-12bz^2c+3bzdc$ by $3cz$. *Anſ.* $3cz-4bz+bd$.

8. Divide $3bc^2d-4c^2d+5cd^2-7c^3d$ by $5cd$.
*Anſ.* $\frac{3}{5}bc-\frac{4}{5}c+d-\frac{7}{5}c^2$.

9. Divide $z^3+4z^4b+6z^2b^2+4zb^3+b^4$ by $z+b$.
*Anſ.* $z^3+3z^2b+3zb^2+b^3$.

10. Divide $c^5-5c^4z+10c^3z^2-10c^2z^3+5cz^4-z^5$ by $c^2-2cz+z^2$. *Anſ.* $c^3-3c^2z+3cz^2-z^3$.

11. Divide $x^5-5x^4y+10x^3y^2-10x^2y^3+5xy^4-y^5$ by $x-y$.
*Anſ.* $x^4-4x^3y+6x^2y^2-4xy^3+y^4$.

12. Divide $x^7-7x^6z+21x^5z^2-35x^4z^3+35x^3z^4-21x^2z^5+7xz^6-z^7$ by $x^3-3x^2z+3xz^2-z^3$.
*Anſ.* $x^4-4x^3z+6x^2z-4xz^3+z^4$.

13. Divide $\frac{1}{2}bc-\frac{2}{5}bcd+cy$ by $\frac{3}{4}c$. *Anſ.* $\frac{2}{3}b-\frac{8}{15}bd+\frac{4}{3}y$.

14. Divide $\frac{2}{3}c^2z^2-\frac{1}{2}cd^2z+\frac{2}{5}czm^2-\frac{7}{8}cz^3$ by $\frac{3}{5}cz$.
*Anſ.* $\frac{10}{9}cz-\frac{5}{6}d^2+\frac{2}{3}m^2-\frac{35}{24}z$.

15. Divide $\frac{125}{512}y^3-\frac{27}{343}p^3$ by $\frac{5}{8}y-\frac{3}{7}p$.
*Anſ.* $\frac{25}{64}y^2+\frac{15}{56}yp+\frac{9}{49}p^2$.

16. Divide $\frac{1}{16}p^4-\frac{1}{3}p^3z+\frac{2}{3}p^2z^2-\frac{16}{27}pz^3+\frac{16}{81}z^4$ by $\frac{1}{2}p-\frac{2}{3}z$.
*Anſ.* $\frac{1}{8}p^3-\frac{1}{2}p^2z+\frac{2}{3}pz^2-\frac{8}{27}z^3$.

## *Involution.*

INVOLUTION is performed as **Multiplication**; but here the quantity firſt given, which is called the root, is the conſtant multiplier, and the products are called powers, viz. the firſt product is the ſquare, or ſecond power; the ſecond is the cube or third power, &c.

*Rule.** 1. Multiply the quantity into itſelf as many times as there are units in the index leſs one, and the laſt product will be the power required.

---

2. When they are both — the quotient is alſo + becauſe +×− produceth − in the dividend.

3, When one of them is + and the other — the quotient is — becauſe −×+ produces − in the dividend, and −×− produces + in the dividend.

* The *n*th power of any product is equal to the *n*th power of each of the factors multiplied together. And the *n*th power of a fraction is equal to the *n*th power of the numerator divided by the *n*th of the denominator.

2. If the quantity be itſelf a ſurd it will be involved, by multiplying its exponent by the exponent of the propoſed power.

3. Quantities compoſed of ſeveral factors multiplied together, are involved by raiſing each factor to the power propoſed.

4. A fraction is involved by raiſing both the numerator and denominator to the power propoſed.

| The ſquare of | The cube of | The Biquadrate |
|---|---|---|
| $c = c^2$ | $x = x^3$ | $y = y^4$ |
| $3bd = 9b^2d^2$ | $7z = 343z^3$ | $cy = c^4y^4$ |
| $\frac{1}{2}c^2 = \frac{1}{4}c^4$ | $3xy = 27x^3y^3$ | $2bz = 16b^4z^4$ |
| $4cd = 16c^2d^2$ | $\frac{1}{2}czx = \frac{1}{8}c^3z^3x^3$ | $\frac{1}{2}c^2z = \frac{1}{16}c^5z^4$ |
| $\frac{7}{8}bcz = \frac{49}{64}b^2c^2z^2$ | $\frac{7}{8}c^2z = \frac{343}{512}c^6z^3$ | $\frac{1}{3}bc = \frac{1}{81}b^4c^4$ |
| $\frac{1}{12}b^3c = \frac{1}{144}b^6c^2$ | $\frac{4}{5}c^2z^3 = \frac{64}{125}z^6c^9$ | |

1. What is the ſquare of $x+z$? $=x^2+2xz+z^2$.
2. What is the cube of $x+z$? $=x^3+3x^2z+3xz^2+z^3$.
3. What is the fourth power of $x+z$? $=x^4+4x^3z+6x^2z^2+4xz^3+z^4$.
4. What is the ſeventh power of $x+z$?

*Anſ.* $x^7+7x^6z+21x^5z^2+35x^4z^3+35x^3z^4+21x^2z^5+7xz^6+z^7$.

*Note.* Any power of a reſidual differs not from the ſame power of a binomial only in the ſigns + and — which will take place alternately.

There is alſo a rule or theorem, given by Sir Iſaac Newton, whereby any power of a binomial $x+y$, or a reſidual $x-y$, may be expreſſed in ſimple terms, without the trouble of thoſe tedious multiplications which are required otherwiſe.

*Theo.* $$\overset{1}{1} \times \overset{2}{\frac{m-0}{1}} \times \overset{3}{\frac{m-1}{2}} \times \overset{4}{\frac{m-2}{3}} \times \overset{5}{\frac{m-3}{4}} \times \overset{6}{\frac{m-4}{5}} \times \overset{7}{\frac{m-5}{6}}$$
$$\times \overset{8}{\frac{m-6}{7}} \times \overset{9}{\frac{m-7}{8}} \times \overset{10}{\frac{m-8}{9}} \times \overset{11}{\frac{m-9}{10}} \times \overset{12}{\frac{m-10}{11}}, \text{ \&c.}$$

*Note,* $m$ is the exponent of the power; that is $m=7$ in the ſeventh power; 6 in the ſixth, &c.

The ſum of the coefficients in every power, is equal to the number 2, raiſed to that power: Thus, $1+1=2$ for the firſt power; $1+2+1=4=2^2$ for the ſquare; $1+3+3+1=8=2^3$ for the cube, or third power, &c.

So that if $x+y$ was to be raiſed to the 5th power, the terms, without the coefficients are $a^5$, $a^4x$, $a^3x^2$, $a^2x^3$, $ax^4$, $x^5$; and the coefficients will be $1, 5, \frac{5\times4}{2}: \frac{10\times3}{3}. \frac{10\times2}{4}. \frac{5\times1}{5}$ and therefore the 5th power is $a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5$.

2. What is the 8th power of $x+y$?

*Anſ.* $x^9+8z^7y+28z^6y^2+56z^5y^3+70z^4y^4+56z^3y^5+28z^2y^6+8zy^7+y^8$.

3. What is the 10th power of $z-y$?

*Anſw.* $z^{10}-10z^9y+45z^8y^2-120z^7y^3+210z^6y^4-252z^5y^5+210z^4y^6-120z^3y^7+45z^2y^8-10z^9y+y^{10}$

4. What is the 12th power of $z+y$?

*Ans.* $z^{12}+z^{11}y+66z^{10}y^2+220z^9y^3+495z^8y^4+792z^7y^5+924z^6y^6+792z^5y^7+495z^4y^8+220z^3y^9+66z^2y^{10}+12zy^{11}+y^{12}$.

To ſquare any binomial, trinomial, quadrinomial, or multinomial. A uſeful

*Rule.* Take the ſquare of each of the given quantities and connect them together with ſign $+$ and take a double product of the firſt given quantity into all the following quantities, and a double product of the ſecond given quantity into all the following quantities, &c. and ſo on until you come to the laſt.

1. Square $z+x$. *Anſ.* $z^2+2zx+x^2$.
2. Square $x-y$. *Ans.* $x^2-2xy+y^2$.
3. Square $x+y-z$. *Ans.* $z^2+x^2+y^2+2zx-2zy-2xy$
4. Square $z-x+y-p$. *Ans.* $z^2+x^2+y^2+p^2-2zx+2zy-2zp-2xy+2xp-2yp$.
5. Square $z+x-y+m-n+p$. *Anſ.* $z^2+x^2+y^2+m^2+n^2+p^2+2zx-2zy+2zm-2zn+2zp-2xy+2xm-2xn+2xp-2ym+2yn-2yp-2mn+2mn-2np$.

## *Evolution**

IS the converſe of Involution, and in ſimple powers the root is evident by inſpection, but in compound powers, (viz. ſeveral

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* Any even root of an affirmative quantity may be either $+$ or $-$; thus; the ſquare root of $a^2$ is either $+a$ or $-a$: for $+a\times+a=+a^2$ and $-a\times-a=+a^2$.

And

ſeveral quantities connected with the ſigns — and +) you muſt take the roots of the ſimple powers, and connect them together with their proper ſigns and you will probably have the root required, otherwiſe a ſurd, which may be proved by Involution.

| The ſquare root of | The cube root of |
|---|---|
| $c^2=c$ | $z^3=z$ |
| $4x^2z^4=2az^2$ | $c^3z^3=cz$ |
| $\frac{1}{36}b^2y^2=\frac{1}{6}by$ | $512b^3y^3=8by$ |
| $\frac{4}{49}c^2x^6=\frac{2}{7}cx^4$ | $\frac{1}{343}cy=\frac{1}{7}c^2y$ |
|  | $216c^3x^3=6bx$ |

1. What is the ſquare root of $z^2+2zb+b^2$ ?

$$2z+b\,)\;z^2+2zb+b^2\;(\;z+b \quad \textit{Anſ.}$$

$$z^2$$

$$\overline{\phantom{2bz+b^2}}$$

$$2bz+b^2$$

$$2bz+b^2$$

$$\overline{\phantom{2bz+b^2}}$$

$$0 \quad 0$$

2. What is the ſquare root of $x^2-2cx+c^2$ ? *Anſ.* $x-c$

3. Required the root of $z^2+zd+\frac{1}{4}d^2$ ? *Ans.* $z+\frac{1}{2}d$.

4. Required the root of $x^2-\frac{2}{3}px+\frac{1}{9}p$ ? *Anſ.* $x-\frac{1}{3}p$.

5. Required the root of $x^2+x+\frac{1}{4}$ ? *Ans.* $x+\frac{1}{2}$.

6. Required the root of $z^2+2bz+2cz+b^2+2bc+c^2$ ? *Ans.* $z+b+c$.

7. What is the ſquare root of $z^2-2cz-2dz+c^2+2cd+d^2$ ? *Ans.* $z-c-d$.

8. What is the ſquare root of $z^2-cz+pz+\frac{1}{4}c^2-\frac{1}{2}cp+\frac{1}{4}p$ ? *Anſ.* $z-\frac{1}{2}c+\frac{1}{2}p$.

9. Required the ſquare root of $x^4+2cx^2-2dx^2-2mx^2+c^2+d^2+m^2-2cd+2dm-2cm$ ? *Ans.* $x^2+c-d-m$.

10. What is the ſquare root of $z^2-\frac{2}{3}dz-\frac{2}{5}cz+\frac{1}{9}d^2-\frac{2}{15}cd+\frac{1}{25}c^2$ ? *Ans.* $z-\frac{1}{3}d-\frac{1}{5}c$.

11. Required the ſquare root of $z^6-bz^3-cz^3+dz^3+\frac{1}{4}b^2$ $\frac{1}{4}c^2+\frac{1}{4}d^2+\frac{1}{2}bc-\frac{1}{2}ld-\frac{1}{2}cd$ ? *Ans.* $z^3-\frac{1}{2}l-\frac{1}{2}c+\frac{1}{2}d$.

To

---

And an odd root of any quantity will have the ſame ſign as the quantity itſelf: thus, the cube root of $+a^3$ is $+a$ and the cube root of $-a^3$ is $-a$ for $+a\times+a\times+a=+a^3$ and $-a\times-a\times-a=-a^3$.

Any even root of a negative is impoſſible; for neither $+a\times+a$ nor $-a\times-a$ can produce $-a$ .

The *n*th root of an affirmative product is equal to the *n*th root of each of the factors multiplied together: the ſame holds good of a negative product when *n* is odd.—The *n*th root of a fraction is equal to the *n*th root of the numerator divided by the *n*th root of the denominator, attending to the foregoing limitation.

To find the roots of powers in general: 1ſt, of compound quantities,

*Rule.** 1. Find the root of the firſt term, and place it in the quotient.

2. Subtract its power from that term, and bring down the ſecond term for a dividend.

3. Involve the root laſt found to the next lower power, and multiply it by the index of the given power for a diviſor;

4. Divide the dividend by the diviſor, and the quotient will be the next term of the root.

5. Involve the whole root, and ſubtract and divide as before; and ſo on till the whole is finiſhed.

### EXAMPLES.

1. Required the cube root of $x^6+5x^5-40x^3+96x-64$.

$$\begin{array}{l}
x^6+6x^5-40x^3+96x-64(x^2+2x-4 \text{ } \mathit{Anſ.}\\
x^6\\
\hline
3x^4)6x^5\\
\quad 6x^5\\
\hline
x^6+6x+12x^4+8x^3\\
\hline
3x^4)0-12x\\
\hline
x^6+6x^5-40x^3+96x-64\\
\hline
\qquad\qquad 0
\end{array}$$

2. Required the ſquare root of $a^4-2a^3x+3a^2x^2-2ax^3+x^4$. *Anſ.* $a^2-ax+x^2$.

3. What is the cube root of $x^6-6x^5+15x^4-20+x^3+15x^2-6x+1$? *Anſ.* $x^2-2x+1$.

---

* As this method, in high powers, is generally thought too laborious, it may not be improper to obſerve, that the roots of compound quantities may ſometimes be eaſily diſcovered thus: 1. Extract the roots of ſome of the moſt ſimple terms and connect them together by the ſigns + or —, as may be judged moſt ſuitable for the purpoſe. 2. Involve the compound root, thus found, to the propoſed power, and if it be the ſame with the given quantity, it is the root required. 3. But if be found to differ only in ſome of the ſigns, change them from + to —, or from — to +, till its power agrees with the given one throughout: but if not, then the compound quantity is a ſurd, and its root can only be expreſſed by the radical ſign $\surd$.

4. What is the ſquare root of $a^2+2ab+2ac+b^2+2bc+c^2$? *Anſ.* $a+b+c$.

5. Required the biquadrate root of $16a^4-96a^3x+216a^2x^2-216ax^3+81x^4$. *Anſ.* $2a-3x$.

6. What is the fifth root of $32x^5-80x^4+80x^3-40x^2+10x-1$? *Anſ.* $2x-1$.

## FRACTIONAL QUANTITIES.

THE ſame rules are applicable here, as in vulgar fractions; due regard being had to Multiplication, Diviſion, &c. of ſpecies before delivered; ſee page 143, 151, 153, 154, 155, Volume 1ſt.

### *Reduction of Fractions.*

I. To reduce mixed numbers to fractional quantities.

EXAMPLES.

1. Reduce $x+\frac{c}{z}$ to an improper fraction.

$$x+\frac{c}{z}$$

$$\text{Anſ. } \frac{xz+c}{c}=x+\frac{c}{z} \text{ Proof.}$$

Reduce the following mixed quantities to improper fractions.

| | | |
|---|---|---|
| 2. | $z+\frac{d^2-b^2}{x}$ | *Anſwers.* $\frac{zx+d^2-b^2}{x}$ |
| 3. | $z-\frac{c^2+d^2}{c-d}$ | $\frac{zc-zd-c^2+d^2}{c-d}$ |
| 4. | $x-y+\frac{y^2}{x+y}$ | $\frac{x^2-y^2+y^2}{x+y}=\frac{x^2}{x+y}$ |
| 5. | $x-z+c+\frac{z^2+c^2}{x+z-c}$ | $\frac{x+2zc}{x+z-c}$ |
| 6. | $x+y-z+\frac{y^2-2xy}{x-y+z}$ | $\frac{x^2-z^2+2zy-2xy}{x-y+z}$ |
| 7. | $z+x+\frac{dx-x^2}{z-x}$ | $\frac{dx+z^2-2x^2}{z-x}$ |

II. To reduce compound fractional quantities to ſingle ones.

EXAMPLES.

### EXAMPLES.

1. Reduce $\frac{z}{b}$ of $\frac{y}{x}$ to a ſingle fraction. *Anſwers,* $\frac{zy}{bx}$

2. ——— $\frac{z}{x}$ of $\frac{a+z}{b}$ ——————— $\frac{za+z^2}{xb}$

3. ——— $\frac{z}{x}$ of $\frac{x+y}{x-y}$ ——————— $\frac{zx+xy}{x^2-xy}$

4. ——— $\frac{z}{y}$ of $\frac{x+z}{m-n}$ of $5b$ ——— $\frac{5bzx+5bz^2}{ym-yn}$

5. ——— $\frac{x}{y}$ of $\frac{x-1}{m}$ of $\frac{x-z}{m-n}$ of 1 $\frac{x^3-x^2z+xz-x^2}{m^2y-mny}$

III. To reduce fractions of different denominators to fractions of the ſame value, having a common denominator. (See the rule, Volume 1ſt, page 147.)

### EXAMPLES.

1. Reduce $\frac{x}{y}$ and $\frac{c}{z}$ to a common denominator, $\frac{xz}{yz}$ & $\frac{cy}{yz}$

2. ——— $\frac{c}{y}$ & $b$ & $\frac{m}{n}$ to a com. den. $\frac{cn}{ny}$ & $\frac{nby}{ny}$ & $\frac{my}{ny}$

3. ——— $\frac{c}{z}$ & $\frac{m}{y}$ & $\frac{x}{p}$ & $\frac{r}{s}$ $\frac{cyps+zmps+zyxps+rzyp}{zyps}$

4. ——— $x+y+\frac{c}{c+d}$ & $c-\frac{z}{x}$ to a common denominator.

*Anſ.* when added, $\frac{x^2c+xcy+dx^2+dxy+cx+c^2x-cdx-dz}{cx+dx}$

5. ——— $\frac{x+b}{c-d}$ & $\frac{c+d}{x-b}$ & $\frac{x-z}{c+d}$ to a common denominator.

*Anſ.* $\frac{x+b}{c-d}=\frac{cx^2-cb^2+d^2x-db^2}{xc^2-bc^2-d^2x+bd^2}$, & $\frac{x+d}{x-b}=$

$\frac{c^3+c^2d-cd^2-d^3}{xc^2-bc^2-d^2x+bd^2}$, & $\frac{x-z}{c+d}=$

$\frac{cx^2-cx-c^bx+cbz-x^2d+dxz+dbz-dbz}{xc^2-bc^2-xd^2+bd^2}$ when added.

IV. To reduce Fractions to lower terms.

*Rule.* When any quantity is in every term of both the numerator and denominator of a fraction; then, if both be divided by that quantity, the fractions will be reduced to lower (or its loweſt) terms.

EXAMPLES.

1. Reduce $\frac{2c^2z}{4c^3y}$ &c. $=\frac{z}{2cy}$  2. Reduce $\frac{3dxz^2}{9dxy^2}$ &c. $=\frac{z^2}{3y^2}$

3. Reduce $\frac{2cz^2xy}{4c^3by}$ &c. *Anſwer* $\frac{z^2x}{2c^2b}$

4. Reduce $\frac{3d^2x^2-9dcx}{12dxp^2d^2x^2}$ &c. $\frac{dx-3c}{4p^2-2dx}$

5. Reduce $\frac{2c^3x-6cx^2+2cx}{2cx+4c^2x^2-6cx^3}$ &c. $\frac{c^2-3x^2+1}{1+2cx-3x^2}$

6. Reduce $\frac{3c^2y^2-6cy^3+12c^3y}{6cy^3+9c^3-3c^2y^2}$ $\frac{cy-2y^2+4c^2}{2y^2+3c^2-cy}$

## *Addition.*

*Rule.* Proceed in every reſpect as directed in page 151, Volume 1ſt, due regard being had to Addition of Algebra, ſimple quantities.

EXAMPLES.

1. To $\frac{x}{z}$ add $\frac{m}{y}$ *Anſ.* $\frac{zy+my}{xy}$

2. To $\frac{z}{y}$ & $\frac{x}{m}$ add $\frac{P}{q}$ *Anſ.* $\frac{zmq+xyq+Pmy}{myq}$

3. To $s$ add $\frac{4p-s^2}{2s}$ *Anſ.* $\frac{4p+s^2}{2s^2}$

4. To $z+\frac{b}{y}$ add $\frac{c-y}{x}$ *Anſ.* $\frac{zyx+bx+cy-y^2}{yx}$

5. To $c+d+\frac{d^2}{c-d}$ add $\frac{z-d}{c+d}$ *An.* $\frac{cz-cx-dz+dx^2+c^3+dc^2}{c^2-d^2}$

6. Add together $\frac{x+z}{c-d}$ & $\frac{c+d}{x-z}$ & $\frac{m-n}{m+n}$

*Anſwer* $\frac{cxm+cxn-czm+czn-dxm+dxn+dzm-dzn+mc^2-md^2+nc^2-nd^2+mx^2-mz^2+nx^2+z^2}{mxc^2-mxd-mcd+mdz+nxc^2-nxd-ncd+nzd}$

*Subtraction.*

## *Subtraction.*

Due regard muſt be paid to the rules, page 152, Vol. 1ſt, and to the ſubtraction of Algebra.

EXAMPLES.

1. From $\frac{z}{x}$ take $\frac{z}{p}$ *Anſw.* $\frac{pz-zx}{xp}$

2. From $\frac{pz}{xp}$ take $\frac{zx}{px}$ $\frac{pz-zx}{xp}$

3. From $z+y$ take $\frac{ax}{z-y}$ $\frac{z^2-y^2-ax}{z-y}$

4. From $s$ take $\frac{s+d}{2}$ $\frac{s-d}{2}$

5. From $\frac{2z^2+d^2}{2p}$ take $2c$ $\frac{2z^2+d^2-2pc}{2p}$

6. From $\frac{5z^2-3d^2}{2d}$ take $d$ $\frac{2z^2-d^2}{2d}$

7. From $c+\frac{z^2-y^2}{d}$ take $z-\frac{b+d}{z}$

$$\frac{cdz+z^3-zy^2-dz^2+db-d^2}{dz}$$

## *Multiplication.*

* Obſerve the rule, page 154, Volume 1ſt, and multiplication of Algebra, page 73.

EXAMPLES.

1. Multiply $\frac{z}{y}$ by $\frac{m}{x}$. $\begin{array}{c} y \\ x \\ \hline yx \end{array}$ & $\begin{array}{c} m \\ z \\ \hline zm \end{array}$ *Anſ.* $\frac{zm}{xy}$

2. Multiply $\frac{z+y}{m-n}$ by $\frac{z}{y}$ $\frac{z^2+zy}{my-ny}$

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* 1. When the numerator of one fraction, and the denominator of another can be divided by ſome quantity which is common to each, the quotient may be uſed inſtead of them. 2. When a fraction is to be multiplied by an integer, the product is found by multiplying the numerator by it; and if the integer be the ſame with the denominator, the numerator may be taken for the product. 3. When a fraction is to be multiplied by any quantity, it is the ſame thing whether the numerator be multiplied by it, or the denominator divided by it.

3. Multiply $z+\frac{x}{y}$ by $\frac{z+y}{x}$ *Anſwers.* $\frac{z^2y+zy^2+zx+xy}{xy}$

4. Multiply $\frac{z-y+x}{d}$ by $d$ $z-y+x$

5. Multiply $\frac{z-y}{x+y}$ by $z+\frac{y}{p}$ $\frac{pz^2+zy-pyz-y^2}{px+py}$

6. Multiply $\frac{2z-5d+3x}{5d}$ by $5d$ $2z-5d+3x$

7. Multiply $\frac{z+y}{b-c}$ by $b-c$ $z+y$

## *Diviſion.*

Obſerve the rules, page 155, Volume 1ſt, and diviſion of Algebraic quantities.

### EXAMPLES.

1. Divide $\frac{z}{x}$ by $\frac{c}{y}$ $\frac{z}{x}\times\frac{y}{c}=\frac{zy}{xc}$ *Anſwer.*

2. Divide $2z$ by $\frac{c+x}{c-y}$ $\frac{2zc-2zy}{c+x}$

3. Divide $\frac{x+z-y}{m+n}$ by $3d$ $\frac{x+z-y}{3dm+3dn}$

4. Divide $3b+\frac{c-z}{d}$ by $\frac{c-x}{y}$ $\frac{3bdy+cy+zy}{cd-dx}$

5. Divide $\frac{3x-2z}{5c}$ by $\frac{z+b}{5c}$ $\frac{2x-2z}{x+b}$

6. Divide $\frac{3zc-2xd+3yp}{2cz}$ by $\frac{2cz-dx}{2xz}$ $\frac{3zc-2xd+3yp}{2cz-dx}$

7. Divide $\frac{x+c-x}{m-n}$ by $c+\frac{x}{z}$ $\frac{xz+zc-zx}{czm-czn+xm-xn}$

8. Divide $\frac{5z-2y}{2bd}$ by $\frac{2z+y}{2yd}$ $\frac{5z-2y}{2x+y}$

9. Divide $\frac{3zc-2xd+3yp}{2zc}$ by $\frac{2zc-dx}{2cz}$ $\frac{3zc-2xd+3yp}{2cz-dx}$

10. Divide $\frac{2bx-3cz+by}{\frac{4}{3}cz}$ by $\frac{c^2-z^2}{\frac{4}{3}cz}$ $\frac{2bx-3cx+by}{c^2-z^2}$

11. *Divide $\frac{2b^2x-3dz}{c-z}$ by $\frac{2bz-4bz}{c-z}$ $\frac{2b^2x-3dx^2}{2bz-4bz}$

SURD

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* From theſe examples it appears, 1ſt, That fractions having one common

## SURD QUANTITIES.

ROOTS that cannot be expreſſed in natural numbers, are called ſurd roots; and in ſpecies, ſurd roots are ſuch as have no effable expreſſion, except by the radical ſign $\sqrt{\phantom{x}}$; or generally by $\overline{\qquad}\Big)^{\frac{m}{n}}$

### *Addition.*

When the ſurds are alike, (*i. e.* when the radical ſigns are of the ſame dimenſions,) and the quantities under the radical ſigns are alike; then if ſuch ſurds are joined with rational quantities; to the ſum or difference of the rational quantities you are to join the ſurd quantity.

When the ſurds are unlike, or when the ſurds are alike and the quantities under them unlike, they can only be added or ſubtracted by their ſigns.

EXAMPLES.

1. To $2\sqrt{xb}$ add $3\sqrt{xb}$

$$\begin{array}{l} 2\sqrt{xb} \\ 3\sqrt{xb} \\ \hline 5\sqrt{xb} \end{array}$$ *Anſwer.*

2. To $8\sqrt{cx+z^2}$ add $-2\sqrt{cx+z^2}$ *Anſ.* $6\sqrt{cx+z^2}$

3. To $5b\sqrt{cy-bx}$ add $-b\sqrt{cy-bx}$ $4b\sqrt{cy-bx}$

4. To $3\sqrt[3]{xz}$ add $-2\sqrt[3]{xz}$ $\sqrt[3]{xz}$

5. To $5z^2\sqrt[3]{cx-x^2}$ add $-2z\sqrt[3]{cx-x^2}$ $3\sqrt[3]{cx-x^2}$

6. To $3d\sqrt{x^2-by}$ add $-3d\sqrt{x^2-by}$ $0$

7. To $-5\sqrt{xy}$ add $\sqrt{xy}$ $-4\sqrt{xy}$

8. To $\overline{b+c}\sqrt{xz}$ add $\overline{b-c}\sqrt{zx}$ $2b\sqrt{zx}$

9. To

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mon denominator are divided by ſetting down the numerator of the dividend for a new numerator, and the numerator of the diviſor under the numerator of the dividend for a new denominator. 2d. When a fraction is to be divided by any quantity, it is the ſame thing whether the numerator be divided by it, or the denominator be multiplied by it. 3d. When the two numerators or the two denominators, can be divided by ſome common quantity, that quantity may be thrown out of each and the quotients uſed inſtead of the fraction propoſed.

| | *Anſwers.* |
|---|---|
| 9. To $\overline{5x+3c}\sqrt{cz-y^2}$ add $\overline{-5x+3c}\sqrt{cz-y^2}$ | $6c\sqrt{cz-y^2}$ |
| 10. To $5x+2d-c\sqrt{xz}$ add $-2d+c\sqrt{zx}$ | $5x\sqrt{zx}$ |
| 11. To $3\sqrt{zy}$ add $-\sqrt{zy}$ | $2\sqrt{zy}$ |
| 12. To $c+d\sqrt{z^2-xy}$ add $c\sqrt{z^2-xy}$ | $2c+d\sqrt{z^2-xy}$ |
| 13. To $b\sqrt{zd}$ add $c\sqrt{zd}$ | $b+c\sqrt{zd}$ |
| 14. To $b\sqrt{z^2+x^2}$ add $c\sqrt{z^2+x^2}$ | $b+c\sqrt{z^2x^2}$ |
| 15. To $3cz\sqrt{zy}$ add $-\sqrt{zy}$ | $3c+\sqrt{zy}$ |
| 16. To $\sqrt{zx}$ add $-\sqrt{zx}$ | 0 |
| 17. To $c-d\sqrt{z^2-xy}$ add $b\sqrt{z^2-xy}$ | $c-d+b\sqrt{z^2-xy}$ |
| 18. To $2b-p\sqrt[3]{xz}$ add $\sqrt[3]{xz}$ | $2b-p+\sqrt[3]{xz}$ |
| 19. To $b\sqrt{13}$ add $d\sqrt{13}$ | $b+d\sqrt{13}$ |
| 20. To $3\sqrt{5}$ add $d\sqrt{5}$ | $3+d\sqrt{5}$ |
| 21. To $\sqrt{zy}$ add $\sqrt{xy}$ | $\sqrt{zy}+\sqrt{xy}$ |
| 22. To $b\sqrt{z^2-xy}$ add $c\sqrt{x^2-zy}$ | $b\sqrt{z^2-xy}+c\sqrt{x^2-zy}$ |
| 23. To $\sqrt[3]{z^2-xy}$ add $\sqrt{z^2-xy}$ | $\sqrt[3]{z^2-xy}+\sqrt{z^2-xy}$ |

## *Subtraction.*

| | |
|---|---|
| 1. From $3c\sqrt{xy}$ take $c\sqrt{xy}$ | $3c\sqrt{xy}$<br>$c\sqrt{xy}$<br>$2c\sqrt{xy}$ *Anſwer.* |
| 2. From $2b\sqrt{z^2-bz}$ take $-b\sqrt{z^2-bz}$ | *Anſ.* $3b\sqrt{z^2-bz}$ |
| 3. From $2\sqrt[3]{bx-cx}$ take $-\sqrt[3]{bx-cx}$ | $3\sqrt[3]{bx-cx}$ |
| 4. From $2\sqrt{xz}$ take $2\sqrt{xz}$ | 0 |
| 5. From $3d\sqrt{x^2-cz}$ take $c\sqrt{x^2-cz}$ | $\overline{3d-c}\sqrt{x^2-cz}$ |
| 6. From $5z\sqrt{zx-5}$ take $-z\sqrt{zx-5}$ | $6z\sqrt{zx-5}$ |
| 7. From $\sqrt[3]{cx-d^2}$ take $-b\sqrt[3]{cx-d^2}$ | $b+1\sqrt[3]{cx-d^2}$ |
| 8. From $3\sqrt{c^2-y^2}$ take $2\sqrt{c^2-y^2}$ | $\sqrt{c^2-y^2}$ |

9. From

| | *Anſwers.* |
|---|---|
| 9. From $b\sqrt{zy-d^2}$ take $\sqrt{zy-d^2}$ | $b-1\sqrt{zy-d^2}$ |
| 10. From $2m\sqrt{zx}$ take $n\sqrt{zx}$ | $2m-n\sqrt{zx}$ |
| 11. From $-7\sqrt[3]{z^2-b^2}$ take $-8\sqrt[3]{z^2-b^2}$ | $\sqrt[2]{z^2-b^2}$ |
| 12. From $3\sqrt{c^2-cd}$ take $b\sqrt{c^2-cd}$ | $3-b\sqrt{c^2-cd}$ |
| 13. From $c+d\sqrt{zy-x^2}$ take $d\sqrt{zy-x^2}$ | $c\sqrt{zy-x^2}$ |
| 14. From $3z-2y\sqrt{zx}$ take $3y\sqrt{zx}$ | $3z-5y\sqrt{zx}$ |
| 15. From $5c-2x\sqrt{zx-xy}$ take $m\sqrt{zx-xy}$ | $\overline{5c-2x}-m\sqrt{zx-xy}$ |
| 16. From $\sqrt{zx}$ take $\sqrt{xy}$ | $\sqrt{zx}-\sqrt{xy}$ |
| 17. Fr $cz\sqrt[3]{x^2-bz}$ take $b\sqrt{x^2-bz}$ | $cz\sqrt[3]{x-bz}-b\sqrt{x^2-bz}$ |
| 18. From $c+d\sqrt{z^2-y^2}$ take $m\sqrt{x^2-c^2}$ | $c+d\sqrt{x^2-y^2}-m\sqrt{x^2-c^2}$ |

## *Multiplication.*

1ſt. When the ſurds are pure, and of the ſame kind, then multiply the quantities under the radical ſign together, attending to the rules of multiplication of Algebra, before delivered; and to the product prefix the common radical ſign.

2d. When a pure ſurd is to be multiplied by a rational quantity; make the rational quantity a coefficient to the ſurd by joining it to the propoſed ſurd; or if you bring the rational quantity under the radical ſign, you muſt firſt involve it to the height of the index of the root, and then multiply it into the quantity under the root

3d. To multiply like ſurds having rational quantities without the radical ſign for coefficients: multiply the rational part by the rational part, and let the product be the coefficient to the product of the ſurd multiplied by the ſurd.

I. To multiply ſurd quantities by ſurd quantities of the ſame kind.

EXAMPLES.

1. Multiply $\sqrt{xy}$ by $\sqrt{x-y}$

$$\begin{array}{r} \sqrt{xy} \\ \sqrt{x-y} \\ \hline \end{array}$$

*Anſ.* $\sqrt{x^2y-y}$

2. Multiply $\sqrt[3]{x+z-y}$ by $\sqrt[3]{x+y}$ *Anſ.* $\sqrt{x^2+zx+zy-y^2}$

3. Multiply $\sqrt{\frac{2z-c^2}{z+x}}$ by $\sqrt{\frac{z-x}{c-d}}$

$$\sqrt{\frac{2z^2-zc^2-2zx+zc^2}{cz+cx-dz-dx}}$$

4. Multiply $\sqrt{\frac{3bx-2cz}{c-d}}$ by $\sqrt{\frac{2x+c}{m-n}}$

*Anſw.* $\sqrt{\frac{6bx^2-4czx+3bcx-2cz}{cm-dm-cn+dn}}$

5. Multiply $\sqrt{z-x}$ by $\sqrt{z-x}$ $\sqrt{x^2-2xz+z^2}$

6. Multiply $\sqrt{x}$ by $\sqrt{x}$ $\sqrt{x^2}=x$

7. Multiply $\sqrt{\frac{3}{5}z-\frac{2}{3}y}$ by $\sqrt{\frac{3}{5}z-\frac{2}{3}y}$

*Anſ.* $\sqrt{\frac{9}{25}z^2-\frac{4}{5}zy+\frac{4}{9}y^2} = \frac{3}{5}z-\frac{2}{3}y$

8. Multiply $\sqrt{\frac{x+z}{c}}$ by $\sqrt{\frac{x+z}{c}}$ $\sqrt{\frac{x^2+2zx+z^2}{c2}}=\frac{x+z}{c}$

From theſe examples it appears, that any ſquare root is ſquared by caſting off the radical ſign.

II. To multiply ſurds by rational quantities.

EXAMPLES.

1. Multiply $\sqrt{zx-x^2}$ by 2. *Anſ.* $2\sqrt{zx-x^2}$

or thus, $\sqrt{4zx-4x^2}$

2. Multiply $\sqrt{zy-xz}$ by $3c$ *Anſwer* $\sqrt{9c^2zy-9c^2xy}$

3. Multiply $\sqrt[3]{\frac{d^2b}{d^3+c^3}}$ by $\frac{c}{d}$ $\sqrt[3]{\frac{c^3d^2b}{d^6+d^3c^3}}=\frac{c}{d}\sqrt{\frac{d^3b}{d^3+c^2}}$

4. Multiply $\sqrt{2cz-dx}$ by $\frac{2}{3}z$ $\sqrt{\frac{4}{9}cz^3-\frac{4}{9}dxz}$

5. Multiply $\sqrt{\frac{cd}{b-d}}$ by $\frac{c}{d}$ $\sqrt{\frac{c^3d}{bd^2-d^3}}$

6. Multiply $\sqrt{x-z}$ by $x+z$ $\sqrt{x^4-2zx^3-z^4}$

7. Multiply $\sqrt[3]{cx-cd}$ by $2z$ *Anſw.* $\sqrt[3]{8z^3cx-8zc^3d}$

8. Multiply $\sqrt{\frac{cx+dx}{x-c}}$ by $\frac{3}{5}cx$ $\sqrt{\frac{9c^3x^3+9c^2dxz}{25x-25c}}$

### III. To multiply ſurds joined to rational quantities.

EXAMPLES.

1. Multiply $c\sqrt{x-z}$ by $d\sqrt{x+z}$.

$$\begin{array}{ll} x-z & \\ x+z & \\ \hline x^2-xz & \\ & xz-z^2 \\ \hline \end{array}$$

*Anſw.* $dc\sqrt{x^2-z^2}$

2. Multiply $3d\sqrt{\frac{z-d}{z+d}}$ by $3x\sqrt{\frac{z+d}{m+n}}$

*Anſw.* $9dx\sqrt{\frac{z^2-d^2}{zm+zn+dn+dm}}$

3. Multiply $2d\sqrt{z-x+y}$ by $x\sqrt{x+y}$

*Anſw.* $2dx\sqrt{zx-x^2-zy+2xy-y^2}$

4. Multiply $3x\sqrt{\frac{z+d}{m+n}}$ by $2c\sqrt{\frac{z-d}{z+d}}$

*Anſw.* $6cx\sqrt{\frac{z^2-d^2}{zm+zn+dm+dn}}$

5. Multiply $\frac{c}{d}\sqrt{\frac{x-z+y}{z+d+p}}$ by $\frac{m}{y}\sqrt{\frac{z+x-y}{z-d-p}}$

*Anſw.* $\frac{cm}{dy}\sqrt{\frac{x^2-z^2+2zy-y^2}{z^2-d^2-2dp-p^2}}$

## *Diviſion.*

1ſt. If the quantities are of the ſame kind; divide the quantities under the radical ſign, and to the quotient ſubjoin the common radical ſign.

2d. When the ſurds are alike, and multiplied by rational quantities; divide the rational by the rational, and let the quotient be a coefficient to the quotient of the ſurd divided by the ſurd, with the common radical ſign.

3d. To divide ſurds by rational quantities; make the rational quantity a denominator to the propoſed ſurd, placed as a numerator. But if the rational quantity be brought under the radical ſign, firſt involve it to the height of the index of the root, and then place it under the quantities under the radical ſign, as a denominator, and let the radical ſign affect them both.

I. To divide ſurd quantities by like ſurds.

### EXAMPLES.

1. Divide $\sqrt{bzy}$ by $\sqrt{by}$ $by\,)\,bzy\,(\,\sqrt{z}$ *Anſw.*
$bzy$

2. Divide $\sqrt{4b^2zx+2bzx^2}$ $\sqrt{2bz}$ *Anſwer* $\sqrt{2zx+x^2}$

3. Divide $\sqrt{z^2-x^2}$ by $\sqrt{z-x}$ $\sqrt{z+x}$

4. Divide $\sqrt{c-d}$ by $\sqrt{m-n}$ $\sqrt{\frac{c-d}{m-n}}$

5. Divide $\sqrt{\frac{c^2-cx}{m-n}}$ by $\sqrt{\frac{m+n}{c^2-x^2}}$ $\sqrt{\frac{c^4-c^2x^2-c^3x+cx^3}{m^2-n^2}}$

6. Divide $\sqrt[3]{x^4-y^4}$ by $\sqrt{x+y}$ $\sqrt{x^3-x^2y+xy^2+x^3}$

7. Divide $\sqrt[3]{z^3-3z^2x+3zx^2-x^3}$ by $\sqrt[3]{z-x}$
$\sqrt[3]{z^2-2zx+x^2}$

II. To divide ſurds joined to rational quantities by like ſurds joined to rational quantities.

### EXAMPLES.

1. Divide $cx\sqrt{cdx-b^2x}$ by $x\sqrt{cd-b^2}$ *Anſ.* $c\sqrt{x}$

2. Divide $by\sqrt{cz-dz}$ by $b\sqrt{z}$ $y\sqrt{c-d}$

3. Divide $bx-cx\sqrt{x^2-z^2}$ by $x\sqrt{x+z}$ *Ans.* $b-c\sqrt{x-z}$

4. Divide $\frac{z\sqrt{b^2c-cz^2}}{b}$ by $c\sqrt{z}$ $\frac{z}{c}\sqrt{\frac{b^2c-cz^2}{bz}}$

5. Divide $\frac{c}{x}\sqrt{\frac{z-y}{c}}$ by $\frac{x}{z}\sqrt{\frac{y}{b}}$ $\frac{cz}{x^2}\sqrt{\frac{bz-by}{cy}}$

III. To

### III. To divide ſurds by rational quantities.

| EXAMPLES. | *Anſwers.* |
|---|---|
| 1. Divide $\sqrt{cz-dz}$ by $3$ | $=\frac{\sqrt{cz-dz}}{3}=\sqrt{\frac{cz-dz}{9}}$ |
| 2. Divide $\sqrt{bdz-bcz}$ by $2b$ | $\sqrt{\frac{bdz-bcz}{4b^2}}$ |
| 3. Divide $\sqrt{cz^2-cd}$ by $c+d$ | $\sqrt{\frac{cz^2-cd}{c^2+2cd+d^2}}$ |
| 4. Divide $\sqrt[3]{cz-y^2}$ by $3x$ | $\sqrt[3]{\frac{cz-b^2}{27x}}$ |
| 5. Divide $\sqrt{\frac{b-z}{c+x}}$ by $\frac{z}{y}$ | $\sqrt{\frac{by^2-zy^2}{cz^2+xz^2}}$ |
| 6. Divide $\sqrt{\frac{cx}{bx}}$ by $\frac{c+x}{b}$ | $=\sqrt{\frac{b^2cx}{c^2bz+2cxbz+bzx^2}}$ |

### *To Square Binomial or Reſidual ſimilar Surd Roots.*

*Rule.* Take the ſquare of each part and connect them together with the ſign + and a double product of the parts, which in a binomial muſt have the ſign +, and in a reſidual, the ſign – .

| EXAMPLES. | *Anſwers.* |
|---|---|
| 1. Square $\sqrt{x}+\sqrt{z}$ | $=x+z+2\sqrt{xz}$ |
| 2. Square $\sqrt{z}-\sqrt{y}$ | $z+y-2\sqrt{zy}$ |
| 3. Square $\sqrt{z+x}+\sqrt{z-x}$ | $2z+2\sqrt{z^2-x^2}$ |
| 4. Square $\sqrt{x+y}-\sqrt{x-y}$ | $2x-2\sqrt{x^2-y^2}$ |
| 5. Square $\sqrt{z+x+d}+\sqrt{z-x}$ | $2z+d+2\sqrt{z^2+dz-x^2-dz}$ |
| 6. Square $\sqrt{x-z+y}-\sqrt{x-z-y}$ | $2x-2z-2\sqrt{x^2-2xz+z^2-y}$ |
| 7. Square $\sqrt{\frac{x+b}{2}}+\sqrt{\frac{x-b}{2}}$ | $x+\sqrt{x^2-b^2}$ |
| 8. Square $\sqrt{\frac{x+c}{2}}-\sqrt{\frac{x-c}{2}}$ | $x-\sqrt{x-c}$ |

| | *Answers.* |
|---|---|
| 9. Square $\sqrt{\frac{z+d}{4}}+\sqrt{\frac{z-d}{4}}$ | $\frac{z}{2}+\sqrt{\frac{z^2-d^2}{4}}$ |
| 10. Square $\sqrt{\frac{x^2+3d}{3d}}-\sqrt{\frac{x^2-3d}{3d}}$ | $\frac{2x^2}{3d}.-\sqrt{\frac{4x^4-36d^2}{9a^2}}$ |
| 11. Square $\sqrt{\frac{y+d-c}{2}}+\sqrt{\frac{y+c+d}{3}}$ | $\frac{5y+5d-c}{6}+\sqrt{\frac{2y^2+4dy+2d^2-2c^2}{3}}$ |
| 12. Square $\sqrt{\frac{x^2+3b}{4}}-\sqrt{\frac{x^2-3b}{4}}$ | $\frac{x}{2}-\sqrt{\frac{x^4-9b^2}{4}}$ |
| 13. Square $s+\sqrt{4p-s^2}$ | $4p+\sqrt{16ps^2-4s^4}$ |
| 14. Square $z-\sqrt{2d^2+z^2}$ | $2z^2+2d^2-\sqrt{8z^2d^2+4z^3}$ |
| 15. Square $d-\sqrt{\frac{z^2-d^2}{2}}$ | $z^2-\sqrt{\frac{4dz^2-4d^4}{4}}$ |
| 16. Square $z+\sqrt{\frac{p^2-q^2+z^2}{3}}$ | $2z^2+p^2-q^2+\sqrt{\frac{4z^2p^2-4z^2q^2+4z^4}{9}}$ |
| 17. Square $\sqrt{7}+\sqrt{3}$ | $10+2\sqrt{21}$ |
| 18. Square $\sqrt{11}-\sqrt{6}$ | $17-\sqrt{264}$ |
| 19. Square $2+\sqrt{7}$ | $11+\sqrt{112}$ |

20. Square $7-\sqrt{11}$

*Anſ.* $7+11-14\sqrt{11}=18-14\sqrt{11}=18-\sqrt{2256}$.

## *To Multiply a Binomial Surd by its correſponding Reſidual.*

A binomial and its correſponding reſidual conſiſt of the very ſame parts; but in a binomial the parts are connected with the ſign +, and in the reſidual they are connected with the ſign —, and when multiplied by ſuch, ſubtract the ſquare of the negative part of the reſidual, from the ſquare of its poſitive part, the remainder will be equal to the product ſought.

To multiply a binomial by its correſponding reſidual.

EXAMPLES.

*Anſwers.*

1. Multiply $\sqrt{z}+\sqrt{x}$ by $\sqrt{z}-\sqrt{x}$ $z-x$

2. Multiply $\sqrt{x+b}-\sqrt{x-b}$ by $\sqrt{x+b}-\sqrt{x-b}$. $2b$

3. Multiply $\sqrt{z+x-b}+\sqrt{x-b}$ by $\sqrt{x+z-b}-\sqrt{x-b}$ *Ans.* $z$

4. Multiply $\sqrt{x^2+cd}+\sqrt{x^2+cd-d^2}$ by $\sqrt{x^2+d}$ $-\sqrt{x^2+cd-d^2}$ $d^2$

5. Multiply $z+\sqrt{d^2-z^2}$ by $z-\sqrt{\frac{d^2-y^2}{3}}$ $\frac{3z^2-d^2+z^2}{3}$

6. Multiply $\sqrt{z^2-bz}+b$ by $\sqrt{z^2-bx}-b$ $z^2-bz-b^2$

7. Multiply $y+\sqrt{\frac{z^2+d^2}{2}}$ by $y-\sqrt{\frac{z^2+d^2}{2}}$ $\frac{2y^2-z^2-d^2}{2}$

8. Multiply $\sqrt{\frac{x^2-cx+z}{2}}$ by $\sqrt{\frac{x^2+cx-z}{2}}$ $\frac{x^2+cx-z^2}{4}$

9. Multiply $\sqrt{\frac{d^2+4p+d^2}{4}}$ by $\sqrt{\frac{d^2+4p-d}{4}}$ $\frac{d^2+4p-d^4}{16}$

## SIMPLE EQUATIONS.

AN Equation is when two equal quantities, differently expreſſed, are compared together by means of the ſign = placed between them.

### *Reduction of Equations, by Addition and Subtraction.*

*Rule.* When in a given equation, a quantity with a negative ſign is tranſpoſed to the other ſide of the equation, it muſt there have a poſitive ſign, and then it is added to both ſides; but when a quantity with a poſitive ſign is tranſpoſed, it muſt, on the contrary ſide of the equation, have a negative ſign, and then it is ſubtracted from both ſides of the equation.

EXAMPLES.

1. Given $x-c=b$ required $x$ *Answers* $x=b+c$
2. $2z-c+d=m+z$ required $z$ $z=m+c-d$
3. $z+c=d$ required $z$ $z=c-d$
4. $5x-b+d=c+4x$ required $x$ $x=c+b-d$
5. $z-b+c=d$ required $z$ $z=d+b-c$
6. $c+x-b=d$ required $x$ $x=b-c+d$
7. $z-c+d+z=m+z$ required $z$ $z=m+c-d$
8. $3y-b+c=m+2y$ required $y$ $y=m+b-c$
9. $2z-c+d=m-n+z$ required $z$ $z=m-n+c-d$

## *Reduction of Equations, by Multiplication.*

When in a given equation there are fractions, the equation is cleared of thoſe fractions, by multiplying every term or numerator by every denominator except its own, for the terms of the equation, and rejecting the denominators.

EXAMPLES.

*Anſwers.*

1. Given $\frac{x}{b}=c+d$ required $x$ $=bc+bd$
2. $\frac{z}{m}+c=n$ required $z$ $=mn-mc$
3. $\frac{x-c}{b}-d=p$ required $x$ $x=bp+c+bd$
4. $\frac{z+d}{b}=\frac{c+d}{c}$ required the terms wherein $z$ is on one ſide $zc=bc-cd+bd$
5. $\frac{x+b}{m-n}-c=\frac{l-x}{c}$ required the terms wherein $x$ is on one ſide *Anſ.* $cx+mx-nx=bm-nb-cb+mc^2-nc^2$
6. $\frac{x-c}{m+n}=\frac{d-x}{m-n}$ required the $x$'s on one ſide *Anſ.* $2mx=dm+dn+cm-nc$
7. $\frac{c+x}{c-d}-d=\frac{d-x}{d}$ required the $x$'s, &c. $cx=d^2+cd^2-d^3$

8. $cx-$

8. $\frac{cx-d^2}{m-n}-c=\frac{dx}{m}+\frac{cx}{n}$ required the $x$'s on one ſide

*Anſ.* $2cxmn-cxm^2+dxn^2-dxmn=d^2mn+m^2cn-mn^2c$

9. $\frac{x+c}{b-c}-\frac{b}{d}=\frac{d-x}{b+c}$ required the $x$'s on one ſide

*Anſ.* $2bxd=bd^2-cd^2-bcd+b^3-b^2c-c^2d+cb^2-bc^2$

10. $\frac{x+c}{m-n}-1=\frac{c-x}{m}$ required the $x$'s on one ſide

*Ans.* $2xm-nx=m^2-nm-nc$

## *Reduction of Equations, by Diviſion.*

When in any given equation, any quantity, known or unknown, is in every term thereof; then by dividing by that quantity, the equation will be reduced to more ſimple terms. And when all the terms wherein the ſame power of the unknown quantity is on the one ſide of the equation, then by dividing both ſides of the equation by the ſingle or compound, the coefficient of the unknown quantity, the ſame unknown quantity will be cleared.

### EXAMPLES.

1. $cz^2-dz=pz+qz$ required $z$ $\qquad z=\frac{p+q+d}{c}$

2. $\frac{bz+cz}{m}-n=c+d$ required $z$ *Ans.* $z=\frac{mc+md+nm}{b+c}$

3. $\frac{bz-cz}{c-d}-b=\frac{cz+b}{d}$ required $z$ $\qquad z=\frac{2cb-db}{db-cd-c^2}$

4. $\frac{bz+d}{m+n}-1=\frac{bz}{m-n}+\frac{dz}{c}$ required $z$

*Anſ.* $z=\frac{m^2c-dmc+dnc-n^2c}{dn^2-dm^2-2cbn}$

5. $\frac{cx-p}{c-p}+n=m+\frac{c^2-bx}{m-n}-1$.

*Ans.* $x=\frac{cm^2-pm^2-2cmn+2pmn+c^3-c^2p-cm}{cm-cn+cb-pb}$

$\frac{+2mp+cn-2pn+cn^2-pn^2}{cm-cn+cb-pb}$

6. $dx^2-px=cx+x$ then $x=\frac{c+1+p}{d}$

7. $bx^3-dz^2=cz^3+pz^2+z^2$ then $z=\frac{c+p+1+d}{b}$

8. $\frac{cx+b}{p+q}-d=\frac{d-x}{p-q}$ required $x$

*Ans.* $x=\frac{pd+qd-pb+p^2d+pdq+bq-pdq-q^2d}{pc-qc+p-q}$

## *Reduction of Equations by Involution.*

When in a given equation there are ſurds, tranſpoſe all the rational quantities from the ſurds, and raiſe both ſides of the equation to the height of the index of the root; the equation then will be either intirely cleared from ſurds, or reduced to more ſimple terms, *i.e.* fewer numbers of ſurd quantities,) and ſo proceed till the equation is cleared of ſurds.

### EXAMPLES.

1. Given $\sqrt{x+c}=d$ required $x$.
$x+d=d^2$ then $x=d^2-c$ by ſubtraction.

2. $\sqrt{zb-c^2}=d^2$ required $z$. *Anſ.* $z=\frac{d^4+c^2}{b}$

3. $\sqrt{bx-bd}-c=d$ required $x$. $x=\frac{d^2+c^2+2dc+bd}{b}$

4. $\sqrt{zx+b^2}-d+c=bp$ required $z$.
$z=\frac{b^2p^2+d^2+c^2+2pbd-2bpc-2dc-b^2}{x}$

5. $\sqrt{xc}+\sqrt{xb}=c$ required the $x$'s on one ſide.
*Anſ.* $4x^2cb-x^2c-x^2b^2+2c^3x-2c^2xb+2x^2cb=c^4$

6. $\sqrt{zb}=c-\sqrt{zd}$ required the $z$'s on one ſide.
*Anſ.* $2z^2bd-z^2b-z^2d^2+2c^2zb+2c^2zd=c^4$

7. $\sqrt{x+b}+\sqrt{x-b}=d$ required $x$
*Anſ.* $4d^2x=d^4+4b^2$

8. $\sqrt{cx}-\sqrt{bx}=c+d$ required the $x$'s on one ſide.
*Anſ.* $4bcx^2+4bcdx+2bxd^2+2bxc^2+4c^2dx+2cxd^2+$
$2c^3x-2bcx^2-b^2x^2-c^2x^2=c^4+d^4+2c^2d^2+4c^3d$

Change the ſigns of the known quantity, when they happen to be all negatives, into poſitives.

9. Given $\sqrt{z+b}+\sqrt{z-b}=d$ required z. $z=\frac{d^4+4b^2}{4d^2}$

10. $\sqrt{c+z}+\sqrt{c-z}$ required z $4z^2=c^4-4c^3$

11. $\sqrt{x+c}+\sqrt{x-c}=b-c$ required x.

$$x=\frac{b^4+c^4+6b^2c^2+4b^3c+4bc^3+4c^2}{4b-8bc+4c^2}$$

12. $\sqrt{d+2z}-\sqrt{d-2z}+b=c$ required z.

$$Anſ.\ z^2=\frac{4c^2d-c^4-b^4-6b^2c^2+4db^2-8dcb+4c^3b+4c}{16}$$

13. $\sqrt{x+b}-\sqrt{x-b}=\sqrt{cx}$ required x. $x^2=\frac{4b^2}{3c^2}$

14. $\sqrt{\frac{z+c}{2}}+\sqrt{\frac{z-2}{2}}=\sqrt{dz}$ required z. $z^2=\frac{c^2}{2d-d^2}$.

15. $\sqrt{\frac{z^2+3b}{4}}-\sqrt{\frac{z^2-3b}{4}}=\sqrt{z^2c}$ required z.

$$Anſw.\ z^4=\frac{9b^4}{4cb-4c}$$

## *Reduction of Equations by Evolution.*

When the unknown quantity is in any power thereof, on one ſide of the equation, by evolving according to the index of that power the unknown quantity will be cleared.

### EXAMPLES.

1. Given $\frac{cz^2-dz^2}{b}-c^2=bd$ required z.

$$cz^2-dz^2-bc^2=b^2d\ \therefore\ cz^2-dz^2=b^2d+bc^2$$

$$z^2=\frac{b^2d+bc^2}{c-d}\text{ therefore }z=\sqrt{\frac{b^2d+bc^2}{c-d}}$$

2. $cx^3-dx^3+x^3=cm-d^2$ required x. $x=\sqrt[3]{\frac{c^2m-cd^2}{2c-d}}$

3. $\frac{bz^4-q^4}{m+n}=b^4-z^4$ required z. $z=\sqrt{\frac{b^4m+b^4n+q^4}{b+m+n}}$

4. $se-s=ye-a$; $a=3$, $s=1092$, $y=729$, required e.

$$Anſ.\ e=\frac{s-a}{s-y}=3.$$

5. $\frac{y-d}{e-1}+y=s$ required $y$ in the terms of the reſt, and alſo its value in numbers, when $s=1092$, $a=3$, and $e=3$.

*Anſ.* $y=\frac{es-s+a}{e}=729$

6. $s=\frac{ye-a}{e-1}$ required $a$, if $s=1092$, $y=729$, and $e=3$.

*Anſ.* $a=ye-se+s=3$

7. $\frac{ca-d}{f}+a=x$; $c=3$, $d=5$, $f=2$, and $x=40$, required $a$.

*Anſ.* $a=\frac{fx+d}{c+f}=17$

8. $\frac{2z^2-z}{z}=b$; $b=13$, required $z$. *Anſ.* $z=\frac{b+1}{2}=7$

9. $ba=c^2-2ac$; $b=42$, $c=28$, required $a$.

*Anſ.* $a=\frac{c^2}{b+2c}=8$

10. $\frac{s-a}{sy}=e$; $a=3$, $e=3$, $y=729$, required $s$.

*Anſ.* $s=\frac{cy-a}{c-1}=1092$

11. $ca+d=fd$; $c=12$, $d=48$, $f=18$, required $a$.

*Anſ.* $a=\frac{d}{f-c}=8$.

12. $z^2=bz+z$; $b=8$, required $z$. *Anſ.* $z=b+1=9$.

13. $d=f-2fe$; $d=32$, $f=16$ required $e$.

*Anſ.* $e=\frac{f^2-d}{2f}=7$.

14. $b=2a+1-c$; $b=284$, $c=25$, required $a$.

*Anſ.* $a=\frac{b-1+c}{2}=154$.

15. $a-da-c=f+g$; $d=\frac{1}{3}$, $c=10$, $f=\frac{1}{6}$, $g=15$, required $a$.

*Anſ.* $a=\frac{g+c}{d-f}=150$.

16. $\frac{d}{x}=\frac{cx}{b}$; $b=2$, $c=3$, $d=54$, required $x$.

*Anſ.* $x=\sqrt{\frac{bd}{c}}=6$.

17. $c^2+d=f^2+2fc+c^2$; $d=1680$, $f=12$, required $c$.

$$Ans.\ c=\frac{d-f^2}{2f}=64.$$

18. $\frac{c^2z^2}{d^2}=b-z^2$; $d=4$, $c=5$, $b=2624$, required $z$.

$$Ans.\ z=\sqrt{\frac{bd^2}{c^2+d^2}}=32.$$

19. $2bx+3cx+4x-f=g-bx-cx$; $b=12$, $c=5$, $f=10$, $g=5$, required $x$.

$$Ans.\ x=\frac{g+f}{3b+4c+4}=\frac{1}{4}$$

20. $fz+c-dz=fb$; $b=30$, $c=500$, $d=4$, $f=52$, required $z$.

$$Ans.\ z=\frac{fb-c}{f-d}=22\tfrac{1}{12}.$$

21. $z+bz+cz+dz-f=g$; $b=\frac{1}{2}$, $c=\frac{1}{3}$, $d=\frac{1}{4}$, $f=500$, $g=150000$, required $z$.

$$Ans.\ z=\frac{g+f}{1+b+c+d}=60000.$$

22. $de-e+fe+ge-h=3e-3bc-3fe-3ge-3me$; $b=\frac{1}{10}$, $d=\frac{1}{20}$, $f=\frac{3}{4}$, $g=\frac{3}{8}$, $h=3000$, $m=\frac{1}{2}$, required $e$.

$$Ans.\ e=\frac{h}{d-4+4f+4g+3b+3m}=10000.$$

## EXTERMINATION OF UNKNOWN QUANTITIES,

### *Or, the Reduction of two or more equations to a ſingle one.*

OBSERVE which of all the unknown quantities is leaſt involved, and let the value of that quantity be found in each equation by the rules already given, looking upon all the reſt as known; let the values thus found be put equal to each other; (for they are equal, becauſe they expreſs the ſame thing,) whence new equations will ariſe, out of which that quantity will be totally excluded; with which new equations the operation muſt be repeated, and the unknown quantities exterminated, one by one till at laſt an equation is come to containing only one unknown quantity.

**EXAMPLE.** Let the given equations be $x+y=12$, and $5x+3y=50$, to find $x$ and $y$. Now by tranſpoſing $y$ and $3y$ we get $x-12-y$, and $5x=50-3y$, from the laſt of which equation $x=\frac{5-3y}{5}$. Now by equating theſe two values of $x$ we have $12-y=\frac{50-3y}{5}$ and therefore $60-5y=50-3y$ from which $y=\frac{10}{2}=5$ and $x=12-12y-5=7$.

Or, let the values of the unknown quantity, which you would firſt exterminate, be found in that equation wherein it is the leaſt involved, conſidering all the other quantities as known, and let this value and its powers be ſubſtituted for that quantity, and its reſpective powers in the other equations, and with the new equations thus ariſing, repeat the operations till you have only one unknown quantity and one equation.

**EXAMPLE.** Thus $x$ being the firſt equation (in the laſt example) $=12-y$ then by ſubſtituting this value of $x$, in the ſecond that is $60-5y$ muſt be wrote in the room of its equal $5x$; whence will be had $60-5y+3x=50$; from whence $y=\frac{10}{2}=5$, as before.

Or, laſtly, let the given equations be multiplied or divided by ſuch numbers or quantities, whether known or unknown, that the term which involves the higheſt power of the unknown quantity to be exterminated, may be the ſame in each equation, then by adding or ſubtracting the equations, as occaſion ſhall require, that term ſhall vaniſh, and new equations emerge, wherein the number of dimenſions (if not the number of unknown quantities) will be diminiſhed.

**EXAMPLE.** By multiplying the firſt equation by 5, we ſhall have $5x+5y=60$, from whence ſubtracting the ſecond equation, viz. $5x+3y=50$ there remains $2y=10$, whence $y=5$ and $x$ by the firſt or ſecond equation will be 7 ſtill the ſame as before.

*Note.* The firſt of theſe three ways is the moſt commonly uſed, but the laſt of them is, for the general part, the moſt eaſy and expeditious in practice.

### EXAMPLES.

5 15

1. $5x$ [illegible] $y=$ [illegible] and $4x-5y=5$, quere the value of $x$ and $y$?

*Anſ.* $x=10$ and $y=7$.

2. Let $5x-3y=150$ and $10x+15y=825$, quere $x$ and $y$?
*Anſ.* $x=45$ and $y=25$.

3. Let $\frac{x}{z}+\frac{y}{3}+\frac{z}{4}=62$. $\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47$ and $\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38$, quere $x$, $y$ and $z$?
*Anſ.* $x=24$, $y=60$ and $z=120$.

4. Let $\frac{x}{4}+\frac{y}{5}=15$, $\frac{x}{6}+\frac{y}{9}=9$, quere $x$ and $y$?
*Anſ.* $x=24$ and $y=45$.

5. $\frac{x+y}{5}+\frac{x}{3}-8=\frac{2y-x}{4}+27$ and $\frac{x}{2}-12=\frac{y}{4}+8$, quere $x$ and $y$? *Anſ.* $z=60$ and $y=40$.

6. $y+100=2x+2z$, $y+100=2x+2z$, and $z+100=3x+3y$; required, $x$, and $z$. *Anſ.* $x+9\frac{1}{11}$, $y=45\frac{5}{11}$ and $z=63\frac{7}{11}$.

7. $x-y=2$, and $xy+5x-6y=120$; to exterminate $x$.
*Ans.* $y^2-9y=110$.

8. Let $x+y=s$, and $x^2-y^2=d$; required $x$ and $y$.
*Ans.* $x=\frac{s^2+d}{2s}$ and $y=\frac{s^2-d}{2s}$

9. Let $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62$. $\frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47$, and $\frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38$; required $x$, $y$ and $z$. *Ans.* $x=24$, $y=60$ and $z=120$.

## ARITHMETICAL PROGRESSION.

WHEN a rank or ſeries of terms increaſe by a common difference, ſuch ſeries are ſaid to be in Arithmetical progreſſion.

*Example*: 5, 8, 11, 14, 17, &c. where 3 is the common difference: in ſeries as $a$, $a+e$, $a+2e$, $a+3e$, &c. where $e$ is the common difference.

In ſuch a ſeries let $a$ repreſent the Firſt or leaſt term.
$y$ ——— Laſt or greateſt.
$e$ ——— Common difference.
$n$ ——— Number of terms.
$s$ ——— Number of the ſeries.

*Prop.* I. In any Arithmetical ſeries, the ſum of the extremes multiplied by the number of terms is equal to double the ſum of the terms, *i. e.* $\overline{a+y}\times n=2s$ or $na+ny=2s$.

*Demonſtration.*

$$\begin{array}{lcl c l}
a & + & y & = & a+y \\
a+e & + & y-e & = & a+y \\
a+2e & + & y-2e & = & a+y \\
a+3e & + & y-3e & = & a+y \\
a+4e & + & y-4e & = & a+y \\
a+5e & + & y-5e & = & a+y \\
\&c. & & \&c. & & \&c. \\
\hline
s & + & s & = & \overline{a+y}\times n \text{ i.e. } =2s= \\
 & & & & an+yn \text{ Q. E.D.}
\end{array}$$

*Prop.* II. In any Arithmetical ſeries the laſt or greateſt term is equal to the firſt term added to the number of terms leſs by one, multiplied by the common difference, it is $y=a+\overline{n-1}\times e=$ or $y=a+ne-e$. This is evident by viewing any ſeries in ſpecies, for any term being taken for the laſt, it conſiſts of the firſt term together with the number of terms leſs by one, multiplied by the common difference. Now any three of the foregoing 5 parts, viz. $a$, $y$, $e$, $n$, $s$, being given, the reſt may be found by the two theorems deduced from theſe propoſitions, viz. $na+ny=2s$ and $y=a+ne-e$, which ſhall be the buſineſs of the following twenty queſtions:

5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, &c.

$a=5$, $y=49$, $e=4$, $n=12$, $s=324$

*Queſt.* 1. Given $a$, $e$, $n$, Required $y$.
2. ——— $a$, $e$, $n$, ——— $s$.
3. ——— $a$, $e$, $y$, ——— $n$.
4. ——— $a$, $e$, $y$, ——— $s$.
5. ——— $a$, $e$, $s$, ——— $n$.
6. ——— $a$, $e$, $s$, ——— $y$.
7. ——— $a$, $y$, $s$, ——— $n$.
8. ——— $a$, $y$, $s$, ——— $e$.
9. ——— $a$, $n$, $s$, ——— $e$.
10. ——— $a$, $n$, $s$, ——— $y$.

| | | | | | | |
|---|---|---|---|---|---|---|
| *Queſt.* | 11 | Given | 11 | $a, n, y$ | Required | $s$ |
| | 12 | —— | 12 | $a, n, y$ | ——— | $e$ |
| | 13 | —— | 13 | $e, n, y$ | ——— | $s$ |
| | 14 | —— | 14 | $e, n, y$ | ——— | $a$ |
| | 15 | —— | 15 | $e, n, s$ | ——— | $y$ |
| | 16 | —— | 16 | $y, n, s$ | ——— | $a$ |
| | 17 | —— | 17 | $y, n, s$ | ——— | $e$ |
| | 18 | —— | 18 | $s, y, e$ | ——— | $a$ |
| | 19 | —— | 19 | $y, e, s$ | ——— | $n$ |
| | 20 | —— | 20 | $e, n, s$ | ——— | $a$ |

1. Given $a=5$, $e=4$, $n=12$, required $y$?

*Solution.* By Prop. II. $n=a+ne-e=49$, $a=5$, $ne=48$ $a+ne=53$, therefore $a+ne=53-e=4=49$. *Anſ.*

2. Given $a=5$, $e=4$, $n=12$, required $s$?

By Prop. II. $2s=na+ny$ II. $y=a+ne-e$ $ny=2s-na$ $y=\frac{2sn-na}{n}$ $\frac{2s-na}{n}=a+ne-e$ $2s-na=na+n^2e-ne$

$2s=2na+n^2e-ne$ $s=\frac{2na+n^2e-ne}{2}=324$ *Anſ.*

3. Given $a=5$, $e=4$, $s=324$, required $n$?

*Anſ.* By a due proceſs of Prop. I. & II. the ſquare completed, &c. $n=\frac{e-2a+\sqrt{8se+e^2+4a^2-4ae}}{2e}=12$.

4. Given $a=5$, $e=4$, $y=49$ required $n$?

*Ans.* By Prop. II. & I. and due reduction, and ſeeing that $n=\frac{2s}{a+y}=12$ it therefore follows that $s=324$.

5. Given $a=5$, $e=4$, $s=324$ required $n$?

*Anſ.* By Prop. II. due reduction, the ſquare completed, &c. $n=12$.

6. Given $a=5$, $e=4$, $s=324$, required $y$?

*Anſ.* By Prop. I. & II. due equality, proper reduction, the ſquare completed, &c. $y+49$.

7. Given $a=5$, $y=49$, $s=324$, required $n$?

*Anſ.* By Prop. I. $n=\frac{2s}{a+y}=12$ as required.

8. Given $a=5$, $y=49$, $s=324$, required $e$?

*Anſ.* Prop. I. $na+ny=2s$ conſequently $n=\frac{2s}{a+y}$ and by Prop. II. $y=a+ne-e$ hence $e=\frac{y^2-a^2}{2s-a-y}=4$

9. Given $a=5$, $n=12$, $s=324$, required $e$?

*Anſ.* By Prop I. $na+ny=2s$ Prop. II. $y=a+ne-e$ and $y=\frac{2s-na}{n} \therefore \frac{2s-na}{n}=a+ne-e$ hence we find $e=\frac{2s-2na}{n^2-n}=4$.

10. Given $\left\{\begin{matrix}a=5\\n=12\\s=324\end{matrix}\right\}$ Required $y$?

*Anſ.* By Prop. I. $y=\frac{2s-na}{n}=49$

11. Given $\left\{\begin{matrix}a=5\\n=12\\y=49\end{matrix}\right\}$ Required $s$?

*Anſ.* $s=\frac{na+ny}{2}=324$ by Prop. I.

12. Given $a=5$, $n=12$, $y=49$, required $e$?

*Ans.* $e=\frac{y-a}{n-1}=4$ by Prop. II.

13. Given $e=4$, $n=12$, $y=49$, required $s$?

*Ans.* By Prop. I. and II. $s=\frac{2ny-n^2e+ne}{2}=324$

14. Given $e=4$, $n=12$, $y=49$, required $a$?

*Anſ.* By Prop. II. $a=y-ne+e=5$.

15. Given $e=4$, $n=12$, $s=324$, required $y$?

*Anſ.* By Prop. I. & II. $y=\frac{2s+n^2e-ne}{2n}=49$.

16. Given $y=49$, $n=12$, $s=324$, required $a$?

*Ans.* By Prop. I. $a=\frac{2s-ny}{n}=5$

17. Given $y=49$, $n=12$, $s=324$, required $e$?

*Anſ.* By Prop. I. & II. $e=\frac{2ny-2s}{n^2-n}=4$

18. Given $s=324$, $y=49$, $e=4$, required $a$?

*Anſ.* By Prop. I. & II. due Reduction, the ſquare completed, &c. $a=\frac{e+4\sqrt{y^2+6ey-8se+e^2}}{2}=5$.

19. Given $y=49$, $e=4$, $s=324$, required $n$?

*Anſ.* By Prop. I. & II. due Reduction, &c. &c. $n=\frac{e+2y+\sqrt{8se+e^2+4y^2+4ey}}{2e}=12$.

20. Given $e=4$, $n=12$, $s=324$, required $a$?

*Ans.* $a=\frac{ne^2-n^2e+2s}{2n}=5$

21. The firſt term of an increaſing Arithmetical ſeries is 3, the common difference 2, and the number of terms 20; required the ſum of the ſeries? *Ans.* 440.

22. The firſt term of a decreaſing Arithmetical ſeries is 100, the common difference 3, and the number of terms 34; required the ſum of the ſeries? *Ans.* 1717.

23. Required the ſum of the natural numbers, 1, 2, 3, 4, 5, 6, &c. continued to 1000 terms? *Anſ.* 500500.

24. Required the ſum of the odd numbers, 1, 3, 5, 7, 9, &c. continued to 101 terms. *Anſ.* 10201.

*Note.* The ſum of any number ($n$) of the Arithmetical ſeries of odd numbers, 1, 3, 5, 7, 9, &c. is equal to the ſquare ($n^2$) of that number, that if 1, 3, 5, 7, 9, &c. be the numbers, then will 1, $2^2$, $3^2$, $4^2$, $5^2$, &c. be the ſums of 1, 2, 3, &c. terms, for $0+1$, or the ſum of 1 term$=1^2$ or 1, $1+3$, or the ſum of two terms$=2^2$ or 4; $4+5$ or the ſum of 3 terms$=3^2$ or 9, &c.

Whence it is plain, that let $n$ be any number whatever, the ſum of $n$ terms will be $n^2$.

## GEOMETRICAL PROGRESSION.

WHEN a rank or ſeries increaſes by a common multiplier, or decreaſes by a diviſor, ſuch a ſeries is ſaid to be a ſeries ∺ or in continued proportion, Geometrical. Example in numbers 3, 12, 48, 192, 768, 3072, 12288, &c. in ſpecies $a$, $ae^2$, $ae^3$, $ae^4$, $ae^5$, $ae^6$, $ae^7$, &c. In the Numerical ſeries, 4 is the common ratio, being the common multiplier increaſing, and the common diviſor decreaſing. In ſpecies, $e$ is the common ratio.

In any ſuch ſeries let
$a$ repreſent the First or leaſt term.
$y$ ——— Laſt or greateſt term.
$e$ ——— Common ratio.
$s$ ——— Sum of the ſeries.
$n$ ——— Number of terms.

*Propoſition.*

In any geometrical ſeries continued, the difference of the extremes divided by the common ratio leſs by one, is equal to the ſum of all the ſeries, except the laſt term, and conſequently if the laſt or greateſt term be added to the quotient, it will be equal to the ſum of the ſeries, *i. e.* $\frac{y-a}{e-1}+y=s$

*Demonſtration.*

Suppoſe $y=ae^8$

$$e-1\,)\,ae^8-a\,(\,ae^7+ae^6+ae^5+ae^4+ae^3+ae^2+ae+a=\frac{y-a}{e-1}$$

$$=s-y\ \therefore\ \frac{y-a}{e-1}+y=s \quad \textit{Q. E. D.}$$

$$\begin{array}{l} ae^8-ae^7 \\ \hline ae^7-a \\ ae^7-ae^6 \\ \hline ae^6-a \\ ae^6-ae^5 \\ \hline ae^5-a \\ ae^5-ae^4 \\ \hline ae^4-a \\ ae^4-ae^3 \\ \hline ae^3-a \\ ae^3-ae^2 \\ \hline ae^2-a \\ ae^2-ae \\ \hline ae-a \\ ae-a \\ \hline \end{array}$$

Now any two of the five quantities, viz. $a$, $y$, $e$, $s$, and $n$, being given, the other three can be found, which affords a great variety; but we are circumſcribed in our Appendix, and therefore the following problems only are propoſed, as being the moſt uſeful.

1. Given $a=5$, $e=4$, $s=27305$ required $y$?

*Anſ.* $\frac{y-a}{e-1}+y=s$ hence $y=\frac{se-s+a}{c}=20480$

2. Given $e=5$, $y=20480$, $e=4$, required $s$?

Hence $s=\frac{ey-a}{e-1}=27305$

3. Given $a=5$, $y=20480$, $s=27305$ required $e$?

Hence $e=\frac{s-a}{s-y}=4$

4. Given $y=20480$, $s=27305$, $e=4$, required $a$?

Solution at large, $\frac{y-a}{e-1}+y=s$ this equation turned out of fractions, we get $y-a+ey-y=se-s$ or $ey-se+s=a$ hence $a=ey-se+s=5$, as was required.

## QUADRATIC EQUATIONS.

WHEN after an Equation is reduced, the exponents of the known quantity are as 2 to 1, ſuch equations are called Quadratic; and by ſome adfected quadratics: and for their ſolution the rule is,

*Rule.* Add to each ſide of the equation the ſquare of half the co-efficient of the loweſt power of the unknown quantity, and then by extracting the ſquare root of both ſides of the equation, it will be reduced to a ſimple one, and the unknown quantity may be cleared as before.

*Note.* That before you complete the ſquare, the equation muſt be ſo ordered, that the higheſt power of the unknown quantity muſt have a poſitive ſign, and unity for a co-efficient.

### EXAMPLES.

1. $x^2+2bx=c^2$.

$x^2+2bx+b^2=c^2+b^2$ the ſquare completed.

$x+b=\sqrt{c^2+b^2}$ by extracting the ſquare root.

And $x=\sqrt{c^2+b^2}-b$ by tranſpoſition; the anſwer required.

*Anſwers.*

2. $x^2+cx=b^2$, required $x$? $x=\sqrt{b^2\frac{1}{4}c^2}-\frac{1}{2}c$.

3. $z^2-3cz=bd$, required $z$? $z=\frac{3}{2}c+\sqrt{bd+\frac{9}{4}c^2}$.

4. $z^4$

| | *Anſwers.* |
|---|---|
| 4. $z^4+cz^2=b^2$, required $z$? | $z=\sqrt{-\frac{1}{2}c+\sqrt{b^2+\frac{1}{4}c^2}}$ |
| 5. $z^2-12z=45$, required $z$? | $z=6+\sqrt{45+36}$. |
| 6. $x^2-cx+2dx=d^2+2cd$, required $x$? | $x=cd\sqrt{2d^2+c^2}$ |
| 7. $x^2-cx+dx=m^2$, required $x$? | $x=\frac{1}{2}c+\frac{1}{2}d+\sqrt{m^2+\frac{1}{4}c^2+\frac{1}{4}d^2+\frac{1}{4}cd}$. |
| 8. $z^2-2dz=b^2$, required $z$? | $z=d+\sqrt{z^2+d}$. |
| 9. $z^2-dz=bc$, required $z$? | $z=\frac{1}{2}d+\sqrt{bc+\frac{1}{4}d^2}$. |
| 10. $x^2+5dx=b^2$, required $x$? | $x=-\frac{5}{2}d+\sqrt{b^2+{}^{2}\frac{5}{4}d^2}$. |
| 11. $z^2-\frac{3}{4}bz=cd$, required $z$? | $z=\frac{3}{8}b+\sqrt{cd+\frac{9}{64}b^2}$ |
| 12. $2cz-z^2=bd$, required $z$? | $z=c\pm\sqrt{-bd+c^2}$. |
| 13. $z^2+4bz=cd$, required $z$? | $z=2b+\sqrt{cd+\frac{1}{4}b^2}$. |
| 14. $z^2-\frac{2}{3}dz=cd$, required $z$? | $z=\frac{1}{3}d+\sqrt{cd+\frac{1}{9}d^2}$. |
| 15. $x^2+\frac{5}{6}dx=b^2$, required $x$? | $x=-\frac{5}{12}+\sqrt{b^2d+\frac{25}{144}d^2}$. |
| 16. $x^2-10x=21$, required $x$? | $x=5+\sqrt{46}$. |
| 17. $z^2+2bz+2cz=c^2-2b$, required $z$? | $z=-b-c+\sqrt{2c+b^2}$. |
| 18. $z^2-2mz-2nz=m^2-2mn$, required $z$? | $z=m+n+\sqrt{2m^2+n^2}$. |
| 19. $x^2-cx+dx=m^2$; put $2n=-c+d$, | $\therefore x=\pm n+\sqrt{n^2+m^2}$ |
| 20. $z^2+3bz-2dz=cd$; put $2n=3b-2d$, | $\therefore z=\pm n+\sqrt{n^2+cd}$ |
| 21. $x^2-\frac{2}{3}cx-\frac{1}{2}px=b^2$, required $x$? | *Anſ.* $x+\frac{1}{3}c-\frac{1}{4}p+\sqrt{b^2+\frac{1}{9}c^2+\frac{1}{16}p^2}$ |
| 22. $x^2-bx+cx-dx=m^2$; put $2n=-b+c-d$, | $\therefore x=\pm n+\sqrt{n^2+m^2}$ |
| 23. $x^2-\frac{cx}{m}=\frac{b^2}{m}$, required $x$? | *Anſ.* $x=\frac{c\sqrt{4b^2m+c^2}}{2m}$. |

24. $z^2-\frac{\frac{2}{3}dz}{p}=\frac{c^2}{p}$, required $z$? *Anſ.* $z=\frac{-d+\sqrt{9cp+d^2}}{3p}$

25. $z^2+\frac{2bz}{c}=\frac{d^2}{c}$ required $z$? *Anſ.* $z=\frac{b+\sqrt{d^2c+b^2}}{c}$

26. $x^2+\frac{2dx}{p}-\frac{2cz}{p}=m^2$; put $2n=\frac{2d-2c}{p}$,

$\therefore\ x=\pm n+\sqrt{n^2+\frac{m^2}{p}}$

27. $x^2+\frac{3cdx-bpx+bcx}{q}=\frac{mn}{q}$; put $\frac{3cd-bp+bc}{q}=2n$,

$\therefore\ x=\pm n+\sqrt{n+\frac{nm}{q}}$

28. $z^2-\frac{2bz}{c+d}=\frac{m^2}{c+d}$, required $z$?

*Anſ.* $z=\frac{b+\sqrt{cm^2+dm^2+b^2}}{c+d}$

29. $x^2-\frac{cx}{b-d}=\frac{b^2}{b-d}$, required $x$?

*Anſ.* $x=\frac{c\sqrt{4b^3-4b^2d+c^2}}{2b-2d}$

30. $x^2+\frac{cdx+bdx}{p+q}=\frac{b^2}{p+q}$; put $2n=\frac{cd+bd}{p+q}$ and $m=\frac{b^2}{p+q}$

$\therefore\ x=\sqrt{n^2+m}-n$

31. $x^2-\frac{\frac{1}{2}cdx-\frac{1}{2}bdx}{p-b}=\frac{cd-d^2}{p-b}$; put $2n=\frac{\frac{1}{2}cd+\frac{1}{2}bd}{p-b}$ and $m=$ $\frac{cd-d}{p-b}$

$\therefore x=n\pm\sqrt{m+n^2}$

32. $\frac{bz^2-cz+dz}{m}-n=b$, $\therefore\ z^2-\frac{cz+dz}{bm}=\frac{b+n}{b}$; put $2n=$ $\frac{-c+d}{bm}$, and $m=\frac{b+n}{b}$, $\therefore\ z=\pm n+\sqrt{m+n^2}$

33. $b=cx+\frac{d}{x}$, required $x$? Ans. $x=\frac{b+\sqrt{-4cd+b^2}}{2c}$

34. $c-x=\frac{f}{bx+dx}$; put $\frac{f}{b+d}=m$, $\therefore x=\frac{c}{2}\pm\sqrt{\frac{c^2}{4}-m}$

35. $\frac{cz}{b^2-2bz+z^2}=\frac{d}{z}$, required $z$?*

Ans. $z=\frac{bd+\sqrt{ch^2d}}{c-d}$

36. $x-b=\frac{cdx}{px-mn}$, $\therefore px^2-pbx-mnx+bmn=cdx$, and $x^2-\frac{pb-mn-cd}{p}x=-\frac{bmn}{p}$; put $2n=\frac{bp+mn+cd}{p}$, and $m=\frac{bmn}{p}$, $\therefore x=n\pm\sqrt{n^2-m}$: Or if the value of $x$ is to be exhibited in forms of the original given quantities, without taking the advantage of this commodious method of ſubſtitution, then we have the following expreſſion, viz.

$$x=\frac{mn+bp+cd\pm\sqrt{m^2n^2+b^2p^2+c^2d^2+2mnbp+2mncd+2bpcd-4bpmn}}{2p}$$

37 $d-$

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* From a due conſideration of the foregoing queſtions it appears, that all Quadratic equations, when the ſign of the lower power of the unknown quantity is negative, admit of two ſolutions, for the ſquare root of any quantity may be either + or — ; thus the ſquare root of $x^2$ is $+x$ or $-x$; for $+x\times+x$, or $-x\times-x$, are each equal to $x^2$; but the ſquare root of $-x^2$ or $\sqrt{-x^2}$ is imaginary or impoſſible.

EXAMPLE. 1ſt, Let $x^2+ax=b$. 2nd, Let $x^2-ax=b$. Laſtly, let $x^2-ax=-b$. So in the firſt form $x^2+ax=b$ where $x+\frac{1}{2}a$ is found $=\sqrt{b+\frac{1}{4}a^2}$ the root may be either $+\sqrt{b+\frac{1}{4}a^2}$ or $-\sqrt{b+\frac{1}{4}a^2}$ ſince either of them being multiplied by itſelf will produce $b+\frac{1}{4}a^2$; and this ambiguity is expreſſed by writing the uncertain ſign of $\pm$ before $\sqrt{b+\frac{1}{4}a^2}$; thus $x=\pm\sqrt{b+\frac{1}{4}a^2}-\frac{1}{2}a$.

In this form where $x=\pm\sqrt{b+\frac{1}{4}a^2}-\frac{1}{2}a$, the firſt value of $x$, viz. $x=+\sqrt{b-\frac{1}{4}a^2}-\frac{1}{2}a$, is always affirmative; for ſince $\frac{1}{4}a^2+b$ is greater than $\frac{1}{4}a^2$, the greater ſquare muſt neceſſarily have the greater root; therefore $\sqrt{b+\frac{1}{4}a^2}$ will always be greater than $\sqrt{\frac{1}{4}a^2}$, or its equal $\frac{1}{2}a$; and conſequently $+\sqrt{b+\frac{1}{4}a^2}-\frac{1}{2}a$ will always be affirmative.

The

37. $3y^2=b+cy$; $c=18$, $b=120$, required $y$?

$$\text{Anſ. } y=\frac{c+\sqrt{12b+c^2}}{6}=10.$$

38. $d-bx=\frac{c}{x}$; $b=20$, $c=252$, $d=176$, required $x$?

$$\text{Anſ. } x=\frac{d\pm\sqrt{-4bc+d^2}}{2b}=7, \text{ or } 1\tfrac{4}{5}.$$

39. $\frac{bz}{c^2-2cz+z^2}=\frac{d}{z}$; $c=100$, $d=8$, $b=18$, required $z$?

$$\text{Anſ. } z=\frac{-cd+\sqrt{bc^2d}}{b-d}=\tfrac{1}{5}.$$

40. $2n^2=2s+ne-n^2e$; $e=2$, $s=45$, required $n$?

$$\text{Anſ. } n=\frac{e+\sqrt{16s+8se+e^2}}{4+2e}=5.$$

41. $b-x=\frac{f}{cx+dx}$; $b=21$, $c=\frac{1}{5}$, $d=\frac{1}{3}$, $f=48$, required $x$?

put $\frac{f}{c+d}=m=90$, then $b-x=\frac{m}{x}$,

$$\therefore x=\frac{b}{2}\pm\sqrt{\frac{b^2}{4}-m}=15, \text{ or } 6.$$

The ſecond value, viz. $x=-\sqrt{b+\frac{1}{4}a^2-\frac{1}{2}a}$, will always be negative, becauſe it is compoſed of two negative terms. Therefore, when $x^2+ax=b$ we ſhall have $x=+\sqrt{b+\frac{1}{4}a^2}-\frac{1}{2}a$ for the affirmative value of $x$, and $x=-\sqrt{b+\frac{1}{4}a}-\frac{1}{2}a$ for the negative value of $x$.

In the ſecond form where $x=\pm\sqrt{b+\frac{1}{2}a^2}+\frac{1}{2}a$ the firſt value, viz. $x=+\sqrt{b+\frac{1}{4}a^2}+\frac{1}{2}a$ is always affirmative, ſince it is compoſed of two affirmative terms.

The ſecond value, viz. $x=-\sqrt{b+\frac{1}{4}a^2}+\frac{1}{2}a$ will always be negative; for ſince $b+\frac{1}{4}a^2$ is greater than $\frac{1}{4}a^2$, $\sqrt{b+\frac{1}{4}a^2}$ will be greater than $\sqrt{\frac{1}{4}a^2}$ or its equal $\frac{1}{2}a$; and conſequently $-\sqrt{b+\frac{1}{4}a^2}+\frac{1}{2}a$ is always a negative quantity.

Therefore when $x^2-ax=b$ we ſhall have $x=\pm\sqrt{b+\frac{1}{4}a^2}+\frac{1}{2}a$ for the affirmative value of $x$, and $x=-\sqrt{b+\frac{1}{4}a^2}+\frac{1}{2}a$ for the negative value of $x$; ſo that in both the firſt and ſecond forms the unknown quantity has always two values, one of which is poſitive and the other negative.

42. $y-a=\frac{cgy}{by-bd}$; $a=14$, $b=80$, $c=36$, $d=8$, $h=188$, $g=26$, quere $y$. $hy^2-aby-bdy+abd=cgy$, $\therefore\ y^2-\frac{ab-bd-cg}{b}y=-\frac{abd}{b}$; put $2n=\frac{ab+bd+cg}{b}$ and $m=\frac{abd}{p}$ $\therefore\ y=n\pm\sqrt{n^2-m}=20$, or $2\frac{19}{47}$.

43. $\frac{d}{z}+\frac{b}{z+f}=c$; $b=40$, $d=90$, $f=\frac{1}{3}$, $c=42$, quere $z$? reduced, $z^2+\frac{cf-b-d}{c}z=\frac{df}{c}$; put $\frac{cf-b-d}{c}=2n$; and $\frac{df}{c}=m$, $\therefore\ z=\pm\sqrt{n^2+m}\pm n=3$, or $\frac{5}{21}$.

44. $x^4-2ax^2=b$, quere $x$? complete the ſquare, and $x^4-2ax^2+a^2=b+a^2$, root $x^2-a=\sqrt{b+a^2}$, and $x^2=a\sqrt{b+a^2}$, $\therefore\ x=\sqrt{a+\sqrt{b+a^2}}$.

ARITHMETICAL

In the third form, where $x=\pm\sqrt{\frac{1}{4}a^2-b}+\frac{1}{2}a$, both values of $x$ will be poſitive; ſuppoſing $\frac{1}{4}a^2$ is greater than $b$, for the firſt value, viz. $x=+\sqrt{\frac{1}{4}a^2-b}+\frac{1}{2}a$ will then be affirmative, being compoſed of two affirmative terms.

The ſecond value of $x$, viz. $x=-\sqrt{\frac{1}{4}a^2-b}+\frac{1}{2}a$ is affirmative; for ſince $\frac{1}{4}a^2$ is greater than $\frac{1}{4}a^2-b$, $\sqrt{\frac{1}{4}a^2}$ or $\frac{1}{2}a$ is greater than $\sqrt{\frac{1}{4}a-b}$, and conſequently $-\sqrt{\frac{1}{4}a-b}+\frac{1}{2}a$ will always be an affirmative quantity.

Therefore when $x^2-ax=-b$ we ſhall have $x=+\sqrt{\frac{1}{4}a^2-b}+\frac{1}{2}a$, and alſo $x=-\sqrt{\frac{1}{4}a^2-b}+\frac{1}{2}a$ for the affirmative value of $x$.

But in this third form, if $b$ be greater than $\frac{1}{4}a^2$, the ſolution of the propoſed queſtion will be impoſſible; for ſince the ſquare of any quantity (whether that quantity be affirmative or negative) is always affirmative, the ſquare root of a negative quantity is impoſſible, and cannot be aſſigned.

But if $b$ be greater than $\frac{1}{4}a^2$, $-b$ is a negative quantity; and therefore $\sqrt{\frac{1}{4}a-b}$ is impoſſible or imaginary; conſequently in that caſe $x=\frac{1}{2}a\pm\sqrt{\frac{1}{4}a^2-b}$ is always impoſſible, or imaginary.

## ARITHMETICAL QUESTIONS,

### *To be ſolved Algebraically.*

Suppoſe $x=$ the greater of two unknown numbers, and $y=$ the leſs. Alſo let $2a=$ their ſum, $2b=$ their difference, $2s=$ the ſum of their ſquares, $d=$ the difference of their ſquares, $p=$ the product, and $q=$ the quotient of the greater divided by the leſs. Then by having any two of the ſix quantities, $a$, $b$, $s$, $d$, $p$, and $q$, given, the unknown quantities, $x$ and $y$, and the other four can be found; which affords a variety of 15 different problems; and their ſolutions are below exhibited. The ingenious Mr. Ward, in his Young Mathematician's Guide, ſolves thoſe 15 queſtions, but not indeed in the moſt elegant manner.

Suppoſe $2a=8$, $2b=2$, $2s=34$, $2d=16$, $p=15$, and $y=\frac{5}{3}=1\frac{2}{3}$.

| | Given | | Hence |
|---|---|---|---|
| 1 | $a$ & $b$ | Required $x$ and $y$ and the other four quantities. | $x=a+b$ & $y=a-b$ therefore $x=5$ & $y=3$ |
| 2 | $a$ & $s$ | | $x=a+\sqrt{s-a^2}$ and $y=a-\sqrt{s-a^2}$ ∴ $x=5$ and $y=3$ |
| 3 | $a$ & $d$ | | $x=a+\frac{d}{2a}$ & $y=a-\frac{d}{2a}$ ∴ $x=5$ and $y=3$ |
| 4 | $a$ & $p$ | | $x=a+\sqrt{a^2-p}$ & $y=a-\sqrt{a^2-p}$ ∴ $x=5$ & $y=3$ |
| 5 | $a$ & $q$ | | $x=\frac{2aq}{q+1}$ & $y=\frac{2a}{q+1}$ ∴ $x=5$, and $y=3$ |
| 6 | $b$ & $s$ | | $x=b+\sqrt{s-b^2}$ & $y=\sqrt{s-b^2}-b$ ∴ $x=5$ and $y=3$ |
| 7 | $b$ & $d$ | | $x=b+\frac{d}{2b}$ & $y=\frac{d}{2b}-b$ ∴ $x=5$ and $y=3$ |
| 8 | $b$ & $p$ | | $x=\sqrt{p+b^2}+b$ & $y=\sqrt{p+b^2}-b$ ∴ $x=5$ & $y=3$ |
| 9 | $b$ & $q$ | | $x=\frac{2bq}{q-1}$ & $y=\frac{2b}{q-1}$ ∴ $x=5$ and $y=3$ |
| 10 | $s$ & $d$ | | $x=\sqrt{s+d}$ & $y=\sqrt{s-d}$ ∴ $x=5$ and $y=3$ |
| 11 | $s$ & $p$ | | $x=\sqrt{s+\sqrt{s^2-p^2}}$ & $y=\sqrt{s-\sqrt{s^2-p^2}}$ ∴ $x=5$ & $y=3$ |
| 12 | $s$ & $q$ | | $x=q\sqrt{\frac{2s}{q^2+1}}$ & $y=\sqrt{\frac{2s}{q^2+1}}$ ∴ $x=5$ and $y=3$ |

| | | |
|---|---|---|
| 13 | $d$ & $p$ | $x=\sqrt{\sqrt{p^2+d^2}+d}$ & $y=\sqrt{\sqrt{p^2+d^2}-d}$ $\therefore x=5$ and $y=3$ |
| 14 | $d$ & $q$ | $x=q\sqrt{\frac{2d}{q^2-1}}$ & $y=\sqrt{\frac{2d}{q^2-1}}$ $\therefore x=5$ and $y=3$ |
| 15 | $p$ & $q$ | $x=q\sqrt{\frac{p}{q}}$ & $y=\sqrt{\frac{p}{q}}$ $\therefore x=5$ and $y=3$ |

Having found $x$ and $y$, the other four quantities are eaſily determined; but it may ſometimes be neceſſary to find thoſe four quantities in terms of the two given ones. Thus, in Example 1, where $x=a+b$ and $y=a-b$; then $s=a^2+b^2$, $d=2ab$; $p=a^2-b^2$; and $q=\frac{a+b}{a-b}$ and ſo of the reſt.

N. B. Theſe 15 problems contain a valuable variety of Arithmetical enquiry; in fact, their ſolutions compoſe a whole ſyſtem of Arithmetic, and we preſume that the reader will find that the above method is the beſt that poſſibly can be invented.

*Solution of Problems.*

1. What two numbers are thoſe whoſe difference is 20, and whoſe ſum is 160? *Anſ.* 90 and 70.

2. What two numbers are thoſe whoſe difference is 14, and the quotient of the greater divided by the leſs, is 3? *Anſ.* Greater number 21, and 7 the leſſer.

3. There are two numbers whoſe ſum is 35, and the greater is to the leſs as $5\frac{1}{2}$ to $3\frac{1}{4}$, quere the numbers? *Anſ.* 22 and 13.

*Solution.*

$$b=35,\ c=5\tfrac{1}{2},\ d=3\tfrac{1}{4}.$$

$$x+z=b,\ x : z :: c : d,\ xd=zc,\ z=b-x,\ z=\frac{dx}{c}$$

$$\text{Put}\ \left.\begin{array}{l} x=⊏ \\ z=⊐ \end{array}\right\}\ \text{Number sought.}$$

b=

$$\left.\begin{array}{l} bc=\frac{385}{2}\div\frac{70}{8}=22=x \\ bd=\frac{455}{4}\div\frac{70}{8}=13=z \end{array}\right\} \begin{array}{r} 22 \\ 13 \\ \hline 66 \\ 22\phantom{0} \\ \hline 286 \end{array} \qquad \begin{array}{l} \text{As } 22 : 13 :: 5\frac{1}{2} : 3\frac{1}{4} \\ \phantom{\text{As }} 22 : 13 :: \frac{11}{2} : 1\frac{3}{4} \end{array}$$

$$\begin{array}{r} 13 \\ 11 \\ \hline \end{array}$$

$$\frac{286}{4}=71\frac{1}{2} \qquad \text{Proof, } \frac{143}{2}=71\frac{1}{2}$$

Therefore $\frac{dx}{c}=b-x$, $dx=bc-cx$, $dx+cx=bc$, $=\frac{bc}{d+c}=22$,

$z=b-x=b-\frac{bc}{d+c}=\frac{bd}{d+c}=13$.

4. Find two numbers whoſe difference is 30, and their ratio as $7\frac{1}{4}$ to $3\frac{1}{2}$. *Anſ.* 58 and 28.

5. Find two numbers whoſe product is 550, and whoſe ratio is $6\frac{1}{4}$ to $6\frac{1}{2}$. *Anſ.* 25 and 22.

6. Find two numbers whoſe ratio is as $3\frac{3}{4}$ to 1, and the ſum of whoſe ſquares is 964. *Anſ.* 30 and 8.

7. What number is that whoſe third part, added to its fourth will make 21 ? *Anſ.* 36.

8. What number is that whoſe third part exceeds is fourth by 4 ? *Anſ.* 48.

9. What number is that whoſe fifth part leſs 4, is equal to its half leſs 25 ? *Anſ.* 70.

10. Find two numbers whoſe ratio is $11\frac{1}{4}$ to 3 and the difference of whoſe ſquare is 836. *Anſ.* 30 and 8.

11. Find two numbers the difference of whoſe cubes is 1688112, and whoſe ratio is as 29 to 15. *Anſ.* 58 and 30.

12. A. aſked B. to lend him 20*s.*, B. anſwered he had not then ſo much, but if he had as much more, half as much, and $2\frac{1}{2}$ ſhillings, he would have juſt 20*s.*, how much money had B ? *Anſ.* 7*s.*

13. Find a number which being added to itſelf, and multiplied by itſelf, the ſum and product ſhall be equal. *Anſ.* 2.

14. Find a number whoſe ſquare root is three times greater than its cube root. *Anſ.* $\overline{3}|^{6}=729$.

15. A perſon went to an ale-houſe and borrowed as much money as he carried in, and out of the ſum he ſpent 6*d.* and carried the remainder to a ſecond houſe, where he borrowed

as much money as he carried in, and out of the ſum he ſpent 6*d.* and went with the remainder to a third houſe, where he borrowed as much as he carried in, and out of the ſum he ſpent 6*d.* and then he had nothing left: how much money did he carry to the firſt houſe? *Anſ.* $5\frac{1}{4}$*d.*

16. What two numbers are thoſe whoſe difference is 8 and the difference of their ſquares is 208? *Anſ.* 17 and 9.

17. Find two numbers, the product whereof is 108, and the triple of the greater divided by the leſſer is 4. *Anſ.* 12 and 9.

18. What two numbers are thoſe whoſe ſum is 60, and the greater to the leſſer as 9 to 3? *Anſ.* 45 and 15.

19. Find two numbers, to whoſe ſum if you add 8, the whole will be double the greater, and if you ſubtract 4.5 from their difference the remainder will be half the laſt. *Anſ.* 15 and 7.

20. A. alone can build an houſe in 21 days, B. in 28 days; in what time may both together build it? *Anſ.* 12 days.

21. A man and his wife together can drink a veſſel of beer in 15 days, the man alone can drink it in 24 days; in what time will the woman alone drink it? *Anſ.* 40 days.

22. There are 480 ſquare feet to be put into an oblong form, whoſe length and breadth together make 52: how many feet in each ſide? *Anſ.* 40 length and 12 breadth.

23. The continual product of four numbers in arithmetical progreſſion is 945, and the common difference 2: what are thoſe numbers?* *Anſ.* 3, 5, 7, and 9.

24. To find a number which being multiplied by 3, ſubtracting 5 from the product, and the remainder divided by 2, if the number ſought be added to the quotient, that ſum may be 40? *Anſ.* 17.

25. Find a number which being multiplied by 12, and 48 added to the product, as much may be produced, as if the ſame number ſought were multiplied by 18. *Anſ.* 8.

26. If to my months you ſhould add half this ſum,
And one eighth more, and then ſhould ſubtract one,
The reſidue would ſuch a number, be
As twenty-one, being ſquared, aſſuredly?

*Anſ.* 22 years 8 months.

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* This involves a double quadratic, the ſolution whereof is ſhewn in the laſt example of quadratics.

27. To find a number, to which if its double, triple, and quadruple be added, the ſquare of the ſame number may be produced. *Anſ.* 10.

28. To divide the number 16 into two parts, ſo that the ſquare of the greater part may exceed the ſquare of the leſſer by 29. *Ans.* 7 and 9.

29. A general diſpoſing of his army into a ſquare battle, finds that he has 284 ſoldiers over and above, but increaſing each ſide with one ſoldier, he wants 25 to fill up the ſquare, how many ſoldiers had he? *Ans.* 24000.

30. To find the ſide of a ſquare, whoſe area is to the ſum of the ſides as 45 to 12. *Anſ.* 15.

31. To find the ſide of a cube, whoſe ſurface is to the ſolidity in a given ratio as 6 to 11. *Anſ.* 11.

32. Three numbers in geometrical progreſſion, are required, ſo that the difference of the firſt and ſecond may be 6, and of the ſecond and third 15. *Anſ.* 4, 10, and 25.

33. To find a number, from the cube of which you ſubtract 19, and multiply the remainder by that cube, the product ſhall be 216. *Ans.* +3 or —2.

34. Sixty thouſand brave ſoldiers in battle there were,
Placed on a vaſt plain, and in form a long ſquare;
Now how many acres of ground did they ſtand on,
At two yards and three-quarters between man and man;
And how many in rank file will there be,
When their breadth to their length is as two is to three?

*Ans.* 300 in rank, file 200; and 92*a.* 3*r.* 35.2 *per.* Engliſh meaſure, the ground they ſtood on.

35. Old John, who had in credit liv'd,
Tho' now reduc'd a ſum receiv'd;
This lucky hit's no ſooner found,
Than clam'rous duns came ſwarming round.
To th' landlord—baker—many more,
John paid in all, pounds ninety-four.
Half what remained—a friend he lent;—
On Joan and 'ſelf, one fifth he ſpent;
And when of all theſe ſums bereft,
One tenth o'th' ſum receiv'd had left.

Now ſhew your ſkill, ye learned youths,
And by your work the ſum produce?

*Anſ.* 141*l.*

36. A perſon being aſked the hour of the day, anſwered, it is between 5 and 6, and both the minute hand and hour hand are together: required the hour of the day? *Anſ.* $27\frac{3}{11}$ minutes paſt 5.

37. A general who had fought a battle, upon reviewing his army (whoſe foot was thrice the number of his horſe) finds that before the battle $\frac{1}{12}$—120 of his foot had deſerted, and of his horſe $\frac{1}{20}$+120, beſides $\frac{1}{4}$ of his army was ſent into garriſon (reckoning the ſick and wounded) and $\frac{3}{8}$ of his army remained, the reſt who were wanting, being either ſlain or taken priſoners: now if you add 3000 to the number of ſlain, the ſum will be =half the foot he had at the beginning; what were the number of each? *Anſ.* 10000 horſe and 30000 foot.

38. To divide 100 twice into 2 parts, ſo that the major part of the firſt diviſion, may be triple the minor part of the ſecond diviſion, and the major part of the ſecond may be double the minor part of the firſt? *Anſ.* 60, 40, and 20, 80.

39. Divide 30 twice into two parts, ſo that the major part of the firſt diviſion with the minor part of ſecond may be 33, and the ſum of the minor parts ſubtracted from the ſum of the major leaves 14 remaining. *Anſ.* 20 and 10, alſo 17 and 13.

40. A man, his wife, and his ſon's age make 96 years, ſo that the huſband's and ſon's years together make the wife's +15; but the wife's and the ſon's make the huſband +2; what was the age of each? *Anſ.* huſband 47, wife's 40½, and ſon's 8½.

41. Three merchants from different fairs meet together at an inn, where they reckon up their gains, and find them the ſum of 780l.; if you add the gains of the firſt and ſecond, and ſubtract the gain of the third from the ſum, there remains the gain of the firſt +28l., but if you add the gain of the ſecond and third, and from the ſum ſubtract the gain of the firſt, there remains the gain of the third —43l. what was the gain of each. *Ans.* the gain of the firſt 316l. of the ſecond 273l. and third 191l.

42. Three perſons, A. B. C. owe a ſum of money, ſo that A. and B. together owe 210l. B. and C. 290l. and C. and A. 400l. what did each of them owe? *Ans.* A's. debt 160l. B's. 50l. and C's. 240l.

43. Find three numbers, ſo that the firſt and $\frac{1}{2}$ the ſum of the other two, the ſecond and $\frac{1}{3}$ the ſum of the other two, the third and $\frac{1}{4}$ of the ſum of the other two may always make 34? *Anſ.* 10, 22 and 26.

44. Let a ſquare be divided into 9 ſmall ſquares; we are to find and diſpoſe the numbers through the ſeveral ſmall areas, ſo that the ſum of every three taken diagonally or laterally may be always 15:

| | | |
|---|---|---|
| 4 | 9 | 2 |
| 3 | 5 | 7 |
| 8 | 1 | 6 |

## *Queſtions producing Quadratic Equations.*

1. To find a number, which being multiplied by 6, and the product ſubtracted from the ſquare of the number to be found the remainder will be 280. *Ans.* 20.

*Solution.*

$b = 280$
$c = 6$
$x =$ number ſought.

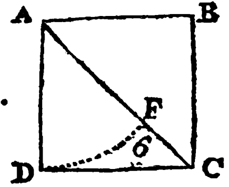
$$\begin{cases} x^2 - cx = b \\ x^2 - cx + \frac{1}{4}c^2 = b + \frac{1}{4}c^2 \text{ the } \square \text{ completed.} \\ x - \frac{1}{2}c = \sqrt{b + \frac{1}{4}c^2} \text{ and } x = \sqrt{b + \frac{1}{4}c^2} + \\ \frac{1}{2}c = 20. \quad \text{Q. E. F.} \end{cases}$$

2. To find a number, which being multiplied by 8, and the product added to the ſquare of the number to be found, the ſum may be 660. *Anſ.* 22.

3. To divide 140 into two parts, ſo that the product of thoſe parts may = the ſquare of 56, *i. e.* 3136. *Ans.* 112 and 28.

4. Let 969 ſoldiers be drawn up into an oblong battle, ſo that the difference of the greater and leſs ſides is 40, required the number of the ſoldiers of each rank in length and breadth. *Ans.* in length 57, and in breadth 17.

5. In the ſquare ABC is given the difference of the diagonal, and the ſide *i.e.* FC=6; required the ſides of the ſquares; *Ans.* 14.4852.

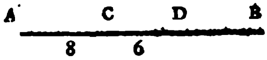


6. A man

6. A man buys ſome ells of cloth for 70s., and finds that if he had 4 ells more he then had bought every ell 2s. cheaper, how many ells did he buy? *Ans.* 10.

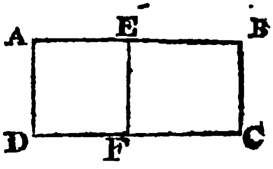
7. A ſet of boon companions dining at an inn, the reckoning in all came to 175 ſhillings, but before the bill was paid two of them ſlunk away, and then the club of thoſe that remained came to 10s. more a man, how many were there in company? *Ans.* 7 men.

8. Let the line AB be divided in C, ſo that AC may be 8 and CD, 6; we are to divide the ſame line AB in D, ſo that the rectangle under AD and CD may be equal to the rectangle AC and CB, or to the product from 8 and 6 which is 48; required the ſegment CD? *Ans.* 4.



9. To divide 21 into two ſuch parts, ſo that if the greater be divided by the leſs, and again the leſs by the greater, and then the firſt quotient being multiplied by 5, and the latter by 25, the numbers produced may be equal. *Ans.* $3\frac{1}{2}$ and $17\frac{1}{2}$.

10. Let there be a rectangular Garden ABCD, the length of which AB is thrice the breadth AD, and reckoning 18 perches from B towards A, *i.e.* BE and drawing EF parallel to AD, let the area of the remaining rectangle FD be given =120 perches, what was the length and breadth of the ſaid garden? *Ans.* 30 and 10.



11. Let 600 ſoldiers be diſpoſed into an oblong battle, which the colonel, willing to make broader, finds that if he takes away 10 ranks from the length, he ſhall augment the breadth with 2 ranks, what was the number of his ſoldiers through every rank in length and breadth? *Ans.* 60 and 10.

12. A man bought a horſe, which he ſold again for 56l. and gained as many pounds in 100 as the horſe coſt him; I demand how much did he give for the horſe? *Ans.* 40l.

13. A draper bought two ſorts of linen for 30 ſhillings, one fine and the other coarſe; an ell of the finer coſt as many ſhillings as he had ells; and alſo 28 ells of the coarſer

at ſuch a price, that 8 ells coſt as many ſhillings as one of the finer; how many ells of the finer linen did he buy, and what price did he give for them both? *Ans.* 4s. for the finer, the coarſer coſt $1\frac{1}{2}$s.

14. Of three proportional numbers, there is the middle term given = 8, and the difference of the ſecond and third = 24; required the ſeveral numbers? *Ans.* 2, 8 and 32.

15. Of four proportional numbers, there is given the third = 12, alſo the ſum of the firſt and ſecond = 8, beſides the ſecond number being ſubtracted from its ſquare, the remainder is four times thereof; required ſaid numbers, *Ans.* 3, 5, 12 and 20.

16. Of four numbers in continued proportion, there is given the ſum of the means = 24, and the ſum of the extremes = 36; required the ſaid numbers, (ſuppoſing that the firſt is leaſt of all)? *Ans.* 4, 8, 16, and 32.

17. Two country women, A. and B. carry 100 eggs together to market, and in the ſale of them, one took as much money as the other; but A, (who had the beſt eggs) ſays to B. had I carried as many eggs as you, I ſhould have had 18d. for them; B. replies, if I had brought as many eggs as you, I ſhould have had but 8d. for them; how many eggs had each? *Anſ.* A. 40, and B. 60.

18. Two country-men, A. and B. ſell their corn at different prices. A ſells 20 buſhels, and B. received for one buſhel as many ſhillings as he ſold buſhels; A. perceives that if he had ſold as many buſhels as B. received ſhillings he ſhould then have received 252 ſhillings: but both together received 176 ſhillings: how many buſhels did B. ſell, and what price had A.? *Anſ.* B. 6 buſhels, and A's price 7s. per buſhel.

19. Two merchants ſell 21 yards of cloth. The firſt ſells one yard for as many ſhillings as are $\frac{1}{5}$ of the number the ſecond had; and the ſecond ſells 1 yard for as many ſhillings as is $\frac{1}{3}$ of the number of yards that the firſt had; the ſale being over, they had taken 48 ſhillings in all; how many yards did each ſell and at what price? *Anſ.* A. had 6 yards at 3s. per yard; B. 15 yards at 2s. per.

20. Two merchants have a parcel of ſilk, the firſt 40 yards, the ſecond 90; the firſt ſells for a crown $\frac{1}{3}$ of an ell more than the ſecond: when the ſale was over, they had taken between them 42 crowns: how many ells did each of them ſell for a crown? *Ans.* A. $3\frac{1}{3}$ ells, and B. 3 ells.

21. To find a number, to the quadruple of which if you add 91, the whole ſhall be to the ſquare of the number as 3 to 4. *Ans.* 14.

22. To find a number, from the double of which, if you ſubtract 12, the ſquare of the remainder leſs 1 ſhall be 9 times the number ſought. *Anſ.* 11.

23. To divide the number 19 into two parts, ſo that the ſum of the ſquares will be 193. *Anſ.* 12 and 7.

24. To divide 7 into two ſuch parts, ſo that the difference of the ſquares which are made from the triple of the leſs part, and the double of the greater may be 17. *Anſ.* 4 and 3.

25. A man buys a piece of linen, and by ſelling it again gains 12s. —$\frac{1}{10}$ part of what he bought it for; and finds by this means that he had gained as much by 100s. as the linen coſt him; at what price was the linen bought and ſold?
*Anſ.* 30s. and ſold it at 39s.

26. A man buys 18 yards of cloth of different ſorts and colours, ſuppoſe red and black, what he bought of each ſort coſt 40 florins, and he pays for every yard of red cloth one florin more than for the black; how many yards of each did he buy? *Anſ.* 10 yards of black, and 8 of red.

27. A man buys 80lb. of pepper, and 36lb. of ſaffron, ſo that for every 8s. he had 14lb. of pepper more than he had of ſaffron for 7l. 6s. and what he laid out amounted to 9l. 8s. how many lb. of pepper had he for 8s. and how many of ſaffron for 7l. 6s.? *Ans.* 20lb. of pepper and 6lb. of ſaffron.

28. A. and B. between them owe 8l. 14s. and they both begin to diſcharge the common debt at the ſame inſtant of time: A. pays 8s. every day, and B. pays the firſt day 1s. the ſecond day 2s. the third 3s. and ſo on: In how many days will they clear the debt, and how much did each owe?

*Anſ.* they will pay it in 12 days; A. pays 4l. 16s. and B. 3l. 18s.

29. A certain man intends to travel as many days as he has ſhillings; it happens that every following day of his journey, he had as many ſhillings as he had the day before, beſides 2s. over and above; and when he came to his journey's end he finds he had in all 2l. 5s.; how many had he at firſt? *Anſ.* 5s.

30. A traveller goes 9 miles every day; 3 days after another follows him, who the firſt day travels 4 miles, the

ſecond 5, the third 6, and ſo on gaining a mile each day; in what time will he overtake the former?

*Ans.* 14.6877 days.

31. Again, two travellers ſet out at the ſame time from two cities, the one from A. and the other from B. which are 20 miles diſtant from one another; the firſt goes five miles a day, and the other 3 miles leſs than the number of days in which they meet; when will they meet? *Ans.* 10 days.

32. A poſt-man ſets out from A. towards B. who travels 8 miles a day; after he had gone 27 miles, another ſets out from B. to meet him, who goes every day $\frac{1}{20}$ of the whole journey or diſtance of the places A. and B. and meets the firſt poſt-man after ſo many days as is $\frac{1}{20}$ of the ſaid diſtance in miles; required the diſtance of A. and B.?

*Ans.* 180 miles.

33. A. aſked his father how old he was, the father replies thus, if you take 4 from my age the remainder will be thrice the number of your years; but if you take 1 from your age, half the remainder will be the ſquare root of my age: required the age of the father and ſon? *Ans.* 49 the father's age, 15 the ſon's age.

34 To find two numbers, the ſum of whoſe ſquares may be 317, and the product if they be multiplied by one another 154. *Ans.* 11 and 14.

35. To find two numbers, the product of which may be 108, and the difference of their ſquares 63. *Ans.* 9 and 12.

36. Two farmers ſell two ſorts of corn, A. ſells 6 buſhels, B. receives in all for his 1l.; now ſays B. to A. if we add the number of my buſhels to the number of your ſhillings the ſum will be 28; ſays A. to B. and if you add the ſquare of my ſhillings to the ſquare of your buſhels the ſum will be 424; how many buſhels did B. ſell, and how many ſhillings did A. receive? *Ans.* 18 buſhels B. ſold, and A. received 10s.

37. To find two numbers, the firſt of which +3, multiplied into the ſecond —3, may produce 121; and on the contrary, the firſt —3 multiplied by the ſecond +3 give 85.

*Ans.* 8 and 14.

38. A debt of 1360l. was diſcharged by ſeveral payments made in ſuch a manner, that the ſecond payment exceeded the firſt by $\frac{1}{2}$ of a pound, and every payment after encreaſing by the ſame exceſs, the laſt payment was 21$\frac{1}{3}$l. what was the firſt payment and laſt, and how many ſeveral payments were

made? *Ans.* there were 120 payments, the firſt was 1l. 10s.

39. A foot-man travels 124 miles in 8 days, at this rate, viz. the ſecond day's journey exceeded the firſt by 3 miles, and the third exceeded the ſecond by 3 miles, and ſo on; how many miles was his firſt day's journey, and how many his laſt? *Ans.* 26 miles the laſt day and 5 the firſt.

40. A draper bought 20 pieces of cloth for 20l. each, and ſold the firſt for a certain number of pounds, the ſecond for 2 more than the firſt, the third for 2 more than the ſecond, and ſo on, he ſold the laſt for 41l. Quere what he ſold the firſt piece for, and what he gained or loſt by the bargain? *Ans.* he ſold the firſt for 3l. and he gained 40l.

41. One diſtributed 456 ſhillings among a number of perſons in this manner, viz. to the firſt he gave 6s. to the laſt 51s. and the number given to each from firſt to laſt was by an equal difference: quere the number of perſons, and what each perſon had? *Ans.* each got 3s. more than the preceding one, and number of perſons 16.

42. A man who had 5 children, being aſked the age of them, anſwered, the age of the eldeſt exceeded the age of the ſecond by 2 years, the third by 4, the fourth by 6, &c. and if the age of the eldeſt was multiplied by the age of the youngeſt, the product would be 128: quere the age of each child? *Ans.* 8 the youngeſt, &c.

43. If the ſum of 6 terms in Arithmetical progreſſion be 48, and the product of the common difference multiplied into the leaſt term = the number of terms? quere the numbers? *Ans.* 3, 5, 7, 9, 11, 13; or 5, 6.2, 7.4, 8.6, 9.8, and 11.

44. Two footmen, A. and B. depart at the ſame time from London towards York, and travel in this manner, viz. A. travels 6 miles per day conſtantly, B. travels 1 mile the firſt day, 2 the ſecond, 3 the third, &c.: quere when B. will overtake A? *Ans.* in 11 days.

45. There are three numbers in Geometrical progreſſion, the difference of the extremes is 16, the mean 6; what are the extremes? *Ans.* 2 and 18.

46. There are three numbers in continued progreſſion, the ſum of the extremes is 20, the mean 6; what are the numbers? *Ans.* 2 and 18 the extremes.

47. There are two numbers, whoſe difference is 15, and if their product be divided by 2, the quotient will give the cube of the leſſer: quere the numbers? *Ans.* 3 and 18.

48. There are two numbers, whoſe difference is 2, and the ſum of their ſquares added to their ſum is 146; what are the numbers? *Ans.* 7 and 9.

49. There are two numbers, whoſe ſum is 14, and the ſum of their ſquares added to their difference is 102; quere the numbers? *Anſ.* 8 and 6.

50. There are two numbers, whoſe ſum is 10, and the proportion which their difference bears to the ſum of their ſquares is as 2 to 29; quere the number? *Anſ.* 3 and 7.

51. There are two numbers, whoſe product is 20, and the ſum of their cubes 189; quere the numbers? *Ans.* 4 and 5.

52. There are two numbers, whoſe product is 20, and the difference of their cubes 161; quere the numbers? *Anſ.* 4 and 5.

53. A merchant having bought certain cloths, ſells them at 17l. 5s. per cloth, and then found that by every 100l. he laid out he gained as many pounds as he paid for one cloth, what was the prime coſt of each cloth? *Ans.* 15l.

54. Each of two captains, whereof one had a leſſer number of ſoldiers in his company than the other by 40, diſtributed equally among the ſoldiers of each company 1200 ſhillings, whereby it happened that the ſoldiers of the leſs company had 5 ſhillings a piece more than thoſe of the greater; quere the number of men in each company, and how many ſhillings each ſoldier had? *Ans.* 120 men in the greater, and 80 in the leſſer; the firſt received 10s. and the other 15s. a piece.

55. Two ſocieties, whereof one exceeds the other by 4, divide equal ſums of pounds; the men of the leſſer ſociety have each 8l. more than thoſe of the greater, and the number of pounds which each ſociety receives, exceeds the number of men of both ſocieties by 172; quere the number of men in each ſociety, and the number of pounds each received? *Ans.* 12 men in the greater, 8 in the leſſer; the firſt received 16l. a man; the ſecond 24l.

56. A grazier bought certain oxen for 270l. and found that if he had paid that ſum for 5 oxen fewer, every ox

would have coſt him 15s. more; quere the number of oxen? *Ans.* 45.

57. Two merchants ſell linen cloth in this manner, viz. each ſells 60 yards, and the firſt ſelling 2 yards leſs for 1l. than the ſecond, receives for his 60 yards 15l. more than the ſecond for his 60 yards; how many yards did each man ſell for a pound? *Ans.* 6 and 4.

58. A merchant buys linen cloth of two ſorts, viz. 90 yards of one ſort together, with 40 of a worſe ſort, for 42l. and he finds that in laying out 1l. upon each ſort, he hath $\frac{1}{3}$ of a yard more of the worſe ſort than of the better; what was the price of a yard of each ſort? *Ans.* beſt 6s. 8d, worſe 6s.

59. Two merchants entered into partnerſhip, the firſt brought in a certain ſum of pounds, which continued in company 12 months, and the ſecond put in 30l. for 17 months, they gained together 18l. 15s. whereof the firſt merchant had 26l. for his principal and gain; how many pounds did the firſt merchant bring into ſtock? *Ans.* 20l.

## DIOPHANTINE QUESTIONS.

1. *RULE.* For the root of the ſquare or cube required, put one or more letters, ſo that when they are involved, either the given number or the higheſt power of the unknown quantity, may vaniſh from the equations; and then, if the unknown quantity be but of one dimenſion, the problem will be ſolved by reducing the equation.

2. But if the unknown quantity be ſtill a ſquare or higher power, ſome other new letters muſt be aſſumed to denote the root; with which proceed as before; and ſo on till the unknown quantity is but of one dimenſion; and from this all the reſt will be determined.

### EXAMPLES.

1. Divide a given ſquare number (100) into two ſuch parts, that each of them may be a ſquare number.

*Solution.* Let $x^2=\square$ be one of the parts, then $100-x^2$ will be the other part, which is alſo to be a ſquare number. Aſſume the ſide of this ſecond ſquare $=2x-10$, then will $100-x^2=\overline{2x-10}|^2$, and that is $200-x=4x^2-40+100$, and by reduction $x=8$ and $2x-10=6$, therefore 64 and 36 are the parts required.

### *A General Solution.*

Let $a^2$ =given ſquare number, $x^2=\Box$ =one of its parts, and $a^2-x^2=$ the other, which is alſo to be a ſquare number, aſſume the ſide of this ſecond ſquare $=rx-a$, then will $a^2-x^2=\overline{rx-a}|^2=r^2x^2-2ar=x+a^2$, and by reduction $x=\frac{2ar}{r^2+1}$ and $rx-a=\frac{2ar^2}{r^2+1}-a=\frac{2ar^2}{r^2+1}-\frac{ar^2-a}{r^2+1}$ $=\frac{ar^2-a}{r^2+1}=$ to a ſquare number, therefore $\overline{\frac{2ar}{r^2+1}}\Big|^2$ and $\overline{\frac{ar-a}{r^2+1}}\Big|^2$ are the parts required, where $a$ and $r$ may be any numbers taken at pleaſure.

*Note.* If $s$ and $r$ be any two unequal numbers, of which $s$ is the greater, then will $2rs$, $s^2-r^2$ and $s^2+r^2$ be the perpendicular, baſe, and hypothenuſe of a right angled triangle.—And from this canon, two ſquare numbers may be found whoſe ſum or difference ſhall be ſquare numbers, and two other ſquare numbers can be found, whoſe difference ſhall be equal the given ſquare number.

2. To find two ſquare numbers, whoſe difference ſhall be equal to any given number (60).

*Ans.* 14 and 16, or $\overline{14}|^2=196$ and $\overline{16}|^2=256$ for the ſquares in numbers, and ſo on for any difference whatſoever.

3. To divide a given number, (13) which is compoſed of two known ſquare numbers (9 and 4) into two other ſquare numbers.

*Solution.* For the ſide of the firſt ſquare ſought, put $rx-3$, and for the ſide of the ſecond $sx-2$, $r$ being the greater number, and $s$ the leſs, then will $\overline{rx-3}|^2+\overline{sx-2}|^2=r^2x^2-6xr+9+s^2x^2-4sx+4=r^2+s^2x^2-6r-4sx+13=13=$ or $r^2+s^2x^2=6r+4sx$ and from this equation by reduction $x$ is found $=\frac{6r+4s}{r^2+s^2}$ Whence $rx-3=\frac{6r^2+4rs}{r^2+s^2}-3=\frac{3r^2+4rs-3s^2}{r^2+s^2}=$ ſide of the firſt ſquare ſought, and $sx-2=\frac{6rs+4s^2}{r^2+s^2}-2=\frac{6rs-2r^2+2s^2}{r^2+s^2}=$ ſide of the ſecond, ſo that if $r$ be taken $=2$ and $s=1$, we ſhall have $\frac{17}{5}$ and $\frac{6}{5}$ for the ſides required in numbers.

*Note.* If $a^2+b^2$ be put equal to the number to be divided, the general ſolution may be given exactly in the ſame manner.

This queſtion is conſidered by Diophantes as a very important one, being made the foundation of moſt of his other problems. In the ſolution of it given above, the value of $r$ and $s$ may be taken equal to any numbers whatever, provided the proportion of thoſe numbers be not the ſame as that of $3$ $(a)$ to $2$ $(b)$ or $3+2$ $(a+b)$ to $3-2$ $(a-b)$.

4. To find two ſquare numbers, whoſe difference ſhall be equal to any given number $(d)$.

*Solution.* Let $d$ be reſolved into any two unequal factors $a$ and $b$; $a$ being the greater and $b$ the leſs. Alſo put $x$ for the ſide of the leſs ſquare ſought, and $x+b=$ ſide of the greater. Then $\overline{x+b}|^2-x^2=x^2+2bx+b^2-x^2=2bx+b^2=d$ $(ab.)$ And if this be divided by $b$, we ſhall have $2x+b=a$. Whence $x=\frac{a-b}{2}=$ ſide of the leſs ſquare ſought. And $x+b=\frac{a-b}{2}+b=\frac{a+b}{2}=$ ſide of the greater. So that by putting $d=60$ and $a\times b=2\times 30$, we ſhall have $\frac{30-2}{2}=14$ and $\frac{30+2}{2}=16$, or $\overline{14}|^2=196$ and $\overline{16}|^2=256$ for the ſquares in numbers; and ſo for any difference of factors whatever.

5. To find two numbers, ſo that if either of them be added to the ſquare of the other, their ſum ſhall be a ſquare number.

*Solution.* Let the numbers ſought be $x$ and $y$. Then $x^2+y=\square$ and $y^2+x=\square$. And if $r-x$ be aſſumed for the ſide of the firſt ſquare $x^2+y$ we ſhall have $x^2+y=r^2-2rx+x^2$, or $y=r^2-2rx$. Whence $2rx=r^2-y$ or $x=\frac{r^2y}{2r}$. Again if $y+s$ be aſſumed for the ſide of the ſecond ſquare, we ſhall have $y^2+\frac{r^2-y}{2r}(y^2+x)=\overline{y+s}|^2=y^2+2sy+s^2$; whence $\frac{r^2-y}{2r}=2sy+s^2$ or $r^2-y=4rsy+2rs^2$. And conſequently $y=\frac{r^2-2rs}{4rs+1}$ and $x=\frac{r^2+y}{2r}=\frac{2r^2s+s^2}{4rs+1}$. So that $\frac{r^2-2rs^2}{4rs+1}$ and $\frac{2r^2s+s^2}{4rs+1}$ are the numbers required; where $r$ and $s$ may be taken at pleaſure, provided $r$ be greater than $2s^2$.

6. To find two numbers, whoſe ſum and difference ſhall be both ſquare numbers.

*Solution.* Let $x$ and $x^2-x$ be the two numbers, then ſince their ſum is evidently a ſquare number, one of the conditions of the queſtions will be anſwered, there remains therefore; only their difference $x^2-2x$ to be made a ſquare. And, if for the ſide of this ſquare there be put $x-r$, we ſhall have $x^2-2rx+r=x^2-2x$, or $2rx-2x=r^2$, whence $x=\frac{r^2}{2r-2}$ and $x^2-x,\ \overline{\frac{r}{2r-2}}\Big|^2=\frac{r^2}{2r-2}$. So that $\frac{r^2}{2r-2}$ and $\overline{\frac{r^2}{2r-2}}\Big|^2-\frac{r}{2r-2}$ are the numbers required, where $r$ may be taken at pleaſure, provided it be greater than 1.

7. To three ſuch numbers, that not only the ſum of all three of them, but alſo the ſum of every two ſhall be a ſquare number.

*Solution.* Let $4x$, $x^2-4x$, and $2x+1$ be the three numbers ſought; then $4x+x^2-4x=x^2$, $x^2-4x+2x+1=x^2-2x+1$ and $4x+x^2-4x+2x+1=x^2+2x+1$, are evidently ſquares. And therefore, three of the conditions mentioned in the queſtion, are accompliſhed.

Whence it remains only to make the quantity $4x+2x+1$ or $6x+1=$ to a ſquare. Let therefore $6x+1=a^2$; and we ſhall have $x=\frac{a^2-1}{6}$ and conſequently $\frac{4a^2-4}{6}$, $\overline{\frac{a^2-1}{6}}^2-\frac{4a^2-4}{6}$ and $\frac{2a^2-2}{6}+1$, or $\frac{2a^2-2}{3}$ $\frac{a^4-26a^2+25}{36}$ and $\frac{a^2+2}{3}$ are the numbers required, where $a$ may be taken at pleaſure, provided it be greater than 5.

Otherwiſe thus, becauſe by the conditions, the ſum of the three is to be a ſquare, and alſo the ſum of every two to be a ſquare, therefore the ſum of theſe three ſquares muſt be double to the firſt ſquare; and hence the queſtion is no more than this, to find three ſquares, whoſe ſum ſhall be double another ſquare. Now let $2\times\overline{a+x}^2=a^2+\overline{a-x}^2+\overline{b-x}^2$; and we have $x=\frac{b^2}{6a+2b}$ ſuppoſe $b=18$, $a=21$, then $x=2$. Hence $\overline{a+x}^2=\overline{23}^2=529$, $a^2=\overline{21}^2=441$, $\overline{a-x}^2=\overline{19}^2=361$, and $\overline{b-x}^2=\overline{16}^2=256$. Then from 529 deduct 441, 361, and 256 and the remainders 88, 168, and 273 are the three numbers required: moreover all their ſquare multiplies give as many other anſwers as you pleaſe.

8. To find three ſquare numbers ſuch, that the ſum of every two of them ſhall be a ſquare number.

*Solution.* Let $x^2$, $y^2$, and $z^2$ be the numbers ſought, then $x^2+z^2=\square$, $y^2+z^2=\square$, and $x^2+y^2=\square$, or $\frac{x^2}{z^2}+1=\square$ $\frac{y^2}{z^2}+1=\square$ and $\frac{x^2}{z^2}+\frac{y^2}{z^2}=\square$ and by putting $\frac{x}{z}=\frac{s^2-1}{2s}$, and $\frac{y}{z}=\frac{r^2-1}{2r}$, we ſhall have $\frac{x^2}{z^2}+1=\frac{s^4+2s^2+1}{4s^2}$ and $\frac{y^2}{z^2}+1=\frac{r^4+2r+1}{4r^2}$ which are both evidently ſquares, and therefore is remains only to make

$x^2$

$\frac{x^2}{z^2}+\frac{y^2}{z^2}=$ a ſquare number.

But $\frac{x^2}{z^2}+\frac{y^2}{z^2}=\overline{\frac{s^2-1}{2s}}^2+\overline{\frac{r^2-1}{2r}}^2=\overline{\frac{s^2-1}{4s^2}}^2+\frac{r^2-1}{4r^2}$

$=\frac{4r^2\times\overline{s^2-1}^2+4s^2\times\overline{r^2-1}^2}{4r^2s^2}=$ ſquare number.

Or $s^2\times\overline{s^2-1}^2+s^2\times\overline{r^2-1}^2=r^2\times\overline{s+1}^2\times\overline{s-1}^2\ s^2\times\overline{r+1}^2\times\overline{r-1}^2=$ſquare number. And by making $\overline{r-1}=\overline{s+1}$ or $r=s+2$ we ſhall have $\overline{s+2}\times\overline{s+1}^2\times\overline{s-1}^2+s^2\times\overline{s+3}^2\times\overline{s+1}^2=$ to a ſquare number.

Or $\overline{s+2}^2\times\overline{s-1}^2+s^2\times\overline{s+3}^2=2s^4+8s^3+6s^2-4s+4=$ to a ſquare number; now let the root of this equation be aſſumed $=\frac{5}{4}-s+2$, then $2s^4+8s^3+6s^2-4s+4=\overline{\frac{5}{4}s^2-s+2}^2=\frac{25}{16}s^4-\frac{5}{2}s^3+5s^2+s^2-4s+4$, or $2s^4+8s^3=\frac{25}{16}s^4-\frac{5}{2}s^3$; or $2s+8=\frac{25}{16}s-\frac{5}{2}$. Whence $s=-24$, and $r=-22$, and $x=\frac{s^2\ 1}{2s}=\frac{575}{48}$; and $\frac{y}{z}=\frac{r^2-1}{2r}=\frac{483}{44}$ or $x=\frac{575z}{48}$ and $y=\frac{483z}{44}$.

In order, therefore to have the anſwer in whole numbers, let $z=528$ and we ſhall have $x=6325$ and $y=5796$ or 528, 5796 and 6325 for the roots of the ſquares required.

*Note.* This queſtion is capable of a great variety of anſwers; but the leaſt roots which have yet been found, in whole numbers are 44, 117 and 240. See Elements de Algebre, par M. Euler, tome 2, page 327, which is particularly calculated for the uſe of thoſe who wiſh to obtain a knowledge of Algebra without the aſſiſtance of a maſter.

9. To find a number $x$ ſuch that $x+1$ and $x-1$ ſhall be both ſquare numbers. *Anſ.* $x=\frac{5}{4}$ or $\frac{85}{36}$ &c.

10. To find a number $x$ ſuch, that $x+128$, and $x+192$ ſhall be both ſquares. *Anſ.* $x=97$.

11. To find a number $x$ ſuch that $x^2+x$ and $x^2-x$ both may be ſquares. *Anſ.* $\frac{25}{24}$.

12. To find two numbers $x$ and $y$ ſuch, that $x+y$, $x^2+y$ and $y^2+x$ may be all ſquares. *Ans.* $x=\frac{1}{6}$ and $y=\frac{1}{12}$.

13. To find three ſquare numbers in Arithmetical progreſſion. *Anſ.* 1, 25 and 49.

14. To find three ſquare numbers in harmonical proportion. *Ans.* 1225, 49 and 25.

15. To find three numbers in Arithmetical progreſſion ſuch, that the ſum of every two of them may be a ſquare number. *Ans.* 120½, 840½, and 1560½.

16. To find three numbers, ſuch that if to the ſquare of every one of them the ſum of the other two be added, the three ſums ſhall be all ſquares. *Ans.* $\frac{8}{3}$, $\frac{16}{3}$ and 1.

17. To find two numbers in proportion as 8 to 15, and ſuch that the ſum of their ſquares ſhall make a ſquare.

*Ans.* 576 and 1080.

18. To find 4 numbers ſuch, that if a ſquare number (100) be added to the product of every two of them, the ſums ſhall be ſquares. *Ans.* 12, 32, 88, and 168.

19. To find two numbers ſuch, that their difference may be equal to the difference of their ſquares, and that the ſum of their ſquares ſhall be a ſquare number. *Ans.* $\frac{4}{7}$ and $\frac{3}{7}$.

20. To find three numbers in Geometrical proportion ſuch, that every one of them being increaſed by a given number (19) ſhall make ſquare numbers.

*Ans.* 81, $\frac{5}{4}$ and $\frac{25}{1296}$.

21. To divide a given number (10) into four ſuch parts, that the ſum of every three of them may be a ſquare number. *Ans.* 1, 6, $\frac{186}{289}$ and $\frac{681}{289}$.

22. *To find three ſquare numbers ſuch, that their ſum being ſeverally added to their three ſides, ſhall make ſquare numbers.

*Ans.* $\frac{44418}{62920}$, $\frac{13254}{62920}$, and $\frac{19881}{62520}$, roots required.

23. To find two numbers ſuch, that their ſum being increaſed and leſſened either by their difference or the difference of their ſquares, the ſums and remainders ſhall be all ſquares. *Ans.* $\frac{49}{50}$ and $\frac{1}{50}$.

24. To find two numbers ſuch, that not only each number, but alſo their ſums and difference being increaſed by unity, ſhall be all ſquare numbers.

*Ans.* 3024. and 5624.

25. To

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* Probably the anſwers to many of theſe queſtions cannot be given in whole numbers.

24. To find three numbers that whether their ſum be added to or ſubtracted from the ſquare of each particular number, the numbers thence ariſing ſhall be all ſquares.

*Ans.* $\frac{406}{96}$, $\frac{518}{96}$, and $\frac{721}{96}$.

25. To find three ſquare numbers ſuch, that the ſum of their ſquares ſhall alſo be a ſquare number.

*Ans.* 9, 16, and $\frac{144}{25}$.

26. To find three ſquare numbers ſuch, that the difference of every two of them ſhall be a ſquare number.

*Ans.* 485809, 34225, and 23409.

27. To divide any given cube number (8) into three other cube numbers. *Ans.* $\frac{64}{27}$, $\frac{125}{27}$, and 1.

28. Two cube numbers (8 and 1) being given, to find two other cube numbers, whoſe difference ſhall be equal to the ſum of the given cubes. *Ans.* $\frac{8000}{343}$ and $\frac{4913}{343}$.

29. To divide a given number (28) compoſed of two cube numbers (27 and 1) into two other cube numbers.

*Anſ.* $\frac{63284705}{11446878}$ and $\frac{28340511}{11446828}$ the roots.

30. To find three cube numbers ſuch, that, if from every one of them a given number (1) be ſubtracted, the ſum of the remainders ſhall be a ſquare. *Anſ.* $\frac{4913}{3375}$, $\frac{21952}{3375}$ and 8.

31. To find three numbers, ſuch, that if they be ſeverally added to the cube of their ſum, the three ſums thence ariſing, ſhall be all cubes. *Anſ.* $\frac{1538}{137464}$, $\frac{18377}{137464}$ and $\frac{23625}{137464}$.

32. To find three numbers in Arithmetical proportion, ſuch, that the ſum of their cubes ſhall be cubes.

*Ans.* 3, 4, 5, or 149, 256, 263.

33. To find three cube numbers, ſuch, that their ſum ſhall be a cube number.

*Anſ.* $3^3$, $4^3$, and $5^3$, or $21^3$, $19^3$, $18^3$, &c.

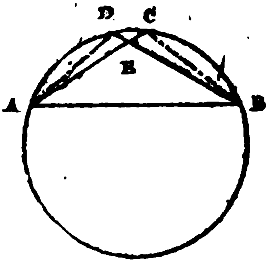
34. To find two numbers, ſuch, that their ſum ſhall be equal to the ſum of their cubes. *Anſ.* $\frac{5}{7}$ and $\frac{8}{7}$.

## GEOMETRICAL QUESTIONS.

1. HAVING given the ſeveral ſides of two angles, ſtanding upon the equal ſegment of a circle, it is required to find the ſegments of the croſſing lines?

$DA=b=23$, $BC=c=34$, $AC=d=37$, $BD=f=47$, $BE=x$, $DE=(BD-DE)=f-x$.

*Solution.*



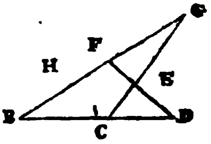
The $\triangleleft$ . AED is ſimilar to the $\triangleright$ AEC by 15, *Euc.* 1, and 21 *E.* 3, by 4 *E.* 6 AD : DE : : BC : CE, *i. e.* $b : \overline{f-x} :: C : \mathrm{CE} = \overline{f-x} \times \frac{c}{b}$ and AE is $= (\mathrm{AC} - \mathrm{CE} = d - \frac{fc - xc}{b}) = \frac{bd - fc + xc}{b}$

Again, By 4 *Euc.* 6. AE : EB : : DE : E that is $\frac{bd - cf + cx}{b} : x :: \left(f - x \times \frac{c}{b}\right) :: 1 : \frac{c}{b}$ therefore $bcd - c^2 f + c^2 x = b^2 x$ and $x = \frac{cf^2 - bcd}{c^2 - b^2} = 40.49 = \mathrm{BE}$

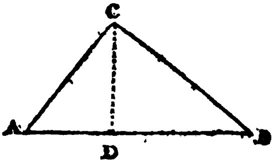
2. Having the ſides of the triangle BDF, and alſo the ſegments BC, FE, given, it is required to find the point G in the ſide BF continued, ſo that GFC ſhall be in a ſtraight line.

FB $= a = 12$, FD $= b = 8$, BD $= c = 16$, FE $= r = 3$, BC $= s = 9$, DE $= (\mathrm{FD} - \mathrm{FE}) = b - r$, CD $= (\mathrm{BD} - \mathrm{BC}$ and put EG $= x$.

*Sol.* Draw EH parallel to BD, then becauſe the $\triangle$ FBD is ſimilar to $\triangleright$ FEH · FD : FB : : FE : FH and alſo FD : DB : : FE : EH *i. e.* $b : a :: r : \mathrm{FH} = \frac{ra}{b}$ and $b : c :: r : \mathrm{EH} = \frac{rc}{b}$ and becauſe $\triangle$ GEH is ſimilar to $\triangleleft$ GBC. GF+FH : HE : : GF+FB : BC *i. e.* $x + \frac{ra}{b} : \frac{rc}{b} :: x + a : s$ therefore $sx + \frac{sra}{b} = \frac{rcx}{b} + \frac{rca}{b}$ conſequently $x = \frac{c - s \times ra}{sb - rc} = 10.5 = \mathrm{FG}$.



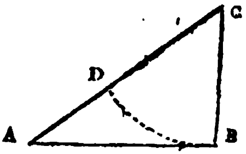
3. The ſides of any oblique triangle being given, it is required to find the ſegments of the baſe made by a perpendicular let fall from the oppoſite angle to the baſe.



Let AC$=b=36$, BC$=c=72$, AB$=d=84$, AD$=x$, then DB $=d-x$

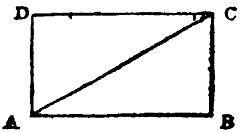
*Anſ.* $x=18.85=$AD.

4. In a right-angled triangle, there is given the baſe and the exceſs of the hypothenuſe above the perpendicular, to find the ſides.



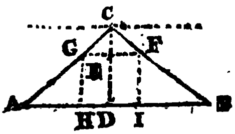
Let AB$=b=96$, AD$=c=48$, BC $=x=$ CD, AC$=$(AD+CD) $=c+x$, then it will follow that $x=\frac{b^2-c^2}{2c}=72=$BC and AD$+x$ $=120=$AC.

5. Two rectangular triangles having one common hypothenuſe, there is given one leg in each triangle and the ſum of the other two legs, to find the ſides ſeverally.



Let BC$=b=72$, AD$=c=36$, AB $+$ DC $=210.47$, AB $=x$, then by a due proceſs $x=\frac{c^2=d^2-b^2}{2d}=$AB$=96$ DC$=d-x$ $=114.47$.

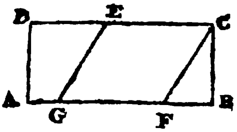
6. In any given triangle, to inſcribe a ſquare.



Let BC$=27=c$, BA$=37=b$, CD$=15=d$, CE$=y$, then BF $=$(BC—CF)$=c-y$ and by proportionality, equality, and proper reductions $y=\frac{cd}{b+d}=7.78=$CF.

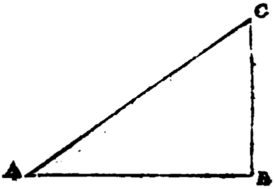
7. In a given rectangular parallelogram to inſcribe a rhombus.

Let AB$=b=41$, BC$=c=15$, FB$=x=$DE, AF$=$(AB—FB)$=b-x=$FC, then $x=\frac{b^2-c^2}{2b}=17.75=$FB$=$DE.



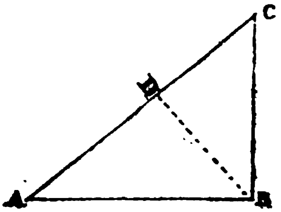
8. The Perimeter and area of a rectangular triangle being given to find the ſides.

Let AB$=$BC$=$AC$=p=288$, the area$=a=3456$, AC$=x$, AB—BC$=y$ AB$=$BC$=p-x$, AB$=\frac{1}{2}p+\frac{1}{2}y-\frac{1}{2}x$, BC$=\frac{1}{2}p-\frac{1}{2}y-\frac{1}{2}x$, and by 47. *E*. 1. and equality, $x=\frac{p^2-4a}{2p}=120=$AC, hence the reſt are found.



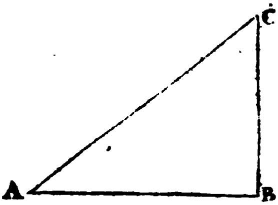
9. The perimeter and perpendicular let fall from the angle to the hypothenuſe of a right-angled triangle being given to find the ſides.

Let AB$=$BC$=$AC$=p$ 288, BD$=$C$=57.6$, AC$=x$, AB$+$BC$=p-x$, AB—BC$=y$ AB$=\frac{1}{2}p+\frac{1}{2}y-\frac{1}{2}x$, BC$=\frac{1}{2}p-\frac{1}{2}y-\frac{1}{2}x$, then p. 4. *Euc*. 6. proper reduction and equality, $x=\frac{p^2}{2p+2c}=120=$AC



10. The ſum of the hypothenuſe and perpendicular, and the difference of the legs of a rectangular triangle, given to find the ſides ſeverally.

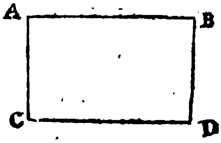
Let AB$+$BC$=b=192$, AB—BC$=c=24$, AC$=x$ ∴ AB$=x-c$, and BC$=b-x$, then per 47. *Euc*. 1, reduction, &c. $x=b+c-\sqrt{2bc}=120=$AC.



11. Having the ſum of the hypothenuſe and one leg, and the difference of the hypothenuſe and the other leg of a right-angled triangle given, to find the other ſides.

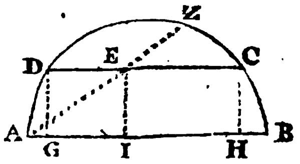
Let AC+BC=$b$=192, (as per laſt figure) AC—AB=$c$ =24, AC=$x$, ∴ AB=$x$—$c$, and BC=$b$—$x$. Then by 47 *Eu.* 1. properties of Quadratics, $x=b+c-\sqrt{2bc}=120$=AC.

12 To find the ſides of a rectangular parallelogram, that ſhall have the ſum of the ſquares of its ſides, and the ſum of its ſides, in a given ratio; and whoſe ſides ſhall be in a given ratio.



Put AB=$x$, AC=$z$, $b=3$, $c=2$, $r=156$, $s=5$: then by hypotheſis, &c. $z=\frac{b+c\times rc}{b^2+c^2\times s}=\frac{rbc+rc^2}{b^2s+c^2s}=24$=AC, and $x=\frac{b^2r+4brc}{b^2s+c^2s}=36$=AB. *Q.E.F.*

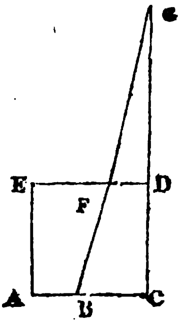
13. Given the diameter of a circle, AB, and a chord line, CD, it is required to draw another chord from A, the extremity of the diameter, which ſhall interſect the former, and make the ſegment AE, equal to the ſegment EC.



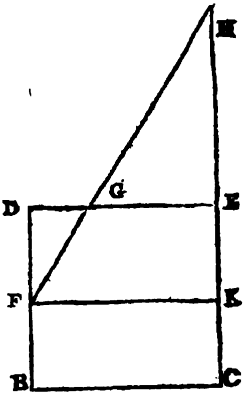
Put AB=$d$, DC=$c$, AE=EC=$x$; ∴ DE=(DC—CE=) $c-x$=GI, AG=$\frac{d-c}{2}=\frac{1}{2}d-\frac{1}{2}c$ AI=(AG+GI=)$\frac{1}{2}d+\frac{1}{2}c-x$, and by proportion, 35 *Eucl* 3. IE²=(DG²=) AG×GB=$\frac{1}{4}d^2-\frac{1}{4}c^2$; then per 47 *Eucl.* 1. &c. $x=\frac{\frac{1}{2}d+\frac{1}{2}dc}{d+c}=\frac{1}{2}d$.

14. From the middle of the ſide of a given ſquare it is required to draw a ſtraight line which ſhall cut the ſide of the ſquare produced, and make the external △ FDG=the given ſquare.

Put AC=$b$=36, BC=$\frac{1}{2}b=\frac{1}{2}$DC,DG =$x$; ∴ GC=(GD+DC=)$x+b$, then per 4 *Eucl.* 6, completing the ſquare &c. $x=2b+2b\sqrt{2}=173.808$.

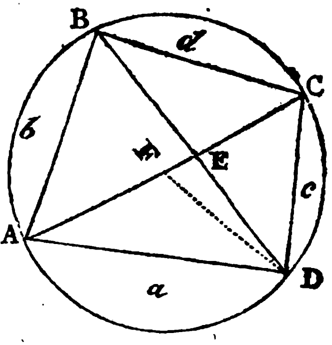


15. From F the middle of the ſide of a given ſquare, it is required to draw a ſtraight line which ſhall cut the oppoſite ſide produced, and make the external △ GEH equal to the given ſquare.



Put CB=FK=$b$=36, HE=$x$, ∴ DF=EK=$\frac{1}{2}$, HK=(H+EK=) $x+\frac{1}{2}b$ ;per 4 *Eucl.* 6, HK : KF :: HE : EG, *i.e.* $x+\frac{1}{2}b : b :: x :$ EG $=\frac{bx}{x+\frac{1}{2}b}$, and, by the queſtion, $\frac{1}{2}\mathrm{HE}+\mathrm{GE}=\overline{\mathrm{EC}}^2$ or $\frac{\frac{1}{2}bx^2}{x+\frac{1}{2}b}=b^2$, whence $x=b+b\sqrt{2}=86.911$:

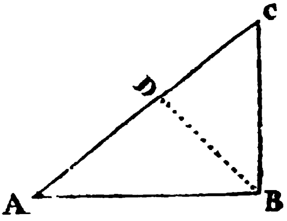
16. The ſides of a quadrilateral in a circle being given to find its diagonal.



Put AB=$a$, BC=CD=$b$, AD=$c$, AE=$x$, then it will follow from the properties of the 15 *Euc.* 1, 21 *Euc.* 3, 4 *Euc.* 6, the Ptolomaic theorem, and quadratics, that

$$x=\sqrt{\frac{a^2c^2b+a^2c\,d}{abc^2+a^2cd+b^2cd+abd^2}}$$

17. If a perpendicular be let fall from the right angle to the hypothenuſe of a right angled triangle, and there being given one leg and the alternate ſegments of the baſe, it is required to find the ſides.

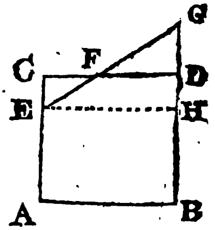


Put AD=$b$=76.8, B=$c$=72, AC=$x$,DC=(AC—AD=)$x-b$, then by the 8 and 4 *Euc.* 6 and quadratics, $x=\frac{b+\sqrt{4c^2+b^2}}{2}=$ 120: hence the reſt are found.

18. In a rectangular triangle there is given the perpendicular let fall from the right angle to the hypothenuſe, and the difference of the ſegments at the baſe made by that perpendicular, to find the ſides.

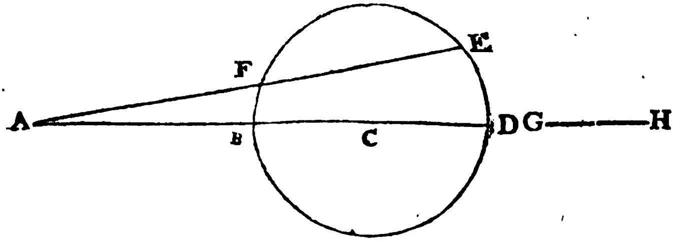
Put $BD=b=57.6$, $AD-DC=c=33.6$, $AC=x$ ∴ $DC=\frac{1}{2}x-\frac{1}{2}c$; then per 8 and 4 *Euc.* 6, completing the ſquare, &c. $x=\sqrt{4b^2+c^2}=120=AC$, from which the other two ſides are found.

19. From a given point in the ſide of a ſquare to draw a right line which ſhall cut the oppoſite ſide of the ſquare produced, and divide the ſide of the ſquare given into a mean and extreme proportion.



Put $CA=b=50$, $CE=c=20=DH$, $DG=x$; ∴ $GH=(GD+DH=)x+c$: then by ſimilar triangles, condition of the queſtion, quadratics, &c. $x=\frac{1}{2}c+\frac{\sqrt{5c^2}}{4}=\frac{1}{2}c+c\sqrt{\frac{5}{4}}=37.326=DG$.

20. From a given point in the plane of a circle to draw a right line ſo that the intercepted chord ſhall be equal to a given right line.



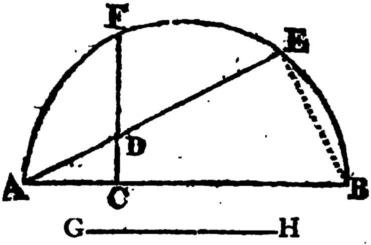
Put $AB=b=28$, $EF=d=GH=34$, $DA=c=70$, $AF=x$, $AE=(AF+FE=)x+d$; then per 36 *Euc.* 3, and completing the ſquare, $x=\frac{-d+\sqrt{4bc+\frac{1}{4}d^2}}{2}=62.53$.

21. Having the diameter of a circle and a verſed ſine of an arc given it is required to draw a chord from the end of the diameter, which ſhall cut the right ſine of the ſame arc, and have its ſegment intercepted between the right ſine and

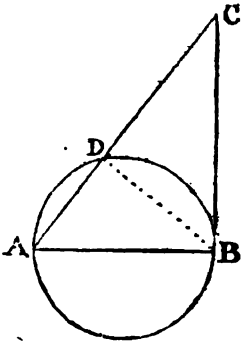
the arc equal to a given right line

Put AB$=b=43$, AC$=d=11$, DE$=$GH$=c=24$, AD$=x$, $\therefore$ AE$=$(AD$+$DE$=$)$x+c$; then by ſimilar triangles, quadratics, &c.

$$x=\frac{c+\sqrt{c^2+4bd}}{2}=12.84.$$



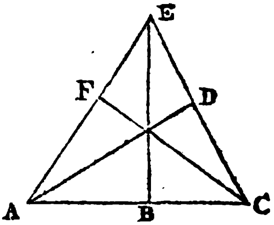
21. From the end of the diameter of a circle, whoſe area and diameter are given, to draw a ſecant meeting a tangent drawn from the other end of the diameter, with this condition, that the rectangle of the ſegments of the ſecant made by the periphery of the circle ſhall be equal to the area of the circle.



Put AB$=d=160$, area of the circle $20106=a$, and BC$=x$. Solution, by 31 *Euc.* 3, the 4& 8 *Euc.* 6, and proceeding as uſual, we have

$$x=\frac{\sqrt{d^2a}}{d^2-a}=306.083=\text{BC}.$$

23. Having the perpendiculars let fall from each angle of a triangle to the oppoſite ſides given to find the oppoſite ſides.

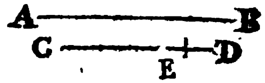


Put AD$=b=21$, CF$=c=20$, EB$=d=19$, AC$=x$, AB$=z$: then by ſimilar triangles, 47 *Euc.* 1, equating the different proceſſes, completing the ſquare, &c. we get

$$x=\sqrt{\frac{4b^4c^2d}{2b^4c^2d^2+2b^2c^2d^4+2b^2c^4d^2-2b^4d^4-c^4d^4}}=24.34.$$

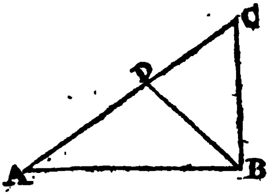
24. Having two right lines given it is required to divide one of them ſo that the rectangle of the undivided line into one of the parts of the divided line ſhall be to the ſquare of the other part of the divided line in a given ratio.

Put



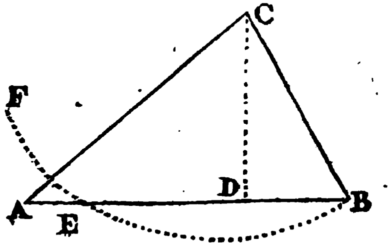
Put $AB=24=b$, $CD=c=16$, $CE=x$; ∴ $ED=c-x$: then proceed according to the queſtion, $x=\frac{2cr\mp bs\pm\sqrt{b^2s^2+4bcrs}}{2r}=10$.

25. In a right angled triangle there is given the difference of the legs, and the perpendicular let fall from the right angle to the hypothenuſe, to find the ſides.



Put $BD=b=57.6$, $AB-BC=c=24$, $AB=x$, $AC=z$; ∴ $BC=x-c$, then per 8 and 4 *Euc.* 6, and 47 *Euc.* 1, ordering the equation, &c. $z=b+\sqrt{b^2+c^2}=120=AE$, $AB=96=x$, and $BC=72$.

26. In any oblique triangle there is given the perpendicular let fall from the oppoſite angle to the baſe, the difference of the ſegments made of the baſe by that perpendicular, and the difference of the other two ſides, to find the ſides ſeverally.



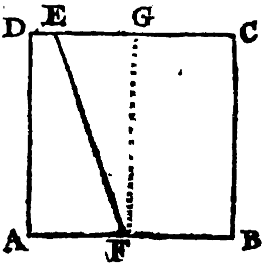
Put $AF=b=2$, $AE=c=3.4$, $CD=d=15.93$, $BC=x$, $BD=z$, ∴ $AC=b+x$, $AD=cz$: then by 47 *E.* 1, paying due regard to the equation, putting the values of $z=$ each other, proper reduction, completing the □, &c. $x=$ $\frac{1}{2}b+\sqrt{\frac{\overline{c^2+4d^2-b^2}\times c^2}{\overline{c^2-b^2}\times 4}}=16=BC$.

27. In any oblique triangle there is given the difference of the ſegments of the baſe made by a perpendicular let fall from the oppoſite angle to the baſe, the difference of the ſides, and the difference between the leſs ſide and the perpendicular, to find the ſides.

Put $AE=c=3.4$, $AF=b=2$, $BC-DC=2.322$, $BC=x$, $BD=z$: ∴ $AC=b+x$, and $AD=c+z$. By the 47 *Euc.* 1, the nature of equality, quadratics, &c. $x=$ $\frac{bc^2+2c^2d-b^3\pm\sqrt{\overline{4c^4d^2-4b^2cd}\times\overline{c^2-b^2}}}{2b^2}=16=BC$.

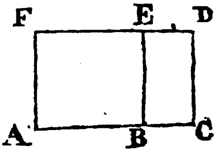
29. If

28. If a right line be drawn any how within a ſquare to cut the oppoſite ſides, and there be given the length of that line, and the alternate ſegments of the oppoſite ſides, it is required to find the ſide of the ſquare.



Put DE$=b$, EF$=c$, FB$=d$, AB $=$DC, ∴ CE$=$(DC$-$ED$=$)$x-b$, EG$=$(EC$-$FB$=$)$x-d-b$, and by putting $b+d=f=16$, EG $=x-f$, then per. 47 *Euc.* 1, ordering the equation, &c. $x=f+\sqrt{2c^2-f^2}=$ 19.83$=$AD.

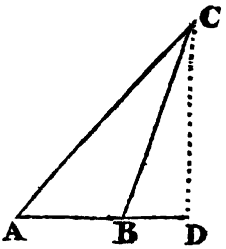
29. There are two rectangular parallelograms of the ſame altitude, and there is given the ſum of their baſes, the area of one of them, and the ratio of the ſides of the other; to find the ſides ſeverally.



Put AC$=$area of AE, EB : BC :: $r : s$ : 9 : 4, BE$=x$, AB, ∴ BC$=\frac{sx}{r}$, AC$=$(AB$+$BC$=$)$z+\frac{sx}{r}=b$, then $x=$

$$\frac{br\pm\sqrt{-4ars+b^2r^2}}{2s}=3.75=\text{BE}.$$

30. In any oblique triangle there is given the perpendicular height, the baſe, and the ſum of the other two ſides, to find the triangle.



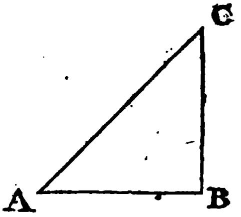
Put CD$=b=16$, AB$=c=12$, AC$+$BC$=d=38$, AC$=x$, BD$=z$, ∴ BC$=d-x$, AD$=c+z$, then per 12 & 13 *Euc.* 2, completing the ſquare, &c. &c. $x=$

$$\left(\tfrac{1}{2}d\pm\sqrt{\frac{2d^2c^2-4b^2c^2-d^4-c^4+\frac{1}{4}d^2}{4d^2-4c^2}}\right.$$

$$=\tfrac{1}{2}d\pm\sqrt{\frac{c^2d^2-4b^2c^2-c^4}{4d^2-4c^2}}\Bigg)=\tfrac{1}{2}d\pm\sqrt{\frac{\overline{d^2-4b}\times c^2}{\overline{d^2-c^2}\times 4}}$$

$=$21.764$=$AC, hence the reſt are found.

31. In



31. In a rectangular triangle there is given the hypothenuſe and the ſum of the legs, to find the triangle.

Put $AC=b=120$, $AB+BC=c=168$, $AB=x$ ∴ $BC=x-c$, hence per 47. *E.* 1. completing the ſquare, &c. $x=\frac{c\pm\sqrt{2b^2-c^2}}{2}=96=AB$.

32. In a right-angled triangle there is given the hypothenuſe and the ſum of the legs, to find the triangle.

Put $AC=b=120$ $AB-BC=c=24$, $AB=x$, $BC=x-c$ hence $x=c+\frac{\sqrt{2b^2-c^2}}{2}=96$

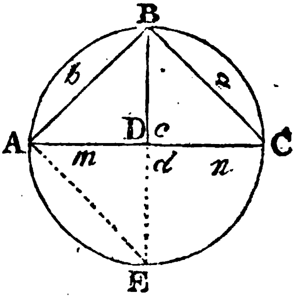
33. In any oblique triangle having the right-line biſecting one of the angles given, and the ſegment of the oppoſite ſide made by that line given; to find the triangle.

Put $AB=b=54$, $BD=c=40$, $BC=d=36$, $AD=x$, $CD=z$. By 3 *Euc.* 6. AB : BC : : AD : CD, or $b : d :: x : z$ ∴ $bz=dx$, and by the Newton. theo. $AD\times DC=AB\times BC+BDq$ or $xz=bd+c^2$ ∴ $\frac{bd+c^2}{z}=x$ & $x=\frac{bz}{d}$ : $\frac{bz}{d}=\frac{bd+c^2}{z}$

Therefore $bz^2=bd^2+c^2d$ hence $z=\sqrt{\frac{bd^2+c^2d}{b}}=48.6$

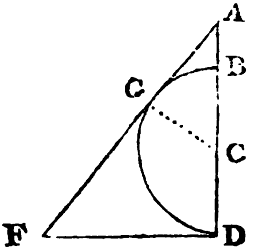
## *The Newtonian Theorem.*

34. In any triangle, if one of the angles be biſected by a right line, (as if the angle ABC be biſected by the right-line BD) then $AB\times BC-BDq=AD\times DC$.



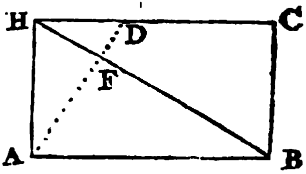
*Demonſtration.* Circumſcribe the triangle with a circle, and continue BD to the periphery in E; draw EA, then the triangle ABE is ſimilar to the triangle DBC ∴ $b : c+d :: c : a$ & $ba=c^2+cd=$(by 35 *E.* 3) $c^2+mn$, *i. e.* $\overline{AB\times BC}-BDq=AD\times DC$. *Q. E. D.*

35. In a right-angled triangle there is given one of the legs and the radius of a ſemicircle inſcribed within it, ſo as to touch the hypothenuſe and the other leg, quere the ſides of the triangle?



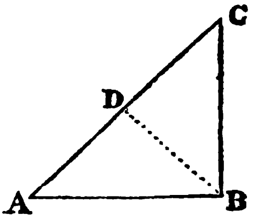
Put BC=CG=CD=$b$=24, AD =C= 72, DF=$x$ ∴ A=(AD —BD=)$c-2b$, then per 4 *Euc.* 6. 37 *Euc.* 3 and proper reduction, $x=\sqrt{\frac{b^2c^2}{c^2-2bc}}$=41.56=DF.

36. In a rectangular parallelogram (AC) there is given the length of AD drawn perpendicular to the diagonal, and the ſegment, CD, to find the ſides of the parallelogram.



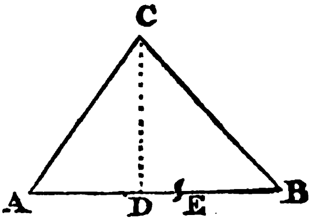
Put AD=$b$=23, CD =$c$=44, FD=$x$, FA=$z$, E=(E+CD=)$x+c$, hence per 47 *Euc.* 1. 4 *Euc.* 6. and proper reduc. &c. $x=\frac{c+\sqrt{c^2+8b^2}}{4}$=8.63=ED, $z=\sqrt{b^2-x^2}$ =21.=8AE, as was required.

37. In any rectangular triangle, there is given the perpendicular, let fall from the right angle, and the ſum of the legs, to find the ſides.



Put BD=$b$=57.6 AB+BC=$c$ =168, AC=$z$ ∴ AB=$\frac{1}{2}c+\frac{1}{2}x$, and BC=$\frac{1}{2}c-\frac{1}{2}x$; then by equality, &c. $z=-b+\sqrt{c^2+b^2}$=120 =AC.

38. In any oblique triangle there is given the perpendicular let fall from oppoſite angle to the baſe, the difference of the ſegment, (made by that perpendicular) and the ratio of the other two ſides, to find the ſides.

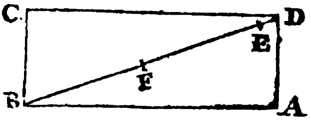


Put AE=$b$=22.5, BD =$c$=20, AB : CD :: $r$ : $s$ 5 : 3, AD=$x$, BC=BE=$z$, DC=$\frac{sx}{r}$ and putting $\frac{s}{r}=\frac{3}{5}$ =$p$,

$=p$, $DC=px$, $AB=(AE+EB=b+z$, then per 47 *Euc.* 1. completing the ſquare, &c. &c.

$$x=\sqrt{\frac{\overline{1+p^2}\times b^2\pm 2b\sqrt{\overline{b^2+2c^2-b^2c}\times\overline{p^2-c^2}}}{1+p^4-2p^2}}=46.$$

39. In a rectangle, ABCD is given the difference between the length AB, and the diagonal BD, that is $DE=2$; likewiſe the difference between the breadth, AD, and the diagonal BD, that is $FD=9$, required the ſides of the rectangle AD, AB?

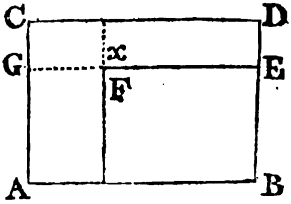


Put $DF=9=a$, $DF=2=b$, $x=AD$, then, per 47 *Euc.* 1. complete the ſquare, reduction, &c. $x=\sqrt{2ab}+b=8=AD$, breadth; then $9+8=17$, the length.

40. Suppoſe the plate of a looking-glaſs is 18 inches by 12, and is to be framed with a frame of equal width, and whoſe area is to be equal to that of the glaſs, the width of the frame is required? *Anſ.* 3.

Put $x=$ the breadth ſought, then $18+2x=$ length, $12+2x=$ breadth, hence $4x^2+60x+216=18\times 12\times 2=432$, ordered, the ſquare completed, &c. $x=3$.

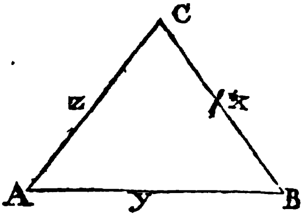
41. A gentleman a garden had,
Five ſcore feet long and four ſcore broad;
A walk of equal breadth, half round,
He made that took up half the ground;
The figure in the margin ſee,
How wide's the walk, I pray tell me?



Put $A=100=a$, $BC=80=b$, $AF=CD$, the breadth of the walk $=x$, then $FG=a-x$, then per the properties of the figure and queſtion, proper ſubſtitution being made, the ſquare completed, &c. we find $x=d+\sqrt{d^2}\frac{ab}{2}=25.969$, feet, the breadth required.

42. Ye young artiſts be ſo kind,
The fencing of a field to find;
Triangular is the form of ground,
Its area twice twelve chains was found;
One at the baſe I knew,
Juſt thirty-ſix and fifty-two;
And cub'd when every ſide
ſhall be,
The ſum you in the margin
ſee.*

* 1728 chains.



Here put S and C for the natural ſine and coſine of the angle ABC, radius $=1$, $a$ for the area of the triangle, $m$ for the ſum of the cubes of the ſides, and $x$, $y$, and $z$, for the ſides as in the figure. Then we ſhall have $sxy=2a$ and $x^2+y^2=z^2+2cxy$ (per Trigonometry), hence $xy=\frac{2a}{8}$ call $p$, and $x^2+y^2=2cp$, for $2cp$ put $b$, then $x^2+y^2=z^2+b$, from the condition of the queſtion and proper reduction, &c. $x$ will be found $=10$, $y=8$, and $z=6$, the ſides required.

43. A pack of cards being laid into any number of heaps, ſo that the ſpots on the bottom card of each heap, added to the number of cards laid thereon, may make 12, by giving the number of heaps, and of the cards left out, to find the number of ſpots on all bottom cards.

Queſtions of this nature are ſolved by the following theo.: Suppoſe $n=$ the number of heaps, $r=$ the number of cards left, $w=$ the whole pack, $x=$ the number of ſpots required, then the theo. will be $13n-w+r=x$, the number of ſpots required.

44. Suppoſe a pack of cards be dealt into ſeven heaps, and there are 12 cards left out, quere the number of ſpots on all the bottom cards?

*Ans.* The number will be found by the above *Theo.* $=51$.

45. One morning in May I went to ſurvey,
As ſoon as bright Sol I eſpy'd;
I meaſured round a four-corner'd ground,
The margin's* the length of each ſide:
The angle at B, together with D,
An hundred and fifty degrees;
The meadow's content is all that I want,
Aſſiſt me, kind youths, if you pleaſe.

* A $B=15.60$
B $C=13.20$
C $D=10.00$
D $A=26.00$

*Anſ.* 21.504 acres.

*Concerning*

## CONCERNING DIVISORS.

IT being often neceſſary, in Arithmetical calculations, to find ſuch multipliers or numbers which may be divided by any number of given diviſors, without any remainders; by which means, many pleaſant queſtions, not reduceable to any other rule in common Arithmetic may be ſolved.

To find the leaſt number that can be divided by any number of diviſors without a remainder.

*Rule.* Multiply all the prime numbers, and the root of ſuch as are ſquare or cube numbers, continually; the product will be the number required.

*Note.* A prime is ſuch a number as cannot be produced by the multiplication of two or more integers.

### EXAMPLES.

1. Required the three leaſt numbers, which, divided by 20, ſhall leave 19 for a remainder; but if divided by 19 ſhall leave 18; if divided by 18, ſhall leave 17; and ſo on, always leaving one leſs than the diviſor, to unity.

By Algebra, thus: Suppoſe $x$=the leaſt number poſſible, and let $a$, $b$, $c$, $d$, &c. repreſent the quotients reſpectively produced, by dividing $x$ by 2, 3, 4, 5, &c. then will the remainders be 1, 2, 3, 4, &c. hence $x=2a+1=3b+2=4c+3=5d+4$; &c. and to $2aA+19$. Now $a=\frac{b+1}{2}$ hence $\frac{b+1}{2}=m$, a whole number $\therefore$ $b=2m-1$, and $a=3m-1$:

Again, $c=m-1+\frac{m}{2}$ Hence $\frac{m}{2}=n$, a whole number. And therefore $m=2n$, $c=3n-1$, and proceeding in this manner we get $A=11639628$, $B-1$; hence the number required $x=232792560\,b-1$ where B repreſents any whole poſitive number; and when $B=1$, then the number required, $x=23279$. The number required $x=232792560-1=$ 232792559. 2d, $232792560\times2-1=465585119$. 3d, $232792560\times3-1=698377679$.

And thus may be performed other numbers, *ad infinitum*, or thus, by the rule: 1, 2, 3, 5, 7, 11, 13, 17 and 19 are prime numbers; alſo, $\sqrt{4}=2$, $\sqrt[3]{8}=2$, and $\sqrt[4]{16}=2$ and all the reſt $\therefore$ $1\times2\times3\times2\times5\times7\times2\times3\times11\times13\times2\times17$

$\times17\times19=232792560$, the leaſt number that can be divided by the given diviſors without a remainder, as before.

2. What is the leaſt number that can be divided by the nine digits without a remainder?

Diviſors, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Now $\sqrt{4}=2$, 6 may be cancelled, being compoſed $2\times3$, and 3, 5, 7, are prime numbers, and $\sqrt[3]{8}=2$, $\sqrt[2]{9}=3$, then per rule, $1\times2\times3\times2\times5\times7\times2\times=32520$.

3. To find the leaſt number of guineas, which being divided by 6, 5, 4, 3, and 1 reſpectively, ſhall leave 5, 4, 3, 2 and 2 reſpectively remaining.

*Solution.* Firſt $1\times2\times3\times2\times5=60$, the leaſt number which will divide by 2. 3, 4, 5, and 6 $\therefore 60-1=59$.

*Q. E. D.*

## MAGICAL SQUARES.

BY a magical ſquare, we underſtand one divided into ſeveral other ſmall equal ſquares, filled with terms of an Arithmetical progreſſion, ſo tranſplanted, that all the ſame line or rank, whether taken perpendicularly, horizontally, or diagonally, make the ſame ſum.

### EXAMPLE.

1. The numbers, 1, 2, 3, 4, 5, 6, 7, 8, and 9 being given, to form them into a magic ſquare, viz. counting each rank perpendicularly, horizontally, or diagonally, that theſe ranks may be equal to each other. Suppoſe it done, and repreſented into proper form by the following ſymbols, placed as follows:

$$\begin{array}{ccc} a & b & c \\ d & e & f \\ g & h & m \end{array}$$

Then by the queſtion we have $\left\{\begin{array}{l} a+e+m=15 \\ b+e+h=15 \\ c+e+g=15 \end{array}\right\}$ Required $e$ the middle number.

Their ſum $=a+b+c+3e+m+h+g=45$.

Again, $a+b+c=15$
Alſo, $m+h+g=15$ } added together, we get $a+b+c+m+h+g=30$;

This, from the fourth equation, gives $3e=15$, then $e=\frac{15}{3}=5$ the middle number. Or thus, by numbers:

Firſt, The ſum of the progreſſional numbers are $1+2+3+4+5+6+7+8+9=45$, then $3=$the number of rows, $\therefore \frac{45}{3}=15=$the ſum of each ſide or rank; and $15\div 3=5=$the middle number as before.

Again, to find the corner figures; and firſt to find the figure repreſented by $a$: beginning with one, I find the corner letter $a$, or any other corner letter, cannot be 1; for if $a$ was 1, then $m$ muſt be$=9$, $b+c1=5-1=14$; as alſo $d+g=15-1=14$; but there remains no two numbers after 5, 1, and 9, whoſe ſum is 14, but 6 and 8. Then if any of theſe figures were $b$, the other would be $c$; and then no figures would remain for the value of either $d$ or $g$, wherefore, $a$ is not equal to 1, or any corner letter$=1$ or 9.

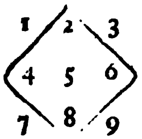
Now $a$ cannot be$=3$, for if it were, then $m$ ſhould$=7$; $b+c=-3=12$, as alſo $d+g+12$; but there remains no two numbers after 5, 3, and 7, whoſe ſum is 12; but $8+4$, which cannot anſwer to $b$ and $c$ and $d$ and $g$; wherefore $a$, or any other corner letter is not $=3$, neither is $m$ nor any other corner $=7$. From what has been ſaid, it is plain, that (if the queſtion propoſed is capable of being ſolved) the corner letters are all even numbers; wherefore, if $a=2$ $m$ will be $=8$, and $c$ muſt be either 4 or 6, let $c=4$, then $g=6$, $b=9$, $d=7$, $f=3$, and $h=1$, and ſo the ſquare will be completed, as was required:

$$\begin{array}{ccc} 2 & 9 & 4 \\ 7 & 5 & 3 \\ 6 & 1 & 8 \end{array}$$

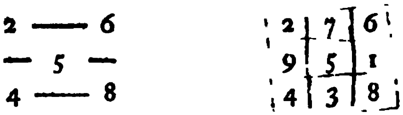
but if $c=6$ ($a$ being$=2$) then $g=4$, $b=7$, $d=9$, $f=1$, and $h=3$, and then the ſquares will ſtand thus:

$$\begin{array}{ccc} 2 & 7 & 6 \\ 9 & 5 & 1 \\ 4 & 3 & 8 \end{array}$$

or they may be found mechanically; thus, ſet them all down progreſſively, about which draw a ſquare corner-way, thus:



Then ſet the four angular figures at the corner, and put the outermoſt alternately; that is, place 1 between 8 and 6; 9 between 4 and 2; 5 between 4 and 8; and 7 between 2 and 6; thus:



There are many ways of conſtructing magic ſquares, as alſo many ſurpriſing properties thereto relating, and it is in itſelf very curious and entertaining; yet it cannot be denied, but that is of very little or no uſe in any other parts of the Mathematics; therefore, they who would ſee more of theſe matters may conſult the Diarian Repoſitory, page 103 and 108; alſo, from 225 to 233, where they will find this ſubject more largely treated of.

## RESOLUTION OF ADFECTED EQUATIONS IN GENERAL.

### *Definitions.*

1. A QUADRATIC Equation has been defined, viz. any one in which there enter the ſecond and firſt powers of the unknown quantity; or where there are two powers of the unknown quantity, the one the double of the other, Thus:

$$\left.\begin{array}{lll} x^2 \pm 2ax = \pm b & \text{and} & x = \mp a \pm \sqrt{a^2 \pm b} \\ x^4 \pm 2ax^2 = \pm b & & x = \sqrt{a \pm \sqrt{a^2 \pm b}} \\ x^6 \pm 2ax^3 = \pm b & & x = \sqrt[3]{a \pm \sqrt{a^2 \pm b}} \end{array}\right\}$$

By completing the ſquare, &c. as in Quadratics, the Root is determined.

2. A Cubic Equation is that in which the cube, or third power of the unknown quantity is concerned; thus:

$x^2$

$$\left.\begin{array}{l} x^3 \pm ax^2 \pm bx - c = 0 \\ x^3 \pm dx^2 \pm e = 0 \\ x^3 \pm fx \pm n = 0 \end{array}\right\}$$

Where $x$ is the unknown quantity and the value of $x$ in terms of the other quantities is called the root of the equation.

3. A Biquadratic Equation is that which contains the fourth power of the unknown quantity, Thus:

$$x^4 \pm ax^3 \pm bx^2 \pm cx \pm d = 0$$
$$x^4 \pm ax^3 \pm n = 0$$

*&c.*

4. And in general, an equation is ſaid to be of that kind or denomination, which the higheſt power of the unknown quantity points out; whether all the lower powers of that quantity do or do not enter the equation; but thoſe in which all the powers from the higheſt downward, enter, are of a more complete form than thoſe where ſome are wanting, as in the two biquadratics above, the firſt is more perfect in its kind than the ſecond.

N. B. Every equation has as many roots or values of the unknown quantity as is pointed out by the exponent of its higheſt power; thus, in the equation,

$$x^3 \pm ax^2 \pm bx \pm c = 0,$$

there are three roots or values of $x$; but in general, all theſe roots, except one, are either abſurd or impoſſible. And in that equation, where the powers of the unknown quantity all enter, and fall down in a regular Arithmetical progreſſion, ſo many roots only are poſſible.

### *Problem* I.

To find the root of an adfected equation by the method of approximation and converging ſeries,

*Rule.* Find by trial, a number nearly equal to the root required; call that number $r$, and put $y =$ the difference between the aſſumed root and the true one; for $x$, ſubſtitute its value, $r \pm y$, in the given equation, and a new equation will ariſe, affected only with $y$, and known quantities. Reject all the powers of $y$ above the firſt power, and then find $y$ by a ſimple equation. Then the value of $y$ being added to, or ſubtracted from it, according as $r$ was too great or too ſmall, will give the value of $x$ nearly. Put $r$ again for the value of $x$, and repeat the operation for another value of $y$, and another correction of the root will be obtained, and ſo on to any degree of accuracy required.

### EXAMPLE.

Given $x^3+6x^2-8x-235=0$ quere $x$?

By a few trials, we find $x=$ſomething more than 4.9; put $r\times 4.9$, and $x=r+y$, then the equation, after rejecting all the powers of $y$, above the firſt, becomes

$$r^3+3r^2y+6r^2+12ry-8r-8y-235=0$$

that is, $y=\frac{235-r^3-6r^2+8r}{3r^2+2r-8}=\frac{10.491}{122.83}=101$, nearly, and $r+y\doteq 4.9+101=5.001+x$ nearly; but by trial, this value of $x$ will be found a ſmall matter too great, put $r=5.001$, and $x=r-y$, repeat the operation, and we have

$$y=\frac{235-r^3-6r^2+8r}{-3r^2-12r+8}=.0010003, r-y=5.001-.0010003$$

$=4.999997=x$, nearly $=r$, and for a third operation, $x=r+y$, and $y=\frac{235-r^3-6r^2+8r}{3r^2+12r-8}=.00000030001$, &c.

hence $x=r+y=5.000003001$, which gives the root true to 5 places of decimals, and if the operation be once more repeated, the root will be found true to 20 places, &c. whence $x=5$ nearly.

### *Problem* II.

To find the root of an adfected equation by common Arithmetic; that is, by the method of Trial and Error, or *Double Poſition*.

*Rule.* Find by trial, two numbers, as near the true root as poſſible: ſubſtitute theſe numbers ſeverally in the given equation, inſtead of the unknown quantity; and find the errors which ariſe therefrom.

Multiply the difference of the two aſſumed roots by the leſſer error, and divide the product by the difference of the errors, when they are alike, or by their ſum, when they are unlike. Or ſay, as the ſum or difference of the errors is to the difference of the roots, ſo is the leaſt error to the correction of the root.

Add this correction to the root belonging to the leſſer error, if the root be too ſmall; but ſubtract if it be too great: the ſum or difference will be the true root nearly.

Take this root, and the nearer of the two former to the true one; or what is ſtill more convenient; let the two aſſumed roots differ only by an unit in the laſt figure on the right, the one being too ſmall and the other too great: then repeat the operation for another correction to the nearer root; and ſo on, repeating the work until we arrive at any degree of exactneſs required.

Let the former equation be reſumed, viz.

1. Given $x^3+6x^2-8x=235$: quere $x$?

1ſt Operation.

Here, by a few trials, $x$ is found to be more than 4.9 and leſs than 5.1.

| 1ſt *Sup.* | 2d *Sup.* |
|---|---|
| $x=4.9$ and | $x=5.1$ |
| $x^3=117.649$ | 132.651 |
| $6x^2=144.06$ | 156.06 |
| $=8x=-39.2$ | $-40.8$ |
| 222.509 | 247.911 |
| 235. | 235. |
| error, $-12.491$ | er. $+12.911$ |

ſum er. $=25.402$; diff. roots $=.2$. As ſum er. 25.402 : .2 :: 12.491 : .0922; then, $4.9+.0983=4.9983=x$, nearly.

2d Operation.

$x=4.998$ by 1ſt ſup. and $x=5.001$ by 2d do. hence by repeating the work as in the firſt poſition, we obtain .0009999 for the ſecond correction of 5.001, and $5.001-.0009999=5.0000001=$ the root; that is, $x=5$.

2. Given $x^3+x^2+x=100$ quere $x$?

1ſt Operation.

Here it appears that $x$ is between 4 and 5.

| 1ſt *Sup.* | 2d *Sup.* |
|---|---|
| $x=4$ and | $x=5$ |
| $x^3=64$ | 125 |
| $x^2=16$ | 25 |
| $x=4$ | 5 |
| 84 | 155 |
| 100 | 100 |
| Er. $-16$ | error $+55$ |

Sum of errors $=71$.

Diff. aſſumed roots $=1$; then, As 71 : 1 :: 16 : 225 $=$ correction; hence $x=4$ nearly.

2d Operation.

| 1ſt *Sup.* | 2d *Sup.* |
|---|---|
| $x=4.2$ and | $x=4.3$ |
| $x^3=74.088$ | 79.507 |
| $x^2=17.64$ | 18.49 |
| $x=4.2$ | 4.3 |
| 95.928 | 102.297 |
| 100 | 100 |
| Er. $-4.072$ | error $+2.297$ |

ſum whereof $=6.369$ ∴

As 6.369 : 1 :: 2.297 : 0.036 $=$ the 2d correction of the root; and $4.3-0.036=4.264$, nearly.

3d Operation.

$x=4.264$ by 1ſt ſup. and

$x=4.265$ by 2d do.

then by repeating the proceſs as before, we have 0.00043 for the 3d correction, which added to the firſt poſition, will give $x=4.26443$, the root very nearly, and by repeating the operation once more, the value of the root will be obtained truly to 10 places of decimals.

*Remark.* This ſecond method of finding the roots of equations by Poſition, is facile in its operations, and univerſal in its extent, being applicable not only to all kinds of adfected equations; but alſo to thoſe whoſe roots would be ſought for in vain by any other way: for there are equations to be met with ſo embarraſſed with ſurds, that it would be almoſt next to an impoſſibility to approximate the root otherwiſe. Nor can many exponential equations be ſolved without the aid of Poſition, as ſhall be ſhewn immediately:

Given $\sqrt{144x^2-\overline{x^2+20}|^2}+\sqrt{196x^2-\overline{x^2+24}|^2}=114$, quere $x$?

After a few trials, we diſcover that $x$ is between 7 and 8, and very little more than 7.

Then $\sqrt{144x^2-\overline{x^2+20}|^2}+\sqrt{196x^2-\overline{x^2+24}|^2}$.

| | 1ſt *Sup.* | | 2d *Sup.* |
|---|---|---|---|
| | $x=7$ | and | $x=8$ |
| | $=113.290$ | and | $115.759$ |
| ſhould be | $114.$ | | $114.$ |
| Error | $-0.710$ | error | $+1.759$ |

ſum of the errors$=2.469 : 1 :: 0.710 : 0.2$ nearly, and $7.0+.2=7.2=$root firſt approximated, a little too large; hence for the

2d Operation.

$\sqrt{144x^2-\overline{x^2+20}|^2}+\sqrt{196x^2-\overline{x^2+24}|^2}$

| | 1ſt *Sup.* | | 2d *Sup.* |
|---|---|---|---|
| | $x=7.1$ | and | $x=7.2$ |
| | $=113.877$ | and | $114.392$ |
| | $114.$ | | $114.$ |
| Error | $-0.123$ | error | $-0.392$ |

ſum of errors$=.515 : 1 :: 123 : .024=$2d correction; then $7.1+0.024=x$. very nearly; and if the operation be repeated,

repeated we can approach the value of $x$, as near the truth as we pleaſe.

## *Of Exponential Problems.*

An exponential problem is that in which the unknown quantity becomes the exponent of the power of ſome known or unknown quantity. Thus, $x^x=100$, is an exponential equation; and $a\times x^x=b$, is another, &c.

### EXAMPLE.

Given $x^x=100$, quere $x$?

From the nature of Logarithms, it is evident, that $x\times$log. of $x$=log. of $100=2$. Then by a few trials, $x$ will be found to be between 3.5 and 3.6.

| | 1ſt *Sup.* | | 2d *Sup.* |
|---|---|---|---|
| | $x=3.5$ | and | $x=3.6$ |
| Log. of | $x=0.5440680$ | — | 0.5563025 |
| $\times$d | $x=3.5=1.904238$ | — | 2.002689 |
| ſhould | be$=2.$ | — | 2. |
| error | $-\ 0.095762$ | error | $+\ 0.002689$ |

ſum errors$=0.098451:1::0.002689:0.00273=$firſt correction $\therefore\ 3.6-0.00273=3.59727=x$ nearly, and by repeating the operation, we find $x=3.59728474747$, &c.

The two foregoing methods of ſolving adfected equations, are univerſal; particularly that by Poſition, which we find applicable to exponentials alſo. There are indeed other curious rules ſuited to the ſolution of equations under particular circumſtances; ſuch as Newton's method by diviſors; Cardan's, for cubic equations; Des Cartes, for biquadratics, &c. And although theſe ſeveral methods are highly worthy the ſtudy of the Algebraiſt, and reflect great honor on their reſpective inventors; yet, as they cannot be comprized in the compaſs of our Appendix; we have been obliged reluctantly to omit them, and give thoſe alone which are conciſe and applicable to all caſes. However, the author intends, if Providence permits, to enlarge this Appendix; ſo that it is at preſent, little more than the ſkeleton of a large work on Algebra, now ready for preſs, and which he propoſes to publiſh ſhortly, in which the various methods of thoſe excellent writers ſhall be handled at large.

# INSTRUCTIONS

# FOR

# BOOK-KEEPING.

BOOK-KEEPING is the *mercantile art* of keeping books of accounts, in a ſyſtematic and regular manner: ſo that the books of a merchant, or thoſe of any man of buſineſs or property, ſhould at all times exhibit the true ſtate of his affairs, ſhew the exact ſum of ready money in hand, the reſpective ſums due him by others, the *quantity and value* of all his property, the amount of his ſeveral debts, if he owe any; and in ſhort, a complete and correct ſet of books of accounts, ſhould afford perfect information on every ſubject for which they may be conſulted.

To attain this great object many plans and projects have been deviſed, according to the taſte and capacity of the Merchant, Trader, Gentleman, Schoolmaſter, &c.: but all have failed except one, ſaid to have been firſt uſed by the Italians, uſually called the method by *Double Entry*. This ſyſtem has been found completely to anſwer the end propoſed; and experience has ſhewn, that he who adheres cloſely to this method, arrives at the higheſt pitch of perfection expected in the art; and that every deflection from it is for the worſe; and the farther the man of buſineſs keeps his books therefrom, the more uncertain are his affairs, and the leſs can he or any other know whether he is in a proſperous or ruinous ſtate. True indeed there may be many merchants at preſent who keep their books by the method commonly ſtiled *Single Entry*; but this plan is only a crippled limb of the great ſtock; for although their weak method can never inform them of the quantity or value of the goods on hand; yet they are obliged to call in the aid of the true and ſcientific method, in perſonal accounts, and give regular debits and credits therein. Upon the whole, this ſyſtem of Book-keeping by Double Entry, being the only perfect one that can poſſibly be diſcovered, we think it uſeleſs to trouble our reader with any other.

The author being circumſcribed, in this Appendix, to a ſmall ſpace, a complete ſyſtem of Book-keeping cannot, in candour, be expected: however, he hopes he has given ſuch rules in general, for journalizing, (which is his ultimate object,) that not only pupils and beginners may profit by; but that many eminent clerks and book-keepers may find worthy their moſt ſerious attention.

## *General Rules for Journalizing by Double Entry.*

1. In the common tranſactions of *buying and ſelling*, on the merchant's own account.

### GENERAL RULES.

I. In buying Goods,

1. The thing received is debtor to the thing given for it; that is,

2. When you buy goods for ready money: make the goods, mentioning quantity and value, debtor to Caſh; and Caſh Cr. by the goods, &c.

3. When you buy goods *on credit*, or as it is uſually termed, *on time*: make the Goods, mentioning quantity and value, Dr. to the perſon from whom you bought them; and that perſon Cr. by the goods.

4. When Goods are bought for your own Note: make Goods Dr. to Notes payable, mentioning intereſting particulars; and Cr. Notes payable, &c.

5. When Goods are bought for a good bill in ~~my~~ poſſeſſion, payable to the holder: make Goods Dr. to Bills receivable; and Bills receivable Cr. by Goods, &c.

II. In ſelling Goods,

1. When Goods are ſold for ready money: make Caſh Dr. to Goods; and Cr. Goods by Caſh, mentioning quantity and value, &c.

2. When goods are ſold on credit: make the Buyer, (naming him) Dr. to Goods; and Goods Cr. by that perſon, mentioning quantity and value, &c.

3. When Goods are ſold for an accepted bill: make bills receivable Dr. to Goods; and Cr. Goods by Bills receivable.

The foregoing are the moſt ordinary ſimple occurences in common buſineſs; but to have a more general and enlarged ſet of inſtructions for all caſes, whether ſimple or complex, we ſhall enter more fully on the ſubject:

## *Of Particular Trade.*

When a merchant, a man in trade, or a gentleman of property, propoſes to open a complete ſet of books, and to proceed to keep his accounts with ſyſtem and accuracy; the firſt thing to be done is to take an inventory of the whole of his property, in which inventory muſt correctly be ſpecified, the quantity of caſh in hand, the quantity and value of all the ſeveral different kinds of wares belonging to him, whether in his own poſſeſſion or in the hands of factors, his ſhips, houſes, lands, &c. together with the exact ſum that every man in his debt owes him; and laſtly, the correct ſum he owes to each of his creditors: And this inventory conſtitutes the firſt entry or tranſaction recorded in his day-book, after which the common occurrences of buying, ſelling, paying or receiving money, &c. are regularly to be regiſtered in the day-book in the order of their date.

The method of journalizing the inventory or firſt article of the day-book is thus: *Sundries* Drs. to *Stock*, mentioning the particular accounts, viz. Caſh for the ready money in hands, wares, naming each particular kind with the quantity and value, ſhip A., ſhip B., &c.; houſe C., houſe D., &c.; lands E., lands F., &c. and then the ſeveral perſons, A., B., C., &c. with the ſums they owe; and laſtly, *Stock* Dr. to *Sundries*, naming each particular perſon, with the exact ſum he owes each Creditor. And the difference between theſe two aggregate ſums, viz. that for which the *Stock* account is debited, and that for which it is credited, points out the exact ſtate of the merchant's affairs at the time of opening his new ſet of books.

The other articles of the day-book are regularly journalized in the order of their date thus:

1. Goods bought for ready money.

*Rule.* Goods are Dr. to Caſh as above, and Caſh Cr. by the Goods; this clears Caſh account of what it was charged to Stock, and brings it on the Goods, which muſt bring in what they coſt or was loſt by them, and thus on all occaſions one account clears the other.

2. Goods bought upon time.

*Rule.* Make ſuch Goods Dr. to the Seller; he Cr. by the Goods.

3. Goods bought, for part ready money and part upon time.

Some would make Goods bought Dr. to the Seller for all, then the Seller Dr. to Caſh for what is paid. This makes the man creditor more than his due, which indeed the other balances. But there can be no reaſon why the ſeller ſhould be credited ſo much, or Caſh by a wrong account.

The correct ſyſtematic way is to make the Goods Dr. to Sundry Accounts, viz. to Caſh for what is paid, and to the Seller for what is owing, referring to the folios of Caſh and Seller's Account. The ſelling man Cr. by Goods bought, ſo much as is left unpaid, and Caſh Cr. by goods for ſo much ready money paid: this gives each account its juſt due.

4. When ſundry parcels of goods are bought for ready money.

*Rule.* The ſeveral (and reſpective goods) muſt be made Drs. to Caſh for the value they ſtand in; and Caſh Cr. by ſundry accounts for the total amount.

5. When ſeveral parcels of wares are ſold for ready money.

*Rule.* Caſh Dr. to Sundry Accounts (referring to the folios where the ſeveral parcels ſold are entered in the Ledger) for their whole value—the reſpective wares creditor by Caſh; each for the reſpective quantity and amount.

6. When wares are bought on time, and booked, and afterwards ready money is paid upon diſcount.

*Rule.* The Seller Dr. to Sundry Accounts, referring to the folios of each of the ſundries, viz. To Caſh for the ſum paid him, deducting the diſcount, and to Profit and Loſs for the diſcount—and each of the ſundries, viz. Caſh and *Profit and Loſs* Cr. by the Seller for the reſpective ſums.

7. When wares are ſold on time and booked, but money received preſently after, for the ſame, allowing diſcount.

*Rule.* Sundry Accounts Dr. to the Buyer, viz. Caſh for the value of the wares, deducting the diſcount, and Profit and Loſs for the diſcount—The Buyer Cr. by Sundries, viz. Caſh for the ſum received, and *Profit and Loſs* for the diſcount.

8. Goods ſold for an accepted bill.

*Rule.* Make Bills Receivable Dr. to the Goods. The Goods (naming them) Cr. By Bills Receivable.

9 When you promiſe a man money for ſomething he was to perform, which is not performed; you are freed from the debt until it is performed.

Some make Promiſe Account Dr. to the perſon to perform, which gives the man credit before the money be juſtly due;

*Rule.* Make Conditional obligations (which is a general title for all accounts of a like nature) Dr. to the Performer his account conditional, by which we ſee that he is not really creditor till the buſineſs is accompliſhed.

10. But when you promiſe from conſideration of indigence, former favours, &c.

*Rule.* Make *Promiſe Account* Dr. to him to whom the promiſe is made; it being an abſolute declaration of the debt.

11. When money is lent on intereſt.

*Rule.* Make the Borrower Dr. to Caſh, and Cr. Caſh by the Borrower.

12. When you borrow money on intereſt,

*Rule.* Make Caſh Dr. to the Lender—and Cr. the Lender by Caſh.

13. When intereſt is paid by you and the principal continue.

*Rule.* Profit and Loſs Dr. to Caſh for the ſum paid—Caſh Cr. by Profit and Loſs for the ſame ſum.

14. When money is received by you for intereſt, and the principal continue.

*Rule.* Caſh Dr. to Profit and Loſs for the intereſt received—Profit and Loſs Cr. by Caſh for the ſame.

15. When you receive a bill from your correſpondent to be carried to the credit of his account, and find it honoured with acceptance,

*Rule.* Make Bills Receivable Dr. to your Correſpondent; and Cr. your Correſpondent by Bills Receivable. Some abſurdly Dr. the Acceptor; but this brings the name of a perſon into your books, with whom you may probably never have the leaſt dealings; becauſe the bill may be diſcounted at the Bank, or paſſed away in payment before it becomes due, and in that caſe you have no farther buſineſs with the Acceptor.

16. There is a certain way of buying and ſelling, though not very uſual, yet practicable; and that is for goods to be delivered at a certain time, and at a price agreed on.—For

this there are ſeveral ways; ſome make Delivery Account in place of the goods not yet received Dr. to the Seller, bound perhaps only upon promiſe to deliver them, (which as our example of promiſing) makes the man Cr. before the money is due, and conſequently no difference between debts abſolutely due by you and thoſe you owe conditionally.

*Rule.* Make General Account of Goods not received, or *Account of Conditional Obligations*, or any other title which may be found moſt anſwerable to the deſign of knowing the quality as well as quantity of goods Dr. to the Seller his *Account conditional.*

17. When wares are to be paid for at ſeveral payments,

*Rule.* Make the Wares (naming them) Dr. to the Seller for their value, mentioning the ſeveral days of payment in the Journal—The Seller Cr. by the Wares for the like ſum.

18. When in lieu of a debt you receive goods, where value is more than the debt due to you, which ſurplus is returned in caſh immediately;

*Rule.* Make Wares bought (naming them) Dr. to Sundry Accounts, viz. to the Seller for ſo much as his debt was, referring to the folio of the Seller's Account; and to Caſh for the ſurplus—The Seller's Account Cr. by Wares, for the amount of the debt, and Caſh Cr. by the Wares for the ſurplus paid.

19. When in payment of a debt you ſell goods to your Creditor, whoſe value exceed his debt, and he returns you the overplus in caſh,

*Rule.* Make Sundry Accounts Dr. to the Wares (naming them) viz. the Buyer for ſo much as his debt was, and Caſh for the overplus returned—Cr. by Sundry Accounts, referring to the folios of Caſh and the Buyer's Account.

20. When you give wares in *barter*, and receive other wares of an equal value for them,

*Rule.* Make Wares received Dr. to the Wares given for the amount—and Cr. Wares given by Wares received, mentioning quantity and value in both caſes.

21. When wares are bought part for wares and part ready money:

*Rule.* Wares bought Dr. to Sundry Accounts, referring to the folios of Wares ſold and Caſh—Wares ſold Cr. by Wares bought for the value of thoſe ſold, and Caſh Cr. by Wares bought for the ſum paid in money.

22. When wares are ſold for a part wares and part ready money,

*Rule.* Make Sundry Accounts Dr. to Wares ſold, viz. Wares bought for what they coſt, and Caſh for the ſum received—Wares ſold Cr. by Sundries, referring to the folios of the wares bought, and Caſh.

23. When you pay money for a ſhip (or part thereof,)

*Rule.* Ship (naming her) Dr. to Caſh, for the ſum paid, mentioning the maſter and what part you have bought—Caſh Cr. by the ſhip, the ſum paid mentioning to whom.

## *Accounts Proper in Foreign Trade.*

*Definition.* PROPER ACCOUNTS in Foreign Trade are, when you ſend your goods beyond the ſea, to ſome correſpondent, for your account, to be ſold.

1. When you ſhip off wares, (which had been poſted),

*Rule.* Make voyage to ſuch a place (mentioning the place whither you ſend them) conſigned to your factor or correſpondent (mentioning his name) Dr. to Sundry Accounts, viz. to the wares ſhipped for their value, (naming the ſhip and maſter's name)—and to Caſh for charges of merchandizes, for the ſum paid on them (naming the wares) and the wares Cr. by the voyage, &c. and Cr. Caſh by the voyage for the charges thereon—All this is in hazard by the voyage, it is therefore charged, and hereby we ſee what goods are at ſea.

2. If you make inſurance upon the voyage,

*Rule.* Make the voyage Dr. to Inſurer for the premium, if not immediately paid; but to Caſh if paid, and Cr. Inſurer by Voyage in the one caſe, or Cr. Caſh in the other. The reaſon of debiting the voyage for the premium is evident, becauſe that ſum is a charge attending it.

If the goods are loſt, and the voyage not inſured, make Profit and Loſs Dr. to Voyage—and Cr. Voyage by Profit and Loſs. But if the Voyage be inſured, make Sundry Accounts Drs. to the voyage, viz. the Inſurer for the nett ſum he is to pay, and Profit and Loſs for all deficiencies—and Cr. Voyage by Sundries, referring to the particular folios for the debits.

3. When money is received upon inſurance,

*Rule.* Make Caſh Dr. to Inſurance Account, expreſſing what ſum you inſured, to whom, and on what account—Inſurance Account Cr. by Caſh, &c.

4. If the goods you inſured are loſt,

*Rule.* Make Inſurance Account Dr. to the perſon to whom you inſured the ſum loſt (or to Caſh if paid) mentioning the ſum inſured by you, to whom, and on what account—and the perſon or Caſh Cr. by Inſurance Account, &c.

6. When you are balancing the Account of Inſurance,

*Rule.* If the ſum on the Cr. ſide exceed that of the Dr. make Inſurance Account Dr. to Profit and Loſs for that exceſs; but if the Dr. ſide exceed the Cr. make Profit and Loſs Dr. to Inſurance Account for that exceſs, and in both caſes per contra creditor.

7. When you lend money on bottomry, that is, on the ſecurity of the ſafe arrival of a ſhip at a certain port,

*Rule* Make *Bottomry* Dr. to Caſh—and Cr. Caſh by Bottomry.

8. When Inſurance is made on ſuch bottomry,

*Rule.* Make Bottomry Dr. to Caſh for the ſum paid—and Caſh Cr. by Bottomry.

9. If my factor writes me he has received and ſold my goods for ready money,

*Rule.* Make factor at ——, (mentioning the place where he lives) my Account Current Dr. to Voyage to ſuch a place, for the known ſale in ſterling money, being the neat proceeds of wares, as by his account, ſold for ſo much foreign coin (mentioning the exchange.) Then make Voyage to ſuch a place, conſigned to ſuch a perſon, (mentioning your factor's name) Cr. by my factor at ſuch a place, my account current, for ſo much ſterling money as you know by his letter the foreign coin of the neat proceeds amounts to.

*Note.* 1. By the above we know what is gained or loſt by the voyage.

2. Account current is when the money may be drawn for at pleaſure, in our own country currency; but my account, or our account with any factor is, when his country currency differs from ours; ſo that he is obliged only to pay in the coin where he is. Therefore, in the Ledger there are commonly two money columns; one for our money, the other for foreign: by keeping a foreigner's account, in this manner, that is with double columns; the inner for the currency of his country, and the outer for that of ours; his account is eaſily balanced, to the ſatisfaction of the parties, and the gain or loſs by the various riſes and falls of exchange quickly come at.

10. When your factor writes to you that he has received your goods formerly conſigned to him, but as yet has made no ſales. Then if you do not wiſh to multiply accounts, you are only to make an entry thereof in your memorandum book, and file the letter; and then wait till you receive his account of the ſales. Some however do make an entry of ſuch intelligence, and debit Wares in the hands of ſuch a one to Voyage for the whole debit amount, and credit the Voyage for the ſame debit. Afterwards credit is given regularly to the goods, for the amount of the ſeveral ſales of which intelligence has been ſent, and the factor debited, if he remit not in the mean time, for the reſpective amounts of the ſeveral ſales. However, as we obſerved above, the better way is to make no entry in the day-book until the intelligence of the whole account of ſales come to hand; then if the factor remit not, make Factor, my account current, Dr. to Voyage for the nett proceeds—and Cr. Voyage by Factor for the nett proceeds; but if he remit the whole or part, the debtor or debtors are eaſily determined from the preceding caſes, and the voyage ſtill Cr. &c.

11. When your factor remits you a bill, and the bill is accepted:

Some make the Acceptor Dr. to the Factor my account current; in an exchange of remittances or drafts in caſes of this nature, ſuch a method may be uſed.—But there is a ſecond way, viz. to make Bills Receivable Dr. to the Remitter; this does not only anſwer the deſign to know the nature of debts, but alſo prevents the introduction of many needleſs accounts.

12. When I have advice that my factor has ſhipped goods upon my account,

*Rule.* Make voyage from ſuch a place Dr. to Factor my account current, for all coſt till on board—for as ſoon as the goods are on board, factors ordinarily give advice, and we become Dr. to them.

13. When they arrive,

*Rule.* Such goods, or ſuch a cargo arriving, as we keep a general or particular account, Dr. to voyage from ſuch a place, for what it was Dr. which clears the voyage, there being no loſs by it. Likewiſe, to Charges Account, for the charges, ſuch as freight, landing, &c. it being on their account.—Some make caſh Cr. for ſuch of the charges as are paid, and charges account only for what is owing; but the former way, (and to bring in caſh once a month or the like, as by the book of petty charges we find convenient,) is the beſt and readieſt; for by it we know the exact account of what charges has been on any account; it alſo ſaves the trouble of a great many poſtings of ſmall parcels in caſh account.

14. If my factor take all or part of my goods for his own uſe, and adviſes me thereof, and that I may draw for part at pleaſure, the reſt he will pay at time,

*Rule.* Make Sundry Accounts Dr. to Goods in do.'s hands, or to Voyage, as the caſe requires, from the former methods, viz. He my Account current, for the ſum I may draw for, and do. My Account current upon time, for the remainder: this diſtinctly anſwers the deſign of giving each account its exact due, and ſhews the quality as well as quantity of debts.

15. When you order your Factor beyond ſea, to ſhip off goods to your correſpondent for his account.

*Rule.* Make your correſpondent my account current Dr. to the factor my account current; and Cr. the Factor my account current by the correſpondent my account current.

16. When you receive the content of a bill here, and thereupon draw the ſame on your Factor to pay to the order of him that paid you.

*Rule.* Make Caſh Dr. to the Factor at ſuch a place, my account current for ſo much ſterling received of ſuch a perſon my bill drawn on ditto. Factor, payable by him to ſuch a perſon at ſuch a time ſo much foreign coin which at ſo much exchange makes ſterling ſo much. Then make Factor at ſuch a place my account current Cr. by Caſh, &c.

17. When you deliver a bill here drawn upon your Factor beyond ſea, and receive not the content till ſome time after.

*Rule.* The perſon to whom you deliver the bill Dr. to Factor at ſuch a place my account current in ſo much ſterling for ſo much foreign coin drawn upon ſuch a Factor payable at ſuch a time to ſuch a perſon or order, the exchange at ſo much ſterling, for ſo much foreign coin makes ——. Factor at ſuch a place my account current Cr. by the perſon to whom you delivered the bill.

18. When you receive advice that your Factor at one place has drawn a bill on your Factor at another place.

*Rule.* Make the drawing Factor my account current Dr. to the paying Factor my account current, for ſo much foreign coin drawn by —— payable at ſuch a time, to ſuch a perſon, ſo much foreign coin; which, at ſuch a rate of exchange, makes ſterling —— Then make the accepting factor my account (mentioning both their names, ſums, &c.) Cr. by the drawing factor, &c.

19. If your correſpondent break, and you agree with him to have no payment until recovery or the like. Some would make Profit and Loſs Dr. to him my or his account current according as the debt was. But the more exact method is to make my account of Deſperate Debts, or my Account Deſperate Dr. to him, my account current according as the the debt is. By this we know the debt is ſtill owing, though payment be uncertain.

20. When we compound with him for part.

*Rule.* Make Sundry accounts Dr. to my factor, his or my account current, Caſh for the ſum received, and Profit and Loſs for the ſum abated. Then Profit and Loſs is juſtly Dr. for the ſum abated.

## *Factorage Accounts in Domeſtic Trade.*

*Definition.* Theſe accounts are when a trade is managed by a Factor or his ſervants for the employer, whom the Factor ſerves on commiſſion.

1. When a Factor receives wares from his employer.

*Rule.* Make account of goods for your employer Dr. to Caſh, for ſo much paid cuſtom, freight, &c. at the receit. Then Caſh Cr. by account of goods, &c. ſo much paid.

2. When wares, received in commiſſion by a Factor, are ſold for ready money.

*Rule.* Make Caſh Dr. to account of goods for the employer for the ſum received. Then make Account of goods for the employer Cr. by Caſh for the ſum.

3. When commiſſion wares are all ſold by the Factor in barter.

*Rule.* Make goods bought in barter Dr. to the employer his account of wares. Account of wares &c. Cr. by wares received in barter for the ſum.

4. When wares in commiſſion are ſold part for ready money and part on time.

*Rule.* Make Sundry accounts Dr. to account of goods, for account of the employer, viz. caſh for the ſum received, the buyer for the ſum on time. Account of goods for the employer Cr. by ſundry accounts for the total value of the goods ſold, referring to the folios of the account of buyer and caſh.

5. When wares are ſent to an employer in return with charges in ſhipping off.

*Rule.* Make Account of wares for account of the owner (or your employer) naming him, his account current Dr. to ſundry accounts, viz. to the goods ſhipped off, naming the value and goods with the name of the ſhip and maſter, and to caſh paid for cuſtom and other charges. Then make Wares ſhipped Cr. by the employer his account current for the value, and Caſh Cr. by the ſame account current for the charges, &c.

*Note.* If theſe goods were bought by order and on the account of the employer, with ready money, and not entered before in your leger.

*Rule.* Make your employer, naming him, his account current Dr. to Caſh, for the value of the goods and charges of ſhipping off. And Caſh Cr. per contra.

6. When you accept a bill drawn on you by your correſpondent at ſo many days ſight, and honor his bill with acceptance.

*Rule.* Make your correſpondent Dr. to Notes payable: and Cr. by Notes payable by the correſpondent.

*Remark.* It is aſtoniſhing that ſuch a tranſaction, which is one of the moſt common occurrences in trade, ſhould be overlooked by 99 out of 100 of our authors on Book-keeping, and generally abſurdly done by thoſe who take notice of it. Dowling, Jackſon, &c. make no entry in the day-book until the bill is paid, whereby the correſpondent's account can not be balanced at any time ſooner than after his bills are all diſcharged, except by an appeal to the bill-book; and then if with this aſſiſtance it be balanced, and debits made to him therefrom, new debits will emerge again from the ſame ſums, &c.

That notes payable and bills accepted by you are of the ſame nature in effect needs not to be demonſtrated.

*Factorage*

## *Factorage Accounts in Foreign Trade.*

*Definition.* Theſe accounts are when a Factor cannot carry on the buſineſs of whom he ſerves on commiſſion without aſſiſtance of foreign correſpondence, for whoſe returns he is accountable to his employer.

When goods ſent to ſea are inſured by you on account of your employer.

*Rule.* Make the employer Dr. to Caſh, (if you paid the inſurance money preſently.) And Caſh Cr. by the employer &c. But if the inſurance money was not to be paid preſently, then employer, &c. Dr. to the inſurer. And inſurer Cr. by the employer, &c.

## *Company Accounts.*

*Definitions.* Company accounts are when a ſtock is employed in common between ſeveral merchants, in the way of trade, and each partner is to have a ſhare of the gain, or bear a part of the loſs in proportion to his ſtock; as taught in our Arithmetic, Volume 2d, page 24.

*Or;* Company trade is either when two, three, or more join for a certain voyage, or any preſent occaſion, or in a ſtanding trade, *i. e.* to carry on all buſineſs in one body conjointly.

If I buy goods, having a commiſſion from another for a part to be on his account, and a part on mine. The uſual way is to make the goods (the whole that is bought,) Dr. to the Seller, if upon time, and to Caſh, if for ready money, naming in company for all. Then my partner is Dr. to his account in company for his ſhare: but I being only as factor for that of his, and obliged to give him a particular account of the ſame, our tranſactions may be kept ſeparately, as not appertaining to company affairs, by making each of them diſtinctly, Dr. or Cr. for what is owing to, or by them reſpectively as in the former methods of accounts; theſe diſtinctions being exactly minded, viz. if the accounts be kept after this method, (or as in company) to make ſuch things in company Dr. for what is due in company, never failing to make partner Dr. to ditto his account in company for his part of what is to be paid in for his part, and ſo Cr. for what comes to his ſhare to receive out, &c.

And if the accounts be kept after the better way (or ſeparately) mine own ſhare, as in particular trade, and my partner, as in factory, there can no more difficulties occur, if ſingle trade and the deſign be underſtood. When ſeveral join ſtocks with me to be employed as the company ſhall think fit, and I am appointed to keep the company's accounts in mine own books: through every paſſage in ſuch company, my affairs are to be conſidered as in ſingle or particular trade, diſtinct from thoſe in company; for it is neceſſary that their accounts be ſo kept, as at any time I may diſtinguiſh between the companies and mine own proper concerns, to the mutual advantage and ſatisfaction of the parties concerned, which can be done no other way than by keeping them ſo, that I may be enabled upon all occaſions, without trouble, to know the exact circumſtances of our reſpective concerns. There muſt likewiſe be ſuch preciſion in the Alphabet, as at balancing the company's accounts, I may clearly ſee what accounts belong to them diſtinctly from mine own, without the leaſt trouble to my memory.

1. When ſeveral perſons join you in company trade, and an equal ſhare, ſome paying in their whole ſhare, ſome paying part, ſome paying intereſt, &c.

*Rule.* Make Sundry Accounts Dr. to Sundry Accounts, for the whole—Then make Caſh in Company Dr. to his account in do. Company, who pays in his ſhare agreed on.

Make His Account with Company Dr. to Sundry Accounts who pays intereſt, viz. to his account in company for his principal (mentioning the time) and to Intereſt Account in Company for the Intereſt.

Make His Account with Company Dr. to his Account in do. Company, who is to pay in a ſhort time.

If you pay in half your own ſhare, make ſundry accounts Dr. to particular ſhare in do. Company for your ſhare, viz. Caſh in company for ½ paid, and Proper concerns with do. company for the remainder not paid.

Then make Adventure in Company Dr. to Sundry Accounts your ſhare, viz. To caſh, for ½ paid to company caſh—and to Company the remainder not paid.

2. When goods are bought upon time for the Company,

*Rule.* Make Goods in Company Dr. to the Seller with do. Company.

3. When a partner pays in his ſhare, part money, part goods,

*Rule.* Make Sundry Accounts Dr. to him with company, viz. Caſh in Company for the ſum paid—Goods in do. for their value.

4. When you buy goods for caſh, for the uſe of a partnerſhip, ½ for yourſelf, and ½ for your partner; being both equally concerned,

*Rule.* Make ſundry Accounts Dr. to Caſh, viz. Goods in Company Dr for your half, and partner Dr for his half: then Cr. Caſh by Sundries, referring to the proper folios for the debits.

5. When the company, by conſent, take in another partner, each quitting equally,

*Rule.* Make Sundry Accounts Dr. to Sundry Accounts the whole quit—Make his account in do. Company, Dr. to do. with do. for the part quit—Make ſundry Accounts Dr. to him with company who pays intereſt, viz. do. his account in do. company the ſum quit and intereſt account in do. company the part he was to pay as intereſt—Make particular ſhare in do. company Dr to proper concerns with do. your part quit. Then make Company Dr. to Adventure in do. for your part quit.

6. When you and three others enter into partnerſhip, all being equally concerned, and each of the three paying into your hands his ¼ of the joint ſtock in caſh; how are you, as the acting partner, to journalize this?

*Rule.* Make Caſh in Company Dr. to Stock in Company, (keeping a ſeparate ſet of company books); then in your own books, make Adventure in Company Dr. to Caſh for your part, removing the caſh from your own cheſt to that of the company's ſtock cheſt. Or thus, Caſh in Company Dr. to Sundry Accounts, viz. to each partner, his account in company for his ¼ part, and to caſh for your part put in. Or thus, Caſh in Company Dr. to Caſh for your part, keeping inner columns in caſh in company account, for the fixed ſtock, and your proportional part put in the outer.

7. To charge coming in with advance for his privilege,

*Rule.* Make him with do. Company Dr. to Sundry Accounts, viz. To do. his account in do. for his ſhare—To Intereſt Account in do company, the ſum for his privilege.

8. When you pay the partners out of the company's caſh, the money they quitted, to make room for another partner.

*Rule.* Make Sundry Accounts Dr. to Caſh in Company, viz. each partner with do. company the ſame quit.

Make proper concerns with do. company your ſum—Then make your own Caſh Dr. to Company your ſum taken out.

9. When the partner received pays,

*Rule.* Make Caſh in Company Dr. to him with do. Company.

10. When company's goods are ſent abroad,

*Rule.* Make Voyage to ſuch a place in company Dr. to Sundries, viz, to goods ſhipped off in company and to charges in do. company the whole.

11. When my factor or correſpondent adviſes of his draft on me,

*Rule.* Make Factor his account current Dr. to Notes payable (when accepted); and credit notes payable by the factor.

12. When the company have returns of goods formerly ſhipped off,

*Rule.* Make Goods in company Dr. to Sundry Accounts, the whole, viz to voyage in company for what the voyage was Dr. and to charges in company for duty, &c. paid.

13. When by conſent of the company I numbered goods among theirs, and drew from their caſh the value,

*Rule.* Make Goods in company Dr. to Caſh in do. company—Then make Caſh Dr. to Sundry Accounts the whole, viz. to Goods for your part, and to partner for his.

14. When company's goods are ſold, part for ready money, and part on time,

*Rule.* Make Sundry Accounts Dr. to Goods in do. company, viz. caſh, and the buyer, (naming him) in do. company the reſpective ſums.

15. When I join another in expectation of gaining by exchange, and accept a bill,

*Rule.* Make Partner, our account of exchange in company Dr. to Notes payable.

16. When adviſed that goods ſhipped for company are arrived,

*Rule.* Make Goods in company in hands of —— Dr. to Voyage to —— in do. company.

17. When ſuch goods are ſold abroad,

*Rule.* Make factor at —— our account current Dr. to Goods in do. company, in hands of do.

18. When I receive a bill from my partner in exchange,

*Rule.* Make Bills Receivable Dr. to partner at —— our account of exchange in company, &c.

19. When I ſell company's goods to their Dr. who pays all off,

*Rule.* Make Caſh in company Dr. to Sundry Accounts, viz. to the goods in company for their value, and to the Debtor with do. company for what he owed, the ſum.

20. When for an accepted bill part money is received, the reſt diſcounted,

*Rule.* Make Sundry Accounts Dr. to Bills Receivable, viz. caſh, for the ſum received, and *Profit and Loſs* for the diſcount.

21. When goods for an account of my factor are ſold on time,

*Rule.* Make Buyer Dr. to Factor his account of wares.

22. When adviſed that company's goods are all ſold on credit,

*Rule.* Make Factor at ſuch a place company their account of ſales Dr. to Goods in do. Factor's hands.

23. When you ſell factor's goods for ready money,

*Rule* Make Caſh Dr. to Factor his account of wares, for the ſum.

24. When caſh is received for goods formerly ſold for my factor and allowance made for prompt payment,

*Rule.* Make Sundry Accounts Dr. to the Payer, viz. caſh for the ſum received, and factor's account of wares (or his acct. current) for the abatement.

25. When the company, by mutual conſent admit another into the partnerſhip, each partner taking up his proportional part of the gain, if any; ſo that the company now encreaſed in number, have no more ſtock employed than formerly. How are you, as the active partner, the new partner, and each of the others, to journalize this?

## *Rules for cloſing the old ſet of books and opening a new ſet.*

1. Let every article or entry in the day-book be carefully compared with the correſpondent entries of the Journal, ſo that there may be a mutual agreement between them in every particular; and that the ſeveral Drs. and Crs. be clearly and accurately pointed out in the journal. Then let a ſimilar ſcrutiny be made between the Journal and Leger: and let all errors be carefully expunged and corrected in the *Leger*, until it is found perfectly to agree with the Journal. Having examined the Leger thus, and being ſatisfied that all the

Journal is fully and accurately poſted, you then proceed to balance the ſeveral accounts in the Leger thus:

2. Having prepared a balance ſheet for the balance account, caſt up the amount of the debit and credit ſides of the caſh account, and the exceſs of the Dr. ſide above that of the Cr. ſhews the exact ſum of ready money in hands; then place that ſum on the Cr. ſide of caſh account; and alſo on the Dr. ſide of the balance account, ſaying By Balance, &c. in the caſh account, and to caſh, &c. in the balance account; the total of both ſides of the caſh account being then equal, that account is now cloſed and balanced. Again, the other *real accounts* of wares, ſhips, houſes, lands, &c. are cloſed, either by the aid of balance account, that of Profit and Loſs, or laſtly, by balance and Profit and Loſs. Thus: when none of the wares are ſold; then credit the account of wares for the whole quantity and total amount of coſt and charges, by balance, &c. and debit the balance acct. for the ſame ſum, mentioning the quantity, &c. When all the wares are ſold; then if the amount of the credit ſide exceed that of the debit; take their difference, which ſhews the gain; and debit the account to Profit and Loſs for the gain, and credit the account of Profit and Loſs by the wares for the ſame ſum.

But if the amount of the debit ſide exceed that of the credit, take their difference, which ſhews the loſs; then credit the account by Profit and Loſs for the loſs, and debit the account of Profit and Loſs to wares for the ſame ſum. Laſtly, if part be ſold and part not; then credit the account by balance for the quantity remaining unſold, valuing it at the rate it coſt you, and debit the account of balance to wares for the ſame ſum, mentioning quantity, and cloſe the account now by the aid of Profit and Loſs account, exactly as in the caſe when all were ſold.

The other *real accounts* of ſhips, houſes, lands, &c. are all cloſed in like manner with thoſe of wares.

N. B. If the total amount of the debit and credit ſides of any account be exactly equal to one another, that account has cloſed already of itſelf.

### *Perſonal Accounts.*

3. All common perſonal accounts are ſimilarly cloſed, viz. by the aid of balance account, provided the accounts are kept by the parties in the ſame country currency. Thus, find the

amount of the debit and credit ſides of the account, and take their difference; then if the debit ſide be the heavier, that difference ſhews the ſum due you by that perſon; but if the credit ſide be the heavier, the difference is the ſum you owe him. Laſtly, debit or credit the account for the difference (as the caſe requires) to or by balance, debit or credit the balance account accordingly, and that account is cloſed.

4. In the various accounts kept with foreigners, wherein exchange is concerned, two columns for the ſeveral amounts of drafts and remittances are uſed, the inner column, for the foreign money, and the outer one for your country currency. And by reaſon of the conſtant fluctuation of the rate of exchange, a conſiderable profit or loſs may ariſe therefrom.

5. Balance the inner columns firſt, valuing the difference at the proper rate of exchange, and carry the ſame to the outer correſpondent column; then debit or credit the account and that of balance, as above. Now if the outer columns be unequal, as generally happens, take their difference, and debit or credit the account for that difference, to or by Profit and Loſs, and carry the ſame to that account, being the ſum you have gained or loſt by the riſe or fall of exchange; and then that account is cloſed.

## *Imaginary Accounts.*

6. The inferior imaginary accounts, ſuch as Inſurance Charges, &c. are all cloſed by the aid of Profit and Loſs account; that account itſelf is cloſed by the aid of Stock Account; and laſtly, balance account is alſo cloſed by the aid of Stock Account, and the difference of the total of the two ſides in balance account, ſhews the nett ſtock or whole property; provided the debit ſide be the heavier, but if the other be the heavier, the difference is the negative property. —Stock account cloſes of itſelf, if all be right.

Then the inventory for a new ſet of books is made out from the balance account.

*FINIS.*

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THE following *Errata* has been diſcovered in the different ſheets as they came from the Preſs; if any other have eſcaped notice, the Author hopes the friendly eye of the literati will not paſs them over without making a remark, and informing him thereof, that they may be corrected in the next edition.

## VOL. I.

Page 1, l. 10, r. ſimple multiplication. p. 16, q. 16, r. 5329602. p. 18, q. 5, r. 6*cwt.* p. 50, line 22, r. $39\frac{1}{3}$. p. 60, q. 10, r. 26m. 4fur. 13per. p. 66, q. 6, r. 4*lb.* p. 71, q. 3, r. 1*d.* p. 71, q. 6, r. 2*d.* p. 74, q. 5, r. 530*l.* 5*s.* p. 81, q. 2, r. 48*l.* 17*s.* $3\frac{1}{4}$*d.* p. 81, line 16, r. *Caſe 2d.* p. 89, q. 8, r. 41*l.* 5*s.* p. 103, q. 4, r. 18*l.* 15*s.* p. 104, q. 5, r. $1\frac{1}{2}$*d.* p. 106, q. 18. r. 84 *moydores.* p. 111, for l. 14, 15, 16, r. gallons multiplied by 2 are pottles, pottles multiplied by 2 are quarts, quarts multiplied by 2 are pints. p. 112, q. 81, r. 4 *days* 15 *hours.* p. 115, q. 15, r. $0\frac{1}{4}$*d.* p. 116, q. 22, r. 2*l.* 14*s.* p. 124, q. 5, r. 2. p. 127, q. 91, r. 2*s:* $1\frac{1}{3}$*d.* p. 129, q. 109, r. 2*h.* $39\frac{6}{8}$ *min.* p. 131, q. 128, r. 90 *yds.* p. 133, q. 134, r. 40 *to* 1. p. 143, q. 7, r. 5. p. 152, q. 3, r. $1\frac{7}{8}$. p. 154, q. 6, r. $\frac{21}{88}$. p. 155, q. 10, r. $9\frac{3}{5}$. ibid. q. 18, r. $\frac{7}{80}$. p. 158, q. 14, r 2*l.* 1*s.* $0\frac{3}{4}$*d.* ibid. q. 15, r. *per cent.* ibid. q. 20. r. $7\frac{111}{264}$. ibid. q. 22, r. $\frac{2}{3}$ of $\frac{3}{10}$. p. 163, l. 4, r. $\frac{1}{5}$. p. 172, q. 9, r. 876 *at.* ibid. q. 12, r. 128*l.* 5*s.* 0*d.* p. 175, q. 7, r. 6*s.* 3*d.* *per.* p. 180, q. 6, r. 7*s.* $0\frac{3}{8}$*d.* p. 187, q. 12, r. 206*l.* 11*s.* 3*d.* p. 189, q. 2, r. 619*l.* 12*s.* 3*d.* ibid. q. 5, r. 309*l.* 15*s.* $11\frac{527}{1358}$*d.* p. 191, q. 8, r. 31*l.* 4*s.* $4\frac{1}{2}$*d.* ibid. q. 9, r. 57*l.* 2*s.* 2*d.* ibid. q. 12, r. 1402*cwt.* 2*qrs.* 10*lb.* *amt.* 2290*l.* 18*s.* 2*d.* p. 192, q. 12, r. 615*l.* 5*s.* $8\frac{1}{2}$*d.* ibid. q. 13, r. 629*cwt.* 3*qrs.* $22\frac{1}{2}$*lb.* *amt.* 808*l.* 8*s.* 9*d.* p. 193, q. 17, r. 2720*l* 9*s.* 2*d.* ibid. q. 18, r. 311*l* 19*s.* $11\frac{3}{4}$*d.* ibid. q. 19, r. 239*l.* 17*s.* $11\frac{1}{2}$*d.* ibid. q. 20, r. 170*l.* 1*s.* $9\frac{267}{1378}$*d.* p. 194, q. 1, r. 2619*l.* 3*s.* &c. p. 197, q. 10, r. 1*l.* 15*s.* $10\frac{567}{7000}$*d.* p. 229, q. 4, r. 1459*l.* 13*s.* $5\frac{2}{3}$*d.* p. 232, q. 6, r. 16*l.* 17*s.* 6*d.* ibid. q. 9, r. 1414*l.* 13*s* 8*d.* ibid. q. 10, r. 2200*l.* 7*s* 8*d.* p. 235, caſe 3d q. 2, r. 209*l.* 17*s:* 8*d.* p. 244, q. 2, r. 733*lm.* 9*s.* 3*d.* p. 246, q. 7, r. 5*s.* $6\frac{1}{4}$*d.* p. 258, q. 2, r. 19*s.* $0\frac{1}{2}$*d.* ibid. q. 3, r. *remit.* 721*l.* 14*s.* 2*d.* *gained* 12*l.* 12*s.* $4\frac{1}{2}$*d.*

## VOL. II.

P. 5, q. 6, r. 38.79225. p. 7. q. 5, r. .18005. p. 8, q. 7, r. 137.71925. p. 10, l. 12, for *for*, r. +. p. 22, q. 31, r. *buy them.* p. 24, q. 40, r. 153*l.* 16*s.* $8\frac{1}{2}$*d.* p. 44, q. 31, r. 41.17.

The above refers in general to the anſwers; *p.* page; *q.* queſtion; *l.* line; *r.* read.